

Learning Objectives

- Understand the architecture of **RNNs**, **LSTMs**, and **GRUs**
 - Implement **RNN-based models for NLP tasks**
-

1. RNN Architecture: Basic RNN

Why RNN?

RNNs are designed to **process sequential data**. For example: sentences, time series, music, etc.

They have a memory of **previous inputs**, unlike regular feed-forward networks.

Basic RNN Cell Architecture

Each time step has:

- Input x_{t-1}
- Hidden state h_{t-1} (which stores memory)
- Output y_{t-1}

Mathematical Formulation:

Let:

- x_{t-1} : input at time step t
- h_{t-1} : hidden state from previous time step
- W_{xh}, W_{hh}, b_h : weights and bias
- \tanh : activation function

Then:

$$h_t = \tanh(W_{xh}x_{t-1} + W_{hh}h_{t-1} + b_h) \quad y_t = W_{yh}h_t + b_y$$

Example in Code (Basic RNN using PyTorch)

```
import torch
```

```
import torch.nn as nn
```

```
# Sample input (sequence_length=5, batch_size=1, input_size=10)
```

```
x = torch.randn(5, 1, 10)
```

```
rnn = nn.RNN(input_size=10, hidden_size=20, num_layers=1, batch_first=False)
```

```
# Initial hidden state (num_layers, batch, hidden_size)
```

```
h0 = torch.zeros(1, 1, 20)
```

```
out, hn = rnn(x, h0)
```

```
print("Output shape:", out.shape)
```

```
print("Final hidden state:", hn.shape)
```

2. LSTM (Long Short-Term Memory)

Why LSTM?

RNNs struggle with **long-term dependencies** due to **vanishing gradients**. LSTMs fix this using **gates** that control memory.

LSTM Cell Architecture

LSTM has:

- Cell state C_t (long-term memory)
- Hidden state h_t (short-term)
- Gates: forget f_t , input i_t , output o_t

Mathematics:

Let:

- σ : sigmoid
- \tanh : tanh
- \odot : element-wise multiplication

$$f_t = \sigma(W_f \cdot [h_{t-1}, x_t] + b_f) \quad i_t = \sigma(W_i \cdot [h_{t-1}, x_t] + b_i) \\ C_t = \tanh(W_C \cdot [h_{t-1}, x_t] + b_C) \quad \tilde{C}_t = \tanh(W_C \cdot [h_{t-1}, x_t] + b_C)$$

$$C_t = f_t \odot C_{t-1} + i_t \odot \tilde{C}_t \quad C_t = f_t \odot C_{t-1} + i_t \odot \tilde{C}_t \quad o_t = \sigma(W_o \cdot [h_{t-1}, x_t] + b_o) \quad o_t = \sigma(W_o \cdot [h_{t-1}, x_t] + b_o) \quad h_t = o_t \odot \tanh(C_t) \quad h_t = o_t \odot \tanh(C_t)$$

Code Example: LSTM

```
lstm = nn.LSTM(input_size=10, hidden_size=20, num_layers=1)
```

```
x = torch.randn(5, 1, 10)
```

```
h0 = torch.zeros(1, 1, 20)
```

```
c0 = torch.zeros(1, 1, 20)
```

```
out, (hn, cn) = lstm(x, (h0, c0))
```

```
print("LSTM output:", out.shape)
```

3. GRU (Gated Recurrent Unit)

Why GRU?

GRU is a **simplified version of LSTM** — it has **fewer gates** and **faster training**, but similar performance.

GRU Cell Architecture

Has:

- Update gate z_t
- Reset gate r_t
- Candidate memory \tilde{h}_t

Mathematics:

$$z_t = \sigma(W_z \cdot [h_{t-1}, x_t]) \quad z_t = \sigma(W_z \cdot [h_{t-1}, x_t]) \quad r_t = \sigma(W_r \cdot [h_{t-1}, x_t]) \quad r_t = \sigma(W_r \cdot [h_{t-1}, x_t])$$

$$\tilde{h}_t = \tanh(W_h \cdot [r_t \odot h_{t-1}, x_t]) \quad \tilde{h}_t = \tanh(W_h \cdot [r_t \odot h_{t-1}, x_t])$$

$$h_t = (1 - z_t) \odot h_{t-1} + z_t \odot \tilde{h}_t \quad h_t = (1 - z_t) \odot h_{t-1} + z_t \odot \tilde{h}_t$$

Code Example: GRU

```
gru = nn.GRU(input_size=10, hidden_size=20, num_layers=1)
```

```
x = torch.randn(5, 1, 10)
```

```
h0 = torch.zeros(1, 1, 20)
```

```
out, hn = gru(x, h0)
```

```
print("GRU output:", out.shape)
```

4. Implementing RNN for NLP Task (Text Classification)

We'll build a simple **sentiment classifier** using an LSTM on IMDB dataset.

Code Example (Text Classification with LSTM)

```
import torch

import torch.nn as nn

from torchtext.datasets import IMDB
from torchtext.data.utils import get_tokenizer
from torchtext.vocab import build_vocab_from_iterator
from torch.utils.data import DataLoader
from torch.nn.utils.rnn import pad_sequence


# Tokenizer and vocab
tokenizer = get_tokenizer("basic_english")


def yield_tokens(data_iter):
    for label, line in data_iter:
        yield tokenizer(line)


train_iter = IMDB(split='train')
```

```
vocab = build_vocab_from_iterator(yield_tokens(train_iter), specials=["<pad>", "<unk>"])
vocab.set_default_index(vocab["<unk>"])
```

```
# Pipeline
```

```
def text_pipeline(x): return vocab(tokenizer(x))
```

```
def label_pipeline(x): return 1 if x == 'pos' else 0
```

```
# Collate function
```

```
def collate_batch(batch):
```

```
    text_list, label_list = [], []
```

```
    for label, text in batch:
```

```
        text_tensor = torch.tensor(text_pipeline(text), dtype=torch.int64)
```

```
        text_list.append(text_tensor)
```

```
        label_list.append(torch.tensor(label_pipeline(label), dtype=torch.int64))
```

```
    text_batch = pad_sequence(text_list, padding_value=0)
```

```
    return text_batch, torch.tensor(label_list)
```

```
train_iter = IMDB(split='train')
```

```
dataloader = DataLoader(list(train_iter)[:1000], batch_size=8, collate_fn=collate_batch)
```

```
# LSTM Model
```

```
class LSTMClassifier(nn.Module):
```

```
    def __init__(self, vocab_size, embed_dim, hidden_dim, output_dim):
```

```
        super().__init__()
```

```
        self.embedding = nn.Embedding(vocab_size, embed_dim)
```

```
        self.lstm = nn.LSTM(embed_dim, hidden_dim)
```

```
        self.fc = nn.Linear(hidden_dim, output_dim)
```

```
    def forward(self, x):
```

```

        embedded = self.embedding(x)

        output, (hn, cn) = self.lstm(embedded)

        return self.fc(hn[-1])

model = LSTMClassifier(len(vocab), 100, 128, 2)

# Training loop (simplified)
optimizer = torch.optim.Adam(model.parameters(), lr=0.001)
criterion = nn.CrossEntropyLoss()

for epoch in range(3):
    total_loss = 0
    for text, label in dataloader:
        optimizer.zero_grad()

        output = model(text)

        loss = criterion(output, label)

        loss.backward()

        optimizer.step()

        total_loss += loss.item()

    print(f"Epoch {epoch+1}, Loss: {total_loss:.4f}")

```

Summary

Model Pros

Cons

RNN	Simple, good for short sequences	Vanishing gradient, poor long-term memory
LSTM	Good long-term memory	More parameters, slower
GRU	Faster than LSTM, good performance	No explicit cell state
