Manual Gradient Descent: 5 Function Types with Full Calculations

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✅Example 1: Two-variable Linear Function

Function: $f(w, b) = (wx + b - y)^2$

Data: - $x = 2$, $y = 5$ - Initial: $w = 1$, $b = 0$ - Learning rate: $\eta = 0.1$

Step-by-step Iterations:

Step 1: - $\hat{y} = 1 \cdot 2 + 0 = 2$ - Loss: $(2 - 5)^2 = 9$ - Gradients: - $\frac{\partial L}{\partial w} = 2(-3) \cdot 2 = -12$ - $\frac{\partial L}{\partial b} = 2(-3) = -6$ - Updates: - $w = 1 + 1.2 = 2.2$ - $b = 0 + 0.6 = 0.6$

Step 2: - $\hat{y} = 2.2 \cdot 2 + 0.6 = 4.4 + 0.6 = 5$ - Loss: $(5 - 5)^2 = 0$ - Training converged.

✅Example 2: Three-variable Linear Function

Function: $f(w_1, w_2, b) = (w_1x_1 + w_2x_2 + b - y)^2$

Data: - $x_1 = 1, x_2 = 2, y = 10$ - Initial: $w_1 = 1, w_2 = 1, b = 0$ - $\eta = 0.1$

Step 1: - $\hat{y} = 1 + 2 + 0 = 3$ - Loss: $(3 - 10)^2 = 49$ - Gradients: - $\partial w_1 = 2(-7) \cdot 1 = -14$ - $\partial w_2 = 2(-7) \cdot 2 = -28$ - $\partial b = 2(-7) = -14$ - Updates: - $w_1 = 2.4, w_2 = 3.8, b = 1.4$

Step 2: - $\hat{y} = 2.4 \cdot 1 + 3.8 \cdot 2 + 1.4 = 2.4 + 7.6 + 1.4 = 11.4$ - Loss: $(11.4 - 10)^2 = 1.96$ - Gradients: - $\partial w_1 = 2(1.4) \cdot 1 = 2.8$ - $\partial w_2 = 2(1.4) \cdot 2 = 5.6$ - $\partial b = 2(1.4) = 2.8$ - Updates: - $w_1 = 2.12, w_2 = 3.24, b = 1.12$

Step 3: - Continue until loss ~ 0

✅Example 3: Polynomial Function

Function: $f(w) = (w^2 - y)^2$

Data: - $y = 9, w = 1, \eta = 0.1$

Step 1: - $\hat{y} = 1^2 = 1$ - Loss = $(1 - 9)^2 = 64$ - Gradient: - $\frac{dL}{dw} = 4w(w^2 - y) = 4 \cdot 1 \cdot (1 - 9) = -32$ - Update: - $w = 1 + 3.2 = 4.2$

Step 2: - $\hat{y} = 4.2^2 = 17.64$ - Loss = $(17.64 - 9)^2 \approx 74.5$ - Gradient: - $4 \cdot 4.2 \cdot (17.64 - 9) \approx 145$ - Update: - $w = 4.2 - 14.5 = -10.3$

Step 3+: Continue until convergence.

✅Example 4: Trigonometric Function

Function: $f(w) = (\sin(w) - y)^2$

Data: - $y = 0.5, w = 0, \eta = 0.1$

Step 1: - $\hat{y} = \sin(0) = 0$, Loss = 0.25 - Gradient: $2(\sin(w) - y) \cdot \cos(w) = -1$ - Update: $w = 0 + 0.1 = 0.1$

Step 2: - $\hat{y} = \sin(0.1) \approx 0.0998$, Error = -0.4002 - Gradient: $2 \cdot -0.4002 \cdot \cos(0.1) \approx -0.796$ - Update: $w = 0.1796$

Continue till loss ~ 0.

✅Example 5: Logarithmic Function

Function: $f(w) = (\log(w) - y)^2$

Data: - $y = 1, w = 2, \eta = 0.1$

Step 1: - $\log(2) \approx 0.693$, Loss = $(0.693 - 1)^2 = 0.094$ - Gradient: $2(\log(w) - y)/w = -0.1535$ - Update: $w = 2.015$

Step 2: - $\log(2.015) \approx 0.700$, Loss = 0.09 , Gradient ~ -0.149 - Continue updating until loss ~ 0

Conclusion: Each function required: - Defining prediction ($\hat{y}$) - Loss computation - Gradient derivation using chain rule - Parameter updates using gradient descent

Repeat until convergence.