This course material is now made available for public usage.
Special acknowledgement to School of Computing, National University of Singapore for allowing Steven to prepare and distribute these teaching materials.

CS3233
Competitive Programming

Dr. Steven Halim

Week 05 – Problem Solving Paradigms
(Dynamic Programming 2)

Outline

- Mini Contest #4 + Break + Discussion + Admins
- A simple DP problem to refresh our memory (Section 3.5.3)
- DP and its relationship with (implicit) DAG (Section 4.7.1)
  - These are CS2020/CS2010 materials
    - Those who have not taken either module must consult Steven separately
- DP on Math Problems (Section 5.4 and 5.6)
- DP on String Problems (Section 6.5)
- More DP techniques (Section 8.3)
- Pointers to other DP techniques in CP2.9

More DP Problems in Chapter 3-4-5-6-8-9 of CP2.9 Book

NON CLASSICAL DP PROBLEMS

Non Classical DP Problems

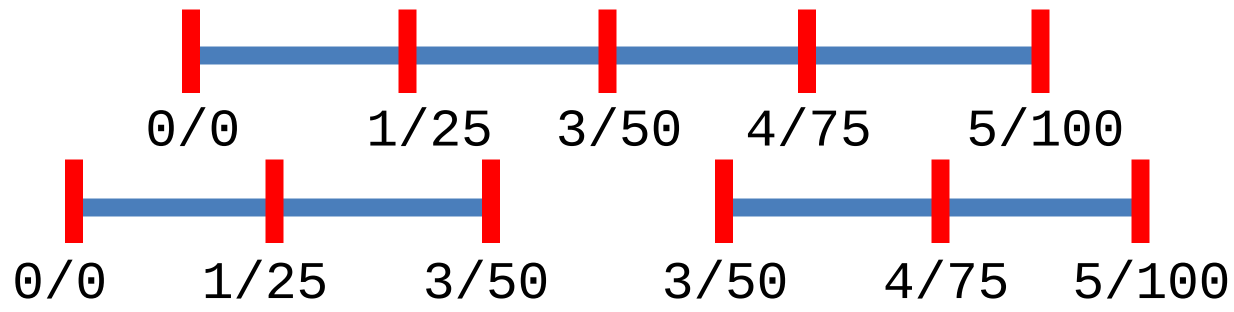
- **My** definition:
  - Not the pure form (or simple variant) of 1D/2D Max Sum, LIS, 0-1 Knapsack/Subset Sum, Coin Change, TSP where the DP **states** and **transitions** can be "memorized"
  - Requires **original*** **formulation** of DP states and transitions
  - Throughout this lecture, we will talk mostly in *DP terms*
    - **State** (to be precise: "*distinct* state")
    - **Space Complexity** (i.e. the number of distinct states)
    - **Transition** (which entail overlapping sub problems)
    - **Time Complexity** (i.e. num of distinct states * time to fill one state)

Refresher: Cutting Sticks

- State: index **(l, r)** where **l, r** ∈ **[0..n+1]** and **l < r**
  - Q: Why these two parameters?
- Space Complexity: **O(n²)** distinct states
- Transition: Try all possible cutting points **i** between **l** and **r**,
  - i.e. cut **(l, r)** into **(l, i)** and **(i, r)** with cost **(A[r] - A[l])**
- Time Complexity: There are **O(n)** possible cutting points, thus overall **O(n² * n) = O(n³)**

A = {25, 50, 75}
n = 3, L = 100

A = {0, 25, 50, 75, 100}



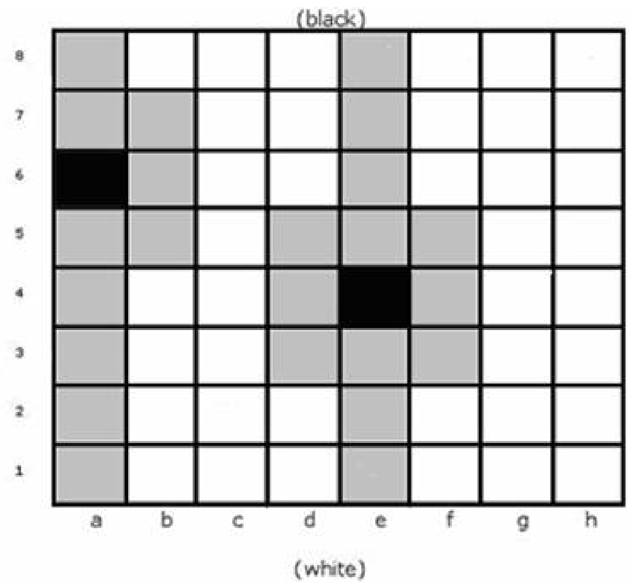
DP on DAG

Overview

- Dynamic Programming (DP) has a close relationship with (usually implicit) Directed Acyclic Graph (DAG)
  - The **states** are the **vertices** of the DAG
  - Space complexity: Number of vertices of the DAG
  - The **transitions** are the **edges** of the DAG
    - Logical, since a recurrence is always **acyclic**
  - Time complexity: Number of edges of the DAG
  - Top-down DP: Process each vertex just once via **memoization**
  - Bottom-up DP: Process the vertices in **topological order**
    - Sometimes, the topological order can be written by just using simple (nested) loops

The Injured Queen Problem

- Like N-queens problem, but the queens are "injured" (can only attack the current column but acts as king otherwise)
- With some of K ($0 \le K \le N$) injured queens positions have been predetermined, count how many possible arrangements of the other (N-K) queens so that no two queens attack each other?



DP on Math Problems

- Some well-known mathematic problems involves DP
  - Some combinatorics problem have recursive formulas which entail overlapping subproblems
    - e.g. those involving Fibonacci number, **f(n) = f(n - 1) + f(n -2)**
  - Some probability problems require us to search the entire search space to get the required answer
    - If some of the sub problems are overlapping, use DP, otherwise, use complete search
  - Mathematics problems involving **static** range sum/min/max!
    - Use dynamic tree DS for dynamic queries

Dice Throwing

- n common cubic dice are thrown ($1 \le n \le 24$)
- What is the probability that the sum of all thrown dices is at least x? ($0 \le x \le 150$)
- Basic probability = # events / |sample space|
- To compute the |sample space| is easy: It is 6^n
- The # events is harder to compute…

CP2.9, Section 6.5

DP on String Problems

- Some string problems involves DP
  - Usually, we do not work with the string itself
  - But we work with the integer indices to represent suffix/prefix/substring
  - Reason: Too costly to pass (sub)strings around as function parameters

String Partition

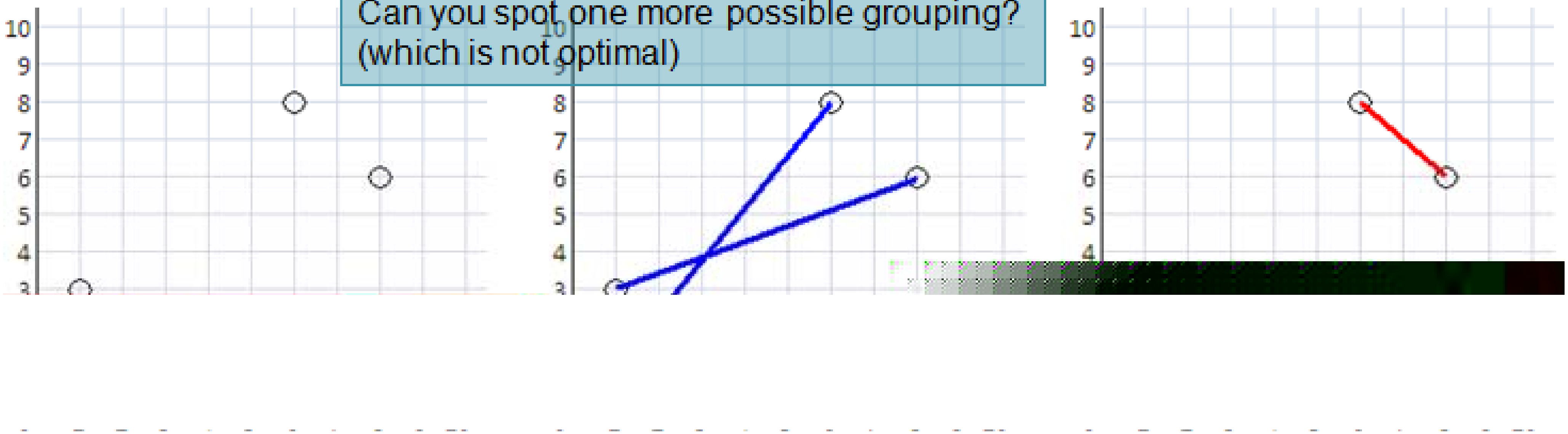
- There are many ways to split a string of digits into a list of non-zero-leading (0 itself is allowed) 32-bit *signed* integers
  – What is the maximum sum of the resultant integers if the string is split appropriately? Examples:
    - 1234554321
      – 1234554321 < 2147483647, so the answer is 1234554321 itself
    - 5432112345
      – 5432112345 > 2147483647, thus 5432112345 must be partitioned
      – There are two ways to partition 5432112345
        » 5 + 432112345 = 432112350, or
        » 543211234 + 5 = 543211239 ← the answer
    - 121212121212
      – 121212121212 > 2147483647, thus 121212121212 must be partitioned
      – The answer is: 1 + 2121212121 + 2 = 2121212124

DP with bitmask

- Bitmask technique can be used to represent *lightweight set of Boolean* (up to 2^{64} if using unsigned long long)
- This is important if one of the DP parameter is a “small set”
- We have seen this form earlier in DP-TSP
- One other useful application (there are many others):
  - ***Finding min weighted perfect matching in small general graph***

Forming Quiz Teams

Can you spot one more possible grouping?
(which is not optimal)



N = 2

Cost = 8.60 + 7.61 = **16.21**

Cost = 2.00 + 2.83 = **4.83**

Common DP States (1)

- Position:
  - Original problem: $[x_1, x_2, \ldots, x_n]$
    - Can be sequence (integer/double array), can be string (char array)
  - Sub problems, break the original problem into
    - Sub problem and Prefix: $[x_1, x_2, \ldots, x_{n-1}] + x_n$
    - Suffix and sub problem: $x_1 + [x_2, x_3, \ldots, x_n]$
    - Two sub problems: $[x_1, x_2, \ldots, x_i] + [x_{i+1}, x_{i+2}, \ldots, x_n]$
  - Example: 1D Max Sum, LIS, etc

Common DP States (2)

- Position**s**:
  - This is similar to the previous slide
  - Original problem: $[x_1, x_2, \ldots, x_n]$ and $[y_1, y_2, \ldots, y_n]$
    - Can be two sequences/strings
  - Sub problems, break the original problem into
    - Sub problem and prefix: $[x_1, x_2, \ldots, x_{n-1}] + x_n$ and $[y_1, y_2, \ldots, y_{n-1}] + y_n$
    - Suffix and sub problem: $x_1 + [x_2, x_3, \ldots, x_n]$ and $y_1 + [y_2, y_3, \ldots, y_n]$
    - Two sub problems: $[x_1, x_2, \ldots, x_i] + [x_{i+1}, x_{i+2}, \ldots, x_n]$ and $[y_1, y_2, \ldots, y_i] + [y_{i+1}, y_{i+2}, \ldots, y_n]$
  - Example: String Alignment/Edit Distance, LCS, Matrix Chain Multiplication (MCM), etc
  - PS: Can also be applied on 2D matrix, like 2D Max Sum, etc

Tips: When to Choose DP

- Default Rule:
  - If the given problem is an **optimization** (max/min) or **counting** problem
    - Problem exhibits optimal sub structures
    - Problem has overlapping sub problems
- In ICPC/IOI:
  - If actual solutions are not needed (only final values asked)
    - If we must compute the solutions too, a more complicated DP which stores *predecessor information* and *some backtracking* are necessary
  - The number of distinct sub problems is small enough (< 1M) and you are not sure whether greedy algorithm works (why gamble?)
  - Obvious overlapping sub problems detected :O

Dynamic Programming Issues (1)

- Potential issues with DP problems:
  - They may be disguised as (or looks like) non DP
    - It looks like greedy can work but some cases fails…
      - e.g. problem looks like a shortest path with some constraints on graph, but the constraints fail *greedy* SSSP algorithm!
  - They may have subproblems but not overlapping
    - DP does not work if overlapping subproblems not exist
      - Anyway, this is still a good news as perhaps Divide and Conquer technique can be applied

Dynamic Programming Issues (2)

- Optimal substructures may not be obvious
  1. Find correct "states" that describe problem
     - Perhaps extra parameters must be introduced?
  2. Reduce a problem to (smaller) sub problems (with the same states) until we reach base cases
- There can be more than one possible formulation
  - Pick the one that works!

DP Problems in ICPC (1)

- The number of problems in ICPC that must be solved using DP are growing!
  - At least one, likely two, maybe three per contest…
- These new problems are **not** the classical DP!
  - They require deep thinking…
  - Or those that look solvable using other (simpler) algorithms but actually must be solved using DP
  - Do not think that you have “mastered” DP by only memorizing the classical DP solutions!

DP Problems in ICPC (2)

- In 1990ies, mastering DP can make you "king" of programming contests…
  - Today, it is a must-have knowledge…
  - So, get familiar with DP techniques!
- By mastering DP, your ICPC rank is probably:
  - from top ~[25-30] (solving 1-2 problems out of 10)
    - Only easy problems
  - to top ~[15-20] (solving 3-4 problems out of 10)
    - Easy problems + brute force + DP problems

For Week 07 homework ☺

(You can do this over recess week too)

BE A PROBLEM SETTER

Be a Problem Setter

- Problem Solver:
  A. Read the problem
  B. Think of a good algorithm
  C. Write 'solution'
  D. Create tricky I/O
  E. If WA, go to A/B/C/D
  F. If TLE/MLE, go to A/B/C/D
  G. If AC, stop ☺

- Problem Setter:
  A. Write a <u>good</u> problem
  B. Write <u>good</u> solution<u>s</u>
     - The correct/best one
     - The incorrect/slower ones
  C. Set a <u>good</u> secret I/O
  D. Set problem settings
- A problem setter <u>must</u> think from a different angle!
  – By setting good problems, you will simultaneously be a better problem solver!!

Problem Setter Tasks (1)

- Write a <u>good</u> problem
  - Options:
    - Pick an algorithm, then find problem/story, or
    - Find a problem/story, then identify a good algorithm for it (harder)
  - Problem description must not be <u>ambiguous</u>
    - Specify input constraints
    - Good English!
    - Easy one: longer,
      Hard one: shorter!
- Write <u>good</u> solution<u>**s**</u>
  - Must be able to solve your own problem!
    - To set hard problem, one must increase his own programming skill!
  - Use the <u>best possible</u> algorithm with lowest time complexity
    - Use the inferior ones 'that barely works' to set the WA/TLE/MLE settings...

Problem Setter Tasks (2)

- Set a good secret I/O
  - Tricky test cases to check AC vs WA
    - Usually 'boundary case'
  - Large test cases to check AC vs TLE/MLE
    - Perhaps use input generator to generate large test case, then pass this large input to our correct solution
- Set problem settings
  - Time Limit:
    - Usually 2 or 3 times the timings of your own best solutions
      - Java is slower than C++!
  - Memory Limit:
    - Check OJ setting^
  - Problem Name:
    - Avoid revealing the algorithm in the problem name

FYI: Be A Contest Organizer

- Contest Organizer Tasks:
  - Set problems of *various* topic
    - Better set by >1 problem setter
  - Must balance the difficulty of the problem set
    - Try to make it fun
    - Each team solves some problems
    - Each problem is solved by some teams
    - No team solve all problems
      - Every teams must work until the end of contest

More References

- **Competitive Programming 2.9**
  - Section 3.5, 4.7.1, 5.4, 5.6, 6.5, 8.3, and parts of Ch9
- **Introduction to Algorithms**, p323-369, Ch 15
- **Algorithm Design**, p251-336, Ch 6
- **Programming Challenges**, p245-267, Ch 11
- http://www.topcoder.com/tc?module=Static&d1=tutorials&d2=dynProg
- Best is practice & more practice!