Homework #4

MEC 529: Introduction to Robotics

Spring 2023

Instructor	Amin Fakhari, Ph.D.
Assigned Date	Friday, Mar. 10, 2022
Due Date	Monday, Mar. 27, 2022

1. (a) (1/10) Write a MATLAB function that returns a normalized screw axis representation $\boldsymbol{\mathcal{S}} = (\boldsymbol{\mathcal{S}}_\omega, \boldsymbol{\mathcal{S}}_v) \in \mathbb{R}^6$ ($\|\boldsymbol{\mathcal{S}}_\omega\| = 1$ or $\boldsymbol{\mathcal{S}}_\omega = \mathbf{0}$, $\|\boldsymbol{\mathcal{S}}_v\| = 1$) of a screw by taking a unit vector $\hat{\boldsymbol{s}} \in \mathbb{R}^3$ in the direction of the screw axis, located at the point $\boldsymbol{q} \in \mathbb{R}^3$, with pitch $h \in \mathbb{R}$. Note that your function should also support a pure translation along the screw axis, i.e., when $h = \inf$. You can also write this function such that it takes any vector $\boldsymbol{s} \in \mathbb{R}^3$ and internally converts it into a corresponding unit vector $\hat{\boldsymbol{s}}$.

   (b) (1/10) For any screw axis $\boldsymbol{\mathcal{S}} = (\boldsymbol{\mathcal{S}}_\omega, \boldsymbol{\mathcal{S}}_v) \in \mathbb{R}^6$ and scalar $\theta \in \mathbb{R}$, we can always find a transformation matrix $\boldsymbol{T} \in SE(3)$ such that $\boldsymbol{T} = e^{[\boldsymbol{\mathcal{S}}]\theta}$, where $[\boldsymbol{\mathcal{S}}] \in se(3)$ (recall that $\boldsymbol{T}$ is equivalent to the displacement obtained by rotating a frame from $\boldsymbol{I}$ about $\boldsymbol{\mathcal{S}}$ by an angle θ or translating the frame from $\boldsymbol{I}$ along $\boldsymbol{\mathcal{S}}$ by amount θ). Write a MATLAB function that takes $\boldsymbol{\mathcal{S}}$ and θ, and returns the corresponding transformation matrix $\boldsymbol{T}$. Note that your function should also support a pure translation along the screw axis, i.e., when $\boldsymbol{\mathcal{S}}_\omega = \mathbf{0}$. For writing this function, you can utilize the functions you have written in Homework #3.

   (c) (1/10) For three arbitrary triple $\{\hat{\boldsymbol{s}}, \boldsymbol{q}, h\}$, compute the screw axis $\boldsymbol{\mathcal{S}}$ using your function in (a) and the corresponding transformation matrix $\boldsymbol{T}$ for a displacement from $\boldsymbol{I}$ along $\boldsymbol{\mathcal{S}}$ by an arbitrary θ using your function in (b). For each case, visualize the initial frame, the transformed frame, and the screw axis. For visualization of the frames, you can use the attached function `triad`, and for visualization of a line corresponding the screw axis $\{\hat{\boldsymbol{s}}, \boldsymbol{q}, h\}$, you can use the line formula as $\boldsymbol{l} = (x, y, z) = \boldsymbol{q} + t\hat{\boldsymbol{s}}$, where $t \in \mathbb{R}$.

2. (a) (1/10) For any transformation matrix $\boldsymbol{T} \in SE(3)$, we can always find a screw axis $\boldsymbol{\mathcal{S}} = (\boldsymbol{\mathcal{S}}_\omega, \boldsymbol{\mathcal{S}}_v) \in \mathbb{R}^6$ ($\|\boldsymbol{\mathcal{S}}_\omega\| = 1$ or $\boldsymbol{\mathcal{S}}_\omega = \mathbf{0}$, $\|\boldsymbol{\mathcal{S}}_v\| = 1$) and scalar $\theta \in \mathbb{R}$ such that $\log \boldsymbol{T} = [\boldsymbol{\mathcal{S}}]\theta$, where $[\boldsymbol{\mathcal{S}}] \in se(3)$. Write a MATLAB function that takes a transformation matrix $\boldsymbol{T}$ and returns the corresponding $\boldsymbol{\mathcal{S}}$ and $\theta \in [0, \pi]$. Note that your function should also support the case $\boldsymbol{R} = \boldsymbol{I}$. I recommend that you first write another function that takes a transformation matrix $\boldsymbol{T}$ and returns/extracts the rotation matrix $\boldsymbol{R} \in SO(3)$ and position vector $\boldsymbol{p} \in \mathbb{R}^3$ (i.e., $\boldsymbol{T} \to (\boldsymbol{R}, \boldsymbol{p})$). You can also utilize the functions you have written in Homework #3.

   (b) (1/10) Write a MATLAB function that returns the screw axis parameters $\{\hat{\boldsymbol{s}}, \boldsymbol{q}, h\}$ by taking a normalized screw axis $\boldsymbol{\mathcal{S}} = (\boldsymbol{\mathcal{S}}_\omega, \boldsymbol{\mathcal{S}}_v) \in \mathbb{R}^6$. Note that your function should also support the case $\boldsymbol{\mathcal{S}}_\omega = \mathbf{0}$. Moreover, note that $\boldsymbol{q}$ is not unique.

   (c) (1/10) Validate your functions in (a) and (b) using the three transformation matrices $\boldsymbol{T}$ computed in 1.(c).

3. (2/10) Two toy cars are moving on a round table as shown in Figure 1. Car 1 moves at a constant speed v_1 along the circumference of the table, while car 2 moves at a constant speed v_2 along a radius; the positions of the two vehicles at $t = 0$ s are shown in the figures. Find T_{01} and T_{02} in terms of t.

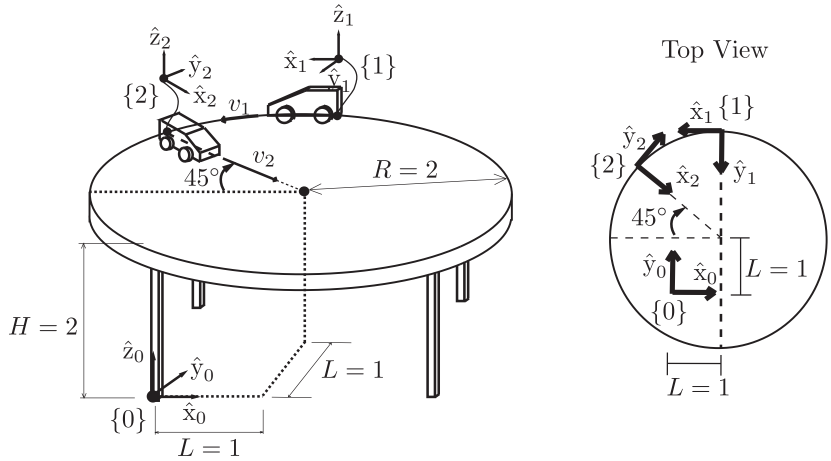

Figure 1: Two toy cars on a round table.

4. (2/10) Figure 2 shows two fingers grasping a can. Frame $\{b\}$ is attached to the center of the can. Frames $\{b_1\}$ and $\{b_2\}$ are attached to the can at the two contact points as shown. The force $\boldsymbol{f}_1 = (f_{1,x}, f_{1,y}, f_{1,z})$ is the force applied by fingertip 1 to the can, expressed in frame $\{b_1\}$. Similarly, the force $\boldsymbol{f}_2 = (f_{2,x}, f_{2,y}, f_{2,z})$ is the force applied by fingertip 2 to the can, expressed in frame $\{b_2\}$. Assume that the system is in static equilibrium.

   (a) Find the total wrench $\boldsymbol{\mathcal{F}}_b$ applied by the two fingers to the can (express your result in frame $\{b\}$).

   (b) Suppose that $\boldsymbol{\mathcal{F}}_{ext}$ is an arbitrary external wrench applied to the can ($\boldsymbol{\mathcal{F}}_{ext}$ is expressed in frame $\{b\}$). Based on your result in (a), find all $\boldsymbol{\mathcal{F}}_{ext}$ that cannot be resisted by the fingertip forces at all.

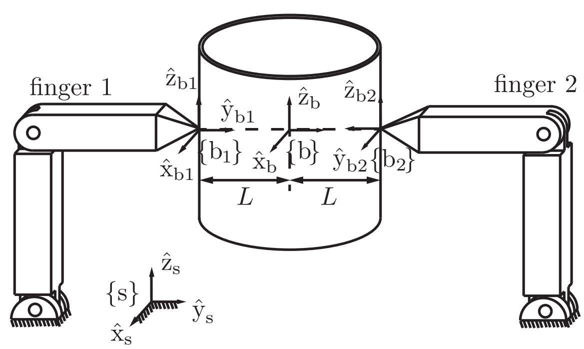

Figure 2: Two fingers grasping a can.

Notes for Questions 1, 2:

- Your report should include a brief description of your results with supporting figures which are usually output of your code. You do not have to include a screenshot of your code in the report.
- Add proper comments to your code, which detail what each part of the code is doing.
- Submit your report and code files in a single Zip file on Brightspace. Name the Zip file as `HW#N_FullName`, where `N` is the homework number and `FullName` is your full name. A report without its supporting code files and code files without a supporting report are NOT acceptable.
- Make sure to submit all the files/functions required to properly execute your code.