Homework #5

MEC 529: Introduction to Robotics

Spring 2023

Instructor	Amin Fakhari, Ph.D.
Assigned Date	Monday, Mar. 27, 2023
Due Date	Friday, Apr. 7, 2023

1. The forward kinematics of a robot refers to the calculation of the pose (position and orientation) of its end-effector frame $\{b\}$ with respect to a fixed space frame $\{s\}$ from its joint variables $\boldsymbol{\theta} \in \mathbb{R}^n$ where n is the number of joints of the robot.

   (a) Write a MATLAB function `FK_SpaceForm` for geometric forward kinematics of an n-DOF open-chain robot that returns the transformation matrix $\boldsymbol{T}_{sb} \in SE(3)$ by taking a matrix $\boldsymbol{S} \in \mathbb{R}^{6\times n}$ which each column of the matrix corresponds to the screw axes $\boldsymbol{\mathcal{S}}_i \in \mathbb{R}^6$ of the robot joints expressed in frame $\{s\}$ when the robot is at its home configuration (i.e., $\boldsymbol{S} = [\boldsymbol{\mathcal{S}}_1, ..., \boldsymbol{\mathcal{S}}_n]$), a transformation matrix $\boldsymbol{M} \in SE(3)$ which is the configuration of $\{b\}$ relative to $\{s\}$ when the robot is in its home configuration, and a vector $\boldsymbol{\theta} \in \mathbb{R}^n$ which is the joint variables. Therefore, $\boldsymbol{T}_{sb} = \texttt{FK\_SpaceForm}(\boldsymbol{S}, \boldsymbol{M}, \boldsymbol{\theta})$.

   (b) Write a MATLAB function `FK_BodyForm` for geometric forward kinematics of an n-DOF open-chain robot that returns the transformation matrix $\boldsymbol{T}_{sb} \in SE(3)$ by taking a matrix $\boldsymbol{B} \in \mathbb{R}^{6\times n}$ which each column of the matrix corresponds to the screw axes $\boldsymbol{\mathcal{B}}_i \in \mathbb{R}^6$ of the robot joints expressed in frame $\{b\}$ when the robot is at its home configuration (i.e., $\boldsymbol{B} = [\boldsymbol{\mathcal{B}}_1, ..., \boldsymbol{\mathcal{B}}_n]$), a transformation matrix $\boldsymbol{M} \in SE(3)$ which is the configuration of $\{b\}$ relative to $\{s\}$ when the robot is in its home configuration, and a vector $\boldsymbol{\theta} \in \mathbb{R}^n$ which is the joint variables. Therefore, $\boldsymbol{T}_{sb} = \texttt{FK\_BodyForm}(\boldsymbol{B}, \boldsymbol{M}, \boldsymbol{\theta})$.

   For writing these functions, you can utilize the functions you have written in Homework #3 and #4. In the next question, you will validate these functions.

2. The PRRRRR spatial open-chain robot manipulator of Figure 1 is shown in its home configuration. Assume that $l_1 = 0.20$ m, $l_2 = 0.20$ m, $l_3 = 0.20$ m, $l_4 = 0.20$ m, $l_5 = 0.10$ m, and $h = 0.35$ m.

   (a) Determine the end-effector transformation matrix $\boldsymbol{M} \in SE(3)$ when the robot is at its home configuration as shown.

   (b) Determine the screw axes $\boldsymbol{\mathcal{S}}_i \in \mathbb{R}^6$ and $\boldsymbol{\mathcal{B}}_i \in \mathbb{R}^6$ of the robot joints expressed the in fixed space frame $\{s\}$ and end-effector frame $\{b\}$, respectively, when the robot is at its home configuration as shown.

   (c) For the joint variable values $\boldsymbol{\theta} = (0.05, \pi/6, \pi/8, \pi/8, -\pi/4, \pi/3)$, use your functions `FK_SpaceForm` and `FK_BodyForm` to compute the end-effector configuration $\boldsymbol{T}_{sb} \in SE(3)$. Confirm that they agree with each other.

   (d) Write a URDF (Universal Robot Description Format) file when the robot is at its home configuration as shown. A sample URDF file corresponding to a 3R spatial robot shown in Figure 2 is attached for your reference (you can open and edit an URDF file using any text editor like Notepad or, for better representation of the code, Notepad++).

   (e) Import your URDF file into MATLAB and show the robot at the configuration $\boldsymbol{\theta}$ given in part (c). A sample MATLAB file for importing a URDF file and showing the robot at a give configuration is attached for your reference. Then, use the attached `triad` function to visually verify that the end-effector configuration $\boldsymbol{T}_{sb}$ computed in part (c) coincides with the $\{b\}$-frame of the robot imported into MATLAB using its URDF file at the same configuration $\boldsymbol{\theta}$.

(f) Repeat parts (c) and (e) for another arbitrary joint variable values $\boldsymbol{\theta}$, and discuss the results in your report.

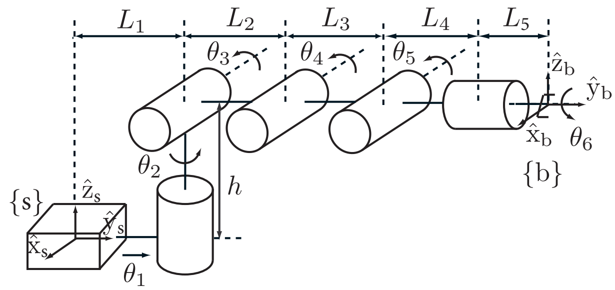

Figure 1: A PRRRRR spatial open-chain robot manipulator at its home configuration.

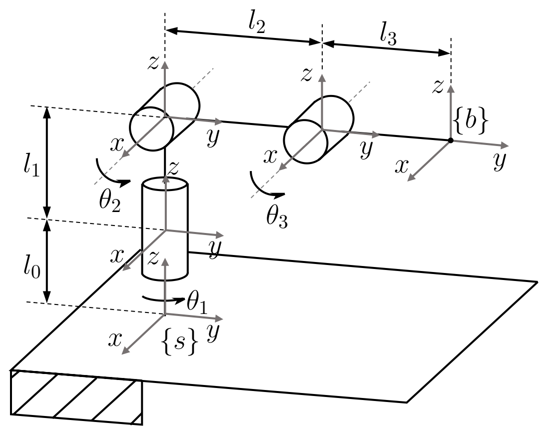

Figure 2: A 3R spatial robot at its home configuration.

3. Figure 3 shows a spatial 6R robot arm and a spatial 3R robot arm at their home configurations. The 6R robot arm has six rotational joints; the first three joints function as a Cartesian positioning device, while the last three joints act as a ZYZ Euler angle-type wrist. Properly assign link frames to the robot arms and find the corresponding Denavit–Hartenberg parameters.

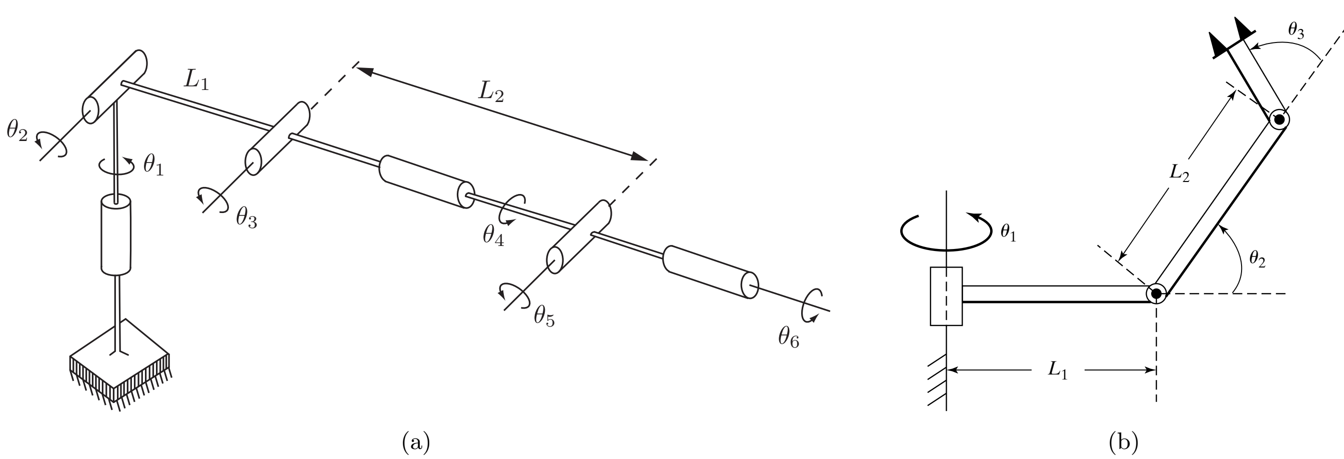

Figure 3: (a) A 6R spatial robot at its home configuration, (b) A 3R spatial robot at its home configuration.

Notes for Questions 2:

- Your report should include a brief description of your results with supporting figures which are usually output of your code. You do not have to include a screenshot of your code in the report.
- Add proper comments to your code, which detail what each part of the code is doing.
- Submit your report and code files in a single Zip file on Brightspace. Name the Zip file as `HW#N_FullName`, where `N` is the homework number and `FullName` is your full name. A report without its supporting code files and code files without a supporting report are NOT acceptable.
- Make sure to submit all the files/functions required to properly execute your code.