SINGULAR VALUE DECOMPOSITION (SVD)

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WHAT IS SVD?

1.)<u>SVD as Factorization</u>.

.It means decomposing a matrix into a product of three simpler matrices. We can co-relate it to other matrix decompositions such as Eigen decomposition, principal components analysis (PCA), and non-negative matrix factorization (NNMF).

WHAT IS SVD?

-It is a method for transforming correlated variables into a set of uncorrelated ones exposing relationship among the original data items.

-It is a method for identifying and ordering the dimensions along which data points exhibit the most variation.

-Method for data reduction.

DEFINITION OF SVD

Singular Value Decomposition (SVD) factors an $m \times n$ matrix A into a product of three matrices, assuming that all values are known:

$$A = U * D * V^T$$

Where,

- ✓ U is an $m \times k$ matrix, V is an $n \times k$ matrix.
- ✓ D is a $k \times k$ matrix, k is the rank of the matrix A.
- ✓ The multiplication (*) is matrix multiplication
- ✓ The superscripted T indicates matrix transposition.

APPLICATION OF SVD

1) **Latent Semantic Analysis.**
2) **Latent Semantic Indexing.**
3) **SVD for General Classification.**
4) **SVD for Clustering and Features.**
5) **SVD for Collaborative Filtering.**

STEPS FOR SVD::

1.) Compute a new matrix W=A.A¯T. Find the Eigen values and Eigen vectors of W.

2.) The square roots of each of the Eigen values of W that are greater than zero are the singular values. These are the **diagonal elements of D**.

3.) Normalize the Eigen vectors of W that correspond to nonzero Eigen values of W that are greater than zero. The columns of U are the normalized eigenvectors.

4.) Now repeat this process by letting W¯1 =A¯T.A. The normalized eigenvectors of this matrix are the columns of V.

STEPS FOR SVD::

Suppose A is any matrix consisting of m rows and n columns ,we want to compute the singular value decomposition of that matrix ,so lets take an example for understanding.

$$A = \begin{bmatrix} 0 & -1 \\ -2 & 1 \\ 1 & 0 \end{bmatrix}$$

STEPS FOR SVD:: NO.→ 1

1.) COMPUTING THE MATRIX W=A.A¯T.

$$A = \begin{bmatrix} 0 & -1 \\ -2 & 1 \\ 1 & 0 \end{bmatrix} \qquad A^T = \begin{bmatrix} 0 & -2 & 1 \\ -1 & 1 & 0 \end{bmatrix}$$

$$W = AA^T = \begin{bmatrix} 0 & -1 \\ -2 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & -2 & 1 \\ -1 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ -1 & 5 & -2 \\ 0 & -2 & 1 \end{bmatrix}$$

STEPS FOR SVD:: NO.→ 2

2.) Calculate the Eigen Values of W which will form the singular values of A producing the diagonal matrix D.

$$D = \begin{bmatrix} \sigma_1 & 0 & \cdots & 0 \\ 0 & \sigma_2 & \cdots & \vdots \\ \vdots & \cdots & \ddots & \vdots \\ 0 & 0 & \cdots & \sigma_n \end{bmatrix}$$

$$\sigma_1 \geq \sigma_2 \geq \cdots \geq \sigma_r > 0$$

Here σ_1 is the singular value of the diagonal matrix which is the square root of the Eigen values of W greater than 0.

STEPS FOR SVD::

- The Eigen Value of W is computed and thus we can find the singular values of A and calculate the diagonal matrix D.

- The Eigen value is calculated by the following formula on W:

$$\det|A - \lambda I| = 0$$

Where I is the n x n Identity matrix and λ is the unknown variable.

STEPS FOR SVD::

- Applying the step in our current example, Eigen values are computed (0,1,6) but we have to consider only the positive values so only (1,6) and singular values are square root of Eigen values. So,

$$\sigma_1 = 1,\ \sigma_2 = \sqrt{6}$$

- Finally we get the value of the diagonal matrix D as:

$$D = \begin{bmatrix} \sqrt{6} & 0 \\ 0 & 1 \end{bmatrix}$$

STEPS FOR SVD:: NO→ 3.

- Now we shall compute the matrix U. From the Eigen value we will calculate the Eigen vector.
- Considering the example stated previously we will now find the U matrix by finding the Eigen vector corresponding to the Eigen value (1,6), which must be normalized.

STEPS FOR SVD::

- The equation for this considering the Eigen value 6 will be::

$$\begin{bmatrix} 1 & -1 & 0 \\ -1 & 5 & -2 \\ 0 & -2 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = 6 \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

- Similarly the corresponding Eigen vector for value 1 is found and value of U is computed. Where v & w are corresponding Eigen vector.

$$v = \begin{bmatrix} -\frac{1}{\sqrt{30}} \\ \frac{5}{\sqrt{30}} \\ -\frac{2}{\sqrt{30}} \end{bmatrix} \quad w = \begin{bmatrix} -\frac{2}{\sqrt{5}} \\ 0 \\ \frac{1}{\sqrt{5}} \end{bmatrix}$$

&

$$U = \begin{bmatrix} -\frac{1}{\sqrt{30}} & -\frac{2}{\sqrt{5}} \\ \frac{5}{\sqrt{30}} & 0 \\ -\frac{2}{\sqrt{30}} & \frac{1}{\sqrt{5}} \end{bmatrix}$$

STEPS FOR SVD:: NO → 4.

- To compute the V matrix we will first find the transpose of W matrix and it is found to be as follows:

$$W' = A^T A = \begin{bmatrix} 0 & -2 & 1 \\ -1 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & -1 \\ -2 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 5 & -2 \\ -2 & 2 \end{bmatrix}$$

- Similarly the Eigen values are (1,6) and the Eigen vectors are as ::

$$v_1 = \begin{bmatrix} -\frac{2}{\sqrt{5}} \\ \frac{1}{\sqrt{5}} \end{bmatrix}, \quad v_2 = \begin{bmatrix} \frac{1}{\sqrt{5}} \\ \frac{2}{\sqrt{5}} \end{bmatrix}$$

STEPS FOR SVD::

- Based on that we calculate V, and then we will find the Transpose of V, in this case it is found to be same.
- Using the formula of SVD we will calculate the value of matrix and it is found to be of the same value as matrix A.

$$V = \begin{bmatrix} -\frac{2}{\sqrt{5}} & \frac{1}{\sqrt{5}} \\ \frac{1}{\sqrt{5}} & \frac{2}{\sqrt{5}} \end{bmatrix}$$

$$DV^T = \begin{bmatrix} \sqrt{6} & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} -\frac{2}{\sqrt{5}} & \frac{1}{\sqrt{5}} \\ \frac{1}{\sqrt{5}} & \frac{2}{\sqrt{5}} \end{bmatrix} = \begin{bmatrix} -\frac{2\sqrt{6}}{\sqrt{5}} & \frac{\sqrt{6}}{\sqrt{5}} \\ \frac{1}{\sqrt{5}} & \frac{2}{\sqrt{5}} \end{bmatrix}$$

$$UDV^T = \begin{bmatrix} -\frac{1}{\sqrt{30}} & -\frac{2}{\sqrt{5}} \\ \frac{5}{\sqrt{30}} & 0 \\ -\frac{2}{\sqrt{30}} & \frac{1}{\sqrt{5}} \end{bmatrix} \begin{bmatrix} -\frac{2\sqrt{6}}{\sqrt{5}} & \frac{\sqrt{6}}{\sqrt{5}} \\ \frac{1}{\sqrt{5}} & \frac{2}{\sqrt{5}} \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ -2 & 1 \\ 1 & 0 \end{bmatrix} = A$$

Latent Semantic Analysis and Indexing

- Latent semantic analysis (LSA) is a straightforward application of singular value decomposition to term-document matrices.
- Latent semantic indexing (LSI) applies LSA to information retrieval.

HOW LSI WORKS WITH SVD?

LSI begins with a term-document matrix.

- The terms are just the words occurring in more than one document, with closed-class words like "and" and "to" removed.
- Words are arranged for case by downcasing, but not otherwise stemmed or filtered. The terms may be the result of an arbitrary tokenization, including stemming, stoplisting, case normalization, and even character n-grams.

HOW LSI WORKS WITH SVD?

- There is a row for each term, with the columns indexed by the document ID (0 to 8).
- The terms and term-document matrix would obviously have to be computed from the documents in a general application.

HOW LSI WORKS WITH SVD?

- Next, the singular values, left singular vectors and right singular values are extracted from the SVD matrix. This is the complete latent semantic indexing of the specified document collection.
- This representation makes clear how the terms and documents are cast into the same k-dimensional space for comparison. This is often called the latent semantic space. For comparison, the dimensions are scaled by their corresponding singular values.

HOW LSI WORKS WITH SVD?

- Next we consider the search component of latent semantic indexing. Queries are computed by taking the centroid of the term vectors corresponding to the terms in the query.
- This is then matched against the document vectors using the scaling provided by the singular values.

HOW LSI WORKS WITH SVD?

- The code first finds the terms by splitting on spaces or commas.
- It then initializes the query vector to be the number of dimensions in the latent semantic space, that is, the number of factors in the SVD.
- It then iterates over the terms and adds their term vectors to the query vector. The addition's done by brute force, looking for the term in the list of terms and then adding its vector if found. If the term isn't found in the array of terms, it's ignored.

HOW LSI WORKS WITH SVD?

- The query vector is printed and then scored against the document and term vectors
- Dot product is computed relative to the scales. Dot product is the most natural for the underlying linear model implied by the use of singular value decomposition.

LIMITATIONS ::

- It cannot capture polysemy i.e words with multiple meanings.
- The resulting matrix dimension may be difficult to interpret.

CONCLUSION::

Singular Value Decomposition - term used to describe the process of breaking down a large database to find the document vector (relevance) for various items by comparing them to other items and documents.

Three big steps are

- **Stemming** - Taking in account for various forms of a word on a page
- **Local Weighting** - Increasing the relevance of a given document based on the frequency a term appears in the document
- **Global Weighting** - Increasing the relevance of terms which appear in a small number of pages as they are more likely to be on topic than words that appear in most all documents.
- **Normalization** - Penalizing long copy and rewarding short copy to allow them fair distribution in results.

THANK YOU.

Q&A