Newton dividend Difference Formula:
interpolation formula

$x = x_0, x_1, x_2, x_3 \cdots x_n$

$y = y_0, y_1, y_2, y_3 \cdots y_n$

the divided operator is devided by

Δ or $f(x_0, x_1 \cdots x_n)$ and defined

as $\Delta y_0 = \dfrac{y_1 - y_0}{x_1 - x_0}$

$\Delta y_1 = \dfrac{y_2 - y_1}{x_2 - x_1}$

$\Delta y_2 = \dfrac{y_3 - y_2}{x_3 - x_2}$

$\Delta^3 y_0 = \dfrac{\Delta^2 y_1 - \Delta^2 y_0}{x_3 - x_0}$

$\Delta^3 y_1 = \dfrac{\Delta^2 y_2 - \Delta^2 y_1}{x_4 - x_1}$

$\Delta^2 y_0 = \dfrac{\Delta y_1 - \Delta y_0}{x_2 - x_0}$

$\Delta^2 y_1 = \dfrac{\Delta y_2 - \Delta y_1}{x_3 - x_1}$

$\Delta^2 y_2 = \dfrac{y_3 - y_2}{x_4 - x_2}$

$$f(x_0, x_1) = \frac{f(x_1) - f(x_0)}{x_1 - x_0} = \frac{y_1 - y_0}{x_1 - x_0}$$

$$f(x_0, x_1, x_2) = \frac{f(x_1, x_2) - f(x_0, x_1)}{x_2 - x_0}$$

Formation of divided difference table:

x	y	Δy	$\Delta^2 y$
x_0	y_0	$\Delta y_0 = \frac{y_1 - y_0}{x_1 - x_0}$	$\Delta^2 y_0 = \frac{\Delta y_1 - \Delta y_0}{x_2 - x_0}$
x_1	y_1		
		$\Delta y_1 = \frac{y_2 - y_1}{x_2 - x_1}$	$\Delta^2 y_1 = \frac{\Delta y_2 - \Delta y_1}{x_3 - x_1}$
x_2	y_2		
		$\Delta y_2 = \frac{y_3 - y_2}{x_3 - x_2}$	$\Delta^2 y_2 = \frac{\Delta y_3 - \Delta y_2}{x_4 - x_2}$
x_3	y_3		
		$\Delta y_3 = \frac{y_4 - y_3}{x_4 - x_3}$	$\Delta^2 y_3 = \frac{\Delta y_4 - \Delta y_3}{x_5 - x_3}$
x_4	y_4		
		$\Delta y_4 = \frac{y_5 - y_4}{x_5 - x_4}$	
x_5	y_5		

$x\textcircled{1} \rightarrow$ সব নিচের column, তত ছোট।

(প্রতি start ২ (০⁻ᵢ)

$$\Delta^3 y$$

$$\Delta^3 y_0 = \frac{\Delta^2 y_1 - \Delta^2 y_0}{x_3 - x_0}$$

$$\Delta^3 y_1 = \frac{\Delta^2 y_2 - \Delta^2 y_1}{x_4 - x_1}$$

$$\Delta^3 y_2 = \frac{\Delta^2 y_3 - \Delta^2 y_2}{x_5 - x_1}$$

$$\vdots$$

$$\Delta^4 y_0 = \frac{\Delta^3 y_1 - \Delta^3 y_0}{x_4 - x_0}$$

$$\Delta^4 y_1 = \frac{\Delta^3 y_2 - \Delta^3 y_1}{x_5 - x_1}$$

$$\Delta^5 y_0$$

$$\Delta^5 y_0 = \frac{\Delta^4 y_1 - \Delta^4 y_0}{x_5 - x_0}$$

$\Rightarrow$ It is applicable for unequal interval

$$x = x_0, x_1, x_2 \; - \; - \; - \; x_n$$

$$y = y_0, y_1, y_2 \; - \; - \; - \; - \; y_n$$

$$y = f(x) = y_0 + (x - x_0)\,\Delta y_0 + (x - x_0)(x - x_1)\,\Delta^2 y_0 + (x - x_0)(x - x_1)(x - x_2)\,\Delta^3 y_0 + (x - x_0)(x - x_1)(x - x_2)(x - x_3)\,\Delta^4 y_0$$

$$+ \; - \; - \; - \; -$$

use Newton's divided difference formula
to find $f(x)$, given

x :   0   2   3   6

$f(x)$:   642   704   729   792

Also find $f(4)$ and $f'(4)$.

x		y	Δy	$\Delta^2 y$	$\Delta^3 y$
x_0	0	648			
			$\dfrac{704-648}{2-0}=28$		
x_1	2	704		$\dfrac{25-28}{3-0}=-1$	
			$\dfrac{729-704}{3-2}=25$		$\dfrac{-1-(-1)}{6-0}=0$
x_2	3	729		$\dfrac{21-25}{6-2}=-1$	
			$\dfrac{792-729}{6-3}=21$		
x_3	6	792			

$$f(x) = y_0 + (x - x_0)\Delta y_0 + (x-x_0)(x-x_1)\Delta^2 y_0$$
$$+ (x-x_0)(x-x_1)(x-x_2)\Delta^3 y_0 + \cdots$$

$$= 648 + (x-0)28 + (x-0)(x-2)(-1)$$

$$= 648 + 28x + x^2 - 2x$$
$$= -x^2 + 30x + 648 = 752$$