

Digital Signal Processing Lab - Filter and Signal Processing Questions

1. Signal Generation and Analysis

Unit Step and Ramp Signals

Generate the unit step $u(n)$ and ramp $r(n)$ signals for $n = -10$ to $n = 10$. Plot both signals and determine their properties (amplitude, transition, etc.).

Exponential Signal

Generate the exponential signal $x(n) = 0.5^n$ for $n = 0$ to $n = 20$. Plot the signal and analyze its behavior.

Sinusoidal Signal

Consider the sinusoidal signal $x(n) = \sin(2\pi * 0.1n)$. Plot the signal for $n = 0$ to $n = 50$. Compute and show the frequency of oscillation.

2. Sampling and Aliasing

Sampling of Analog Signal

Consider the continuous-time signal $x(t) = 3\cos(200\pi t)$. Sample this signal at 100 Hz, 300 Hz, and 1000 Hz. Plot the sampled signals and discuss the aliasing effect for each sampling rate.

Effect of Sampling Rate on Signal Reconstruction

Given the analog signal $x(t) = 5\sin(2\pi * 50t) + 3\cos(2\pi * 150t)$, sample at different rates (50 Hz, 150 Hz, and 200 Hz). Plot the discrete-time signals and reconstruct them using interpolation.

3. Convolution and LTI Systems

Convolution of Two Signals

Given two discrete-time signals $x(n) = [1, 3, -2, 4]$ and $h(n) = [2, 1, 3]$, compute the convolution $y(n) = x(n) * h(n)$ using the convolution sum formula. Plot the result.

Impulse Response of an LTI System

The impulse response of a discrete-time system is $h(n) = u(n) - u(n-5)$. For the input $x(n) = u(n)$,

compute the output $y(n) = x(n) * h(n)$. Plot the input and output signals.

4. Correlation and Cross-Correlation

Autocorrelation

Given the signal $x(n) = [1, 2, 3, 4, 5]$, compute the autocorrelation of $x(n)$ using the formula for the discrete-time autocorrelation function. Plot the result.

Cross-Correlation

Given $x(n) = [2, -1, 3, 0, 4]$ and $y(n) = [1, 4, -2, 5, 0]$, compute the cross-correlation between $x(n)$ and $y(n)$. Plot the result.

5. Fourier Transform and Frequency Analysis

Discrete Fourier Transform (DFT) Calculation

Compute the DFT of the signal $x(n) = [1, 0, -1, 0]$. Plot the magnitude and phase of the DFT.

Frequency Response of a System

Consider the discrete-time system with the transfer function $H(z) = 1 / (1 - 0.5z^{-1})$. Compute the frequency response of the system at frequencies $\omega = 0, \pi/4, \pi/2$. Plot the magnitude and phase response.

6. Filter Design and Implementation

FIR Filter Design

Design a 5-point moving average FIR filter and compute its impulse response. Implement this filter for the input signal $x(n) = [3, 2, 1, 4, 5, 6]$, and compute the output signal.

IIR Filter Design

Design a 2nd-order low-pass IIR filter with cutoff frequency 0.2 Hz. Compute the impulse response and apply the filter to the signal $x(n) = [1, 2, 3, 4, 5, 6]$.

7. Discrete Signal Reconstruction

Reconstruction Using Sinc Interpolation

Given the sampled signal $x[n] = [3, 2, 1, 4]$, reconstruct the original continuous signal using sinc

interpolation. Plot the reconstructed signal.

Upsampling and Downsampling

Consider the signal $x(n) = [1, 0, -1, 0]$. Perform upsampling by a factor of 2, followed by downsampling by a factor of 2. Plot the original and final signals.

8. Z-Transform

Z-Transform of a Sequence

Compute the Z-transform of the sequence $x(n) = [1, 2, 3, 4]$. Use the standard formula for the Z-transform and plot its region of convergence (ROC).

Inverse Z-Transform

Given the Z-transform $X(z) = 1 / (1 - 0.5z^{-1})$, find the inverse Z-transform and plot the sequence $x(n)$.

Filter Design Questions

Low-Pass FIR Filter Design

Design a 4-tap FIR low-pass filter with a cutoff frequency of 0.2 times the Nyquist frequency. Plot the frequency response and impulse response of the filter.

High-Pass FIR Filter

Design a 5-tap high-pass FIR filter using the windowing method. Use a Hamming window and a cutoff frequency of 0.4 times the Nyquist frequency. Compute and plot the frequency response.

Band-Pass FIR Filter

Design a band-pass FIR filter with a passband from 0.2 to 0.5 times the Nyquist frequency. Use a 7-tap filter and plot the impulse and frequency responses.

Butterworth Low-Pass Filter

Design a 3rd-order low-pass Butterworth filter with a cutoff frequency of 0.25 times the Nyquist frequency. Compute and plot the frequency response of the filter.

Chebyshev Filter Design

Design a 2nd-order Chebyshev Type I low-pass filter with a passband ripple of 1 dB and a cutoff frequency of 0.4 times the Nyquist frequency. Plot the frequency and phase response.

Biquad Filter Implementation

Design a biquad filter with the following transfer function: $H(z) = (b_0 + b_1z^{-1} + b_2z^{-2}) / (1 + a_1z^{-1} + a_2z^{-2})$, where $b_0 = 0.5$, $b_1 = 0.5$, $b_2 = 0.5$, $a_1 = -1.8$, $a_2 = 0.81$. Apply this filter to a signal and plot the output signal.