

*****Series Summation Formulas*****

[resource: <https://www.mathsisfun.com/algebra/sequences-sums-arithmetic.html>]

1. Arithmetic Progression:

- i) N^{th} term: $a_n = a + (n-1)d$
- ii) Summation: $S_n = (n/2) * (2a + (n-1)d)$

2. Geometric Progression:

- i) N^{th} term: $a_n = ar^{n-1}$
- ii) Summation: $S_n = a * ((1-r^n)/(1-r))$ [for a finite series]
- iii) Summation to infinity: $S = a/(1-r)$ [where $-1 < r < 1$]

3. Multiple combination of Series:

The image shows a handwritten solution for a series summation problem. At the top, a sequence of numbers is written: 3, 8, 14, 21, 29, 38, 48. Below this, the differences between consecutive terms are calculated: 5, 6, 7, 8, 9, 10. Then, the second differences are calculated: 1, 1, 1, 1, 1. Finally, the third differences are calculated: 0, 0, 0, 0. The general term u_n is then derived using the binomial expansion formula:

$$u_n = 3 + 5 \times {}^{n-1}C_1 + 1 \times {}^{n-1}C_2 + 0$$

$$= 3 + 5 \cdot \frac{(n-1)!}{1! (n-2)!} + \frac{(n-1)!}{2! (n-3)!}$$

$$= 3 + 5 \cdot (n-1) + \frac{(n-1)(n-2)}{2}$$

$$= \frac{6 + 10n - 10 + n^2 - 3n + 2}{2}$$

$$= \frac{1}{2} (n^2 + 7n - 2)$$

The summation s_n is then calculated using the formula for the sum of a series:

$$s_n = \frac{1}{2} (\sum n^2 + 7 \sum n - \sum 2)$$

$$= \frac{1}{2} \left\{ \frac{1}{6} n(n+1)(2n+1) + 7 \frac{n(n+1)}{2} - 2n \right\}$$