



What is LSTM?

LSTM is a type of **Recurrent Neural Network (RNN)** designed to **remember long-term dependencies** and avoid problems like **vanishing/exploding gradients**.

LSTM Cell – Mathematical Explanation

Each LSTM cell has three main gates:

1. **Forget Gate**
2. **Input Gate**
3. **Output Gate**

Variables:

- Input at time t : $x_{t,t}$
- Hidden state at time $t-1$: h_{t-1}
- Cell state at time $t-1$: C_{t-1}
- Output hidden state at time t : h_t
- Cell state at time t : C_t

Let's define the weight matrices and biases:

- W_f, W_i, W_C, W_o : weight matrices for forget, input, cell update, and output gates.
- b_f, b_i, b_C, b_o : biases

Equations

1. **Forget Gate:** What to forget from the previous cell state

$$f_t = \sigma(W_f \cdot [h_{t-1}, x_t] + b_f)$$

2. **Input Gate:** What new information to add

$$i_t = \sigma(W_i \cdot [h_{t-1}, x_t] + b_i) \quad \tilde{C}_t = \tanh(W_C \cdot [h_{t-1}, x_t] + b_C) \quad \tilde{C}_t = \tanh(W_C \cdot [h_{t-1}, x_t] + b_C)$$

3. **Cell State Update:**

$$C_t = f_t \cdot C_{t-1} + i_t \cdot \tilde{C}_t$$

4. **Output Gate:** What to output

$$o_t = \sigma(W_o \cdot [h_{t-1}, x_t] + b_o) \quad h_t = o_t \cdot \tanh(C_t) \quad h_t = o_t \cdot \tanh(C_t)$$

σ = sigmoid activation

tanh = hyperbolic tangent activation

Intuition Example

Let's say we're predicting the next word in the sentence:

"The stock market is ____"

The model should remember "**stock market**" to predict something like "volatile" or "bullish". A basic RNN might forget "stock", but LSTM can **retain** this context over long distances.

Code Example – Keras

Let's use LSTM to predict a sequence using a simple time-series dataset.

Predicting Sine Wave Using LSTM

```
import numpy as np
```

```
import matplotlib.pyplot as plt
```

```
from tensorflow.keras.models import Sequential
```

```
from tensorflow.keras.layers import LSTM, Dense
```

```

# Generate sine wave data
x = np.linspace(0, 50, 500)
y = np.sin(x)

# Prepare input/output sequences
def create_dataset(data, step=10):
    X, Y = [], []
    for i in range(len(data) - step):
        X.append(data[i:i+step])
        Y.append(data[i+step])
    return np.array(X), np.array(Y)

step = 10
X, Y = create_dataset(y, step)

# Reshape to [samples, time steps, features]
X = X.reshape((X.shape[0], X.shape[1], 1))

# Build LSTM model
model = Sequential([
    LSTM(50, activation='tanh', input_shape=(step, 1)),
    Dense(1)
])
model.compile(optimizer='adam', loss='mse')
model.fit(X, Y, epochs=20, verbose=1)

# Predict
pred = model.predict(X)

```

```
# Plot original and predicted
plt.plot(Y, label="True")
plt.plot(pred, label="Predicted")
plt.legend()
plt.title("LSTM Sine Wave Prediction")
plt.show()
```

Summary

Component	Purpose
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Forget Gate	Decides what to throw away from the cell state
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Input Gate	Adds new information to the cell state
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Output Gate	Decides what to output from the cell state
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Cell State	Memory of the network
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Hidden State	Output at the current timestep
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