Homework #6

MEC 529: Introduction to Robotics

Spring 2023

Instructor	Amin Fakhari, Ph.D.
Assigned Date	Friday, Apr. 7, 2023
Due Date	Friday, Apr. 21, 2023

1. (a) (1/10) For an n-DOF open-chain robot, the space Jacobian $\boldsymbol{J}_s(\boldsymbol{\theta}) \in \mathbb{R}^{6\times n}$ relates the joint rate vector $\dot{\boldsymbol{\theta}} \in \mathbb{R}^n$ to the spatial twist $\boldsymbol{\mathcal{V}}_s \in \mathbb{R}^6$ via $\boldsymbol{\mathcal{V}}_s = \boldsymbol{J}_s(\boldsymbol{\theta})\dot{\boldsymbol{\theta}}$. Write a MATLAB function `J_SpaceForm` that returns the space Jacobian $\boldsymbol{J}_s(\boldsymbol{\theta}) \in \mathbb{R}^{6\times n}$ by taking a matrix $\mathbf{S} \in \mathbb{R}^{6\times n}$ which each column of the matrix corresponds to the screw axes $\boldsymbol{\mathcal{S}}_i \in \mathbb{R}^6$ of the robot joints expressed in frame $\{s\}$ when the robot is at its home configuration (i.e., $\mathbf{S} = [\boldsymbol{\mathcal{S}}_1, ..., \boldsymbol{\mathcal{S}}_n]$) and a vector $\boldsymbol{\theta} \in \mathbb{R}^n$ which is the joint variables. Therefore, $\boldsymbol{J}_s = \texttt{J\_SpaceForm}(\mathbf{S}, \boldsymbol{\theta})$.

   (b) (1/10) For an n-DOF open-chain robot, the body Jacobian $\boldsymbol{J}_b(\boldsymbol{\theta}) \in \mathbb{R}^{6\times n}$ relates the joint rate vector $\dot{\boldsymbol{\theta}} \in \mathbb{R}^n$ to the end-effector body twist $\boldsymbol{\mathcal{V}}_b \in \mathbb{R}^6$ via $\boldsymbol{\mathcal{V}}_b = \boldsymbol{J}_b(\boldsymbol{\theta})\dot{\boldsymbol{\theta}}$. Write a MATLAB function `J_BodyForm` that returns the body Jacobian $\boldsymbol{J}_b(\boldsymbol{\theta}) \in \mathbb{R}^{6\times n}$ by taking a matrix $\mathbf{B} \in \mathbb{R}^{6\times n}$ which each column of the matrix corresponds to the screw axes $\boldsymbol{\mathcal{B}}_i \in \mathbb{R}^6$ of the robot joints expressed in frame $\{b\}$ when the robot is at its home configuration (i.e., $\mathbf{B} = [\boldsymbol{\mathcal{B}}_1, ..., \boldsymbol{\mathcal{B}}_n]$) and a vector $\boldsymbol{\theta} \in \mathbb{R}^n$ which is the joint variables. Therefore, $\boldsymbol{J}_b = \texttt{J\_BodyForm}(\mathbf{B}, \boldsymbol{\theta})$.

   (c) (2/10) The RRRP robot of Figure 1 is shown in its zero position ($\boldsymbol{\theta} = \mathbf{0}$). Using the fact that columns of the Jacobian is equivalent to the screw axis of the joints when the robot is in an arbitrary configuration, determine the space Jacobian $\boldsymbol{J}_s(\boldsymbol{\theta})$ and the body Jacobian $\boldsymbol{J}_b(\boldsymbol{\theta})$ when $\theta_1 = \theta_2 = 0, \theta_3 = \pi/2, \theta_4 = L$. Then, using your results, verify your functions `J_SpaceForm` and `J_BodyForm` written in Parts (a) and (b) when $L = 0.5$ m.

   Note that for writing functions `J_SpaceForm` and `J_BodyForm`, you can utilize the functions you have written for Homework #3 and #4.

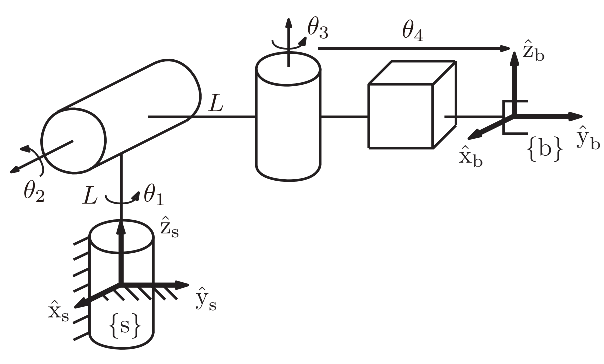

Figure 1: An RRRP spatial open-chain manipulator. The length of the first two links is L.

2. Consider a 2R planar robot as shown in Figure 2. Let $\boldsymbol{\theta} = (\theta_1, \theta_2) \in \mathbb{R}^2$ be the joint angles of the robot, $\boldsymbol{x} = (x, y) \in \mathbb{R}^2$ be the coordinates of the origin of end-effector frame $\{b\}$ with respect to the fixed base frame $\{s\}$, and l_1 and l_2 be the length of the links (assume that $l_1 = l_2 = 0.3$ m).

(a) (1/10) The inverse kinematics of a robot refers to the calculation of the joint variables from a desired configuration of the end-effector frame $\{b\}$ with respect to a fixed space frame $\{s\}$. Using forward kinematics function `FK_2R` and Jacobian function `J_2R` from your Homework #1, write a MATLAB function `IK_MinCoordinate`, based on Newton–Raphson method, that returns the the joint variables $\boldsymbol{\theta} = (\theta_1, \theta_2)$ by taking the minimum-coordinate representation of the desired end-effector configuration $\boldsymbol{x}_d = (x_d, y_d)$, an initial guess $\boldsymbol{\theta}_0 = (\theta_{1,0}, \theta_{2,0})$, and a very small threshold value $\epsilon \in \mathbb{R}$ on the final error. Therefore, $\boldsymbol{\theta} = \texttt{IK\_MinCoordinate}(\boldsymbol{x}_d, \boldsymbol{\theta}_0, \epsilon)$.

(b) (1/10) Verify function `IK_MinCoordinate` (which is based on an iterative numerical method) using function `IK_2R` from your Homework #1 (which is based on an analytic method) for three arbitrary coordinates (x, y) chosen in the robot workspace. Is it possible to find all the inverse kinematics solutions for a given (x, y) using function `IK_MinCoordinate`? If yes, try to find them.

(c) (2/10) The relationship between the space Jacobian $\boldsymbol{J}_s(\boldsymbol{\theta}) \in \mathbb{R}^{6\times n}$, the body Jacobian $\boldsymbol{J}_b(\boldsymbol{\theta}) \in \mathbb{R}^{6\times n}$, and the Jacobian $\boldsymbol{J}_g \in \mathbb{R}^{6\times n}$ defined as $\begin{bmatrix} \boldsymbol{\omega}_s \\ \dot{\boldsymbol{p}} \end{bmatrix} = \boldsymbol{J}_g(\boldsymbol{\theta})\dot{\boldsymbol{\theta}}$ is derived as

$$\boldsymbol{J}_g = \begin{bmatrix} \boldsymbol{I} & \boldsymbol{0} \\ -[\boldsymbol{p}] & \boldsymbol{I} \end{bmatrix} \boldsymbol{J}_s = \begin{bmatrix} \boldsymbol{I} & \boldsymbol{0} \\ -[\boldsymbol{p}] & \boldsymbol{I} \end{bmatrix} [\mathrm{Ad}_{\boldsymbol{T}_{sb}}]\, \boldsymbol{J}_b$$

where $\boldsymbol{\omega}_s \in \mathbb{R}^3$ is the angular velocity of the end-effector frame $\{b\}$ expressed in $\{s\}$, $\boldsymbol{p} \in \mathbb{R}^3$ and $\dot{\boldsymbol{p}} \in \mathbb{R}^3$ are the position and linear velocity of the origin of the end-effector frame $\{b\}$ expressed in $\{s\}$, respectively, $\boldsymbol{I} \in \mathbb{R}^{3\times 3}$ is an identity matrix, $[\mathrm{Ad}_{\boldsymbol{T}_{sb}}] \in \mathbb{R}^{3\times 3}$ is the adjoint map associated with $\boldsymbol{T}_{sb} \in SE(3)$, and $[\boldsymbol{p}] \in so(3)$. Using function `J_2R` from your Homework #1, verify these equations and your functions `J_SpaceForm` and `J_BodyForm` written in Problem 1 for an arbitrary configuration of the 2R planar robot.

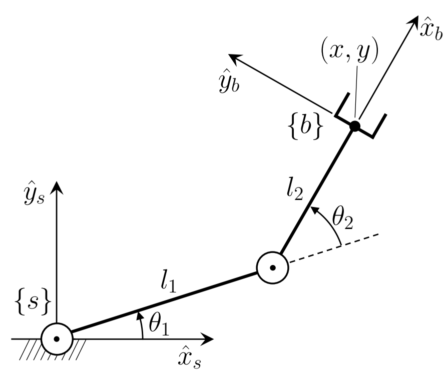

Figure 2: A 2R Planar Robot.

3. (1/10) For the RRRP robot of Figure 1, let $\boldsymbol{p}$ be the coordinates of the origin of $\{b\}$ expressed in $\{s\}$. Using your results in Problem 1.(c), find $\dot{\boldsymbol{p}}$ when $\theta_1 = \theta_2 = 0, \theta_3 = \pi/2, \theta_4 = L$ and $\dot{\theta}_1 = \dot{\theta}_2 = \dot{\theta}_3 = \dot{\theta}_4 = 1$.

4. (1/10) The four-DOF robot of Figure 3 is shown in its home configuration. Joint 1 is a screw joint of pitch h. Given the end-effector position $\boldsymbol{p} = (p_x, p_y, p_z)$ and orientation $\boldsymbol{R} = e^{[\hat{z}]\alpha}$, where $\hat{z} = (0, 0, 1)$ and $\alpha \in [0, 2\pi)$, find an analytical inverse kinematics solution $(\theta_1, \theta_2, \theta_3, \theta_4)$ as a function of p_x, p_y, p_z, and α. How many inverse kinematic solutions will exist for a given end-effector frame pose?

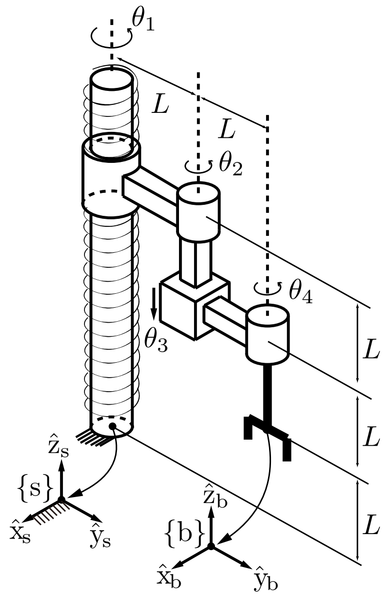

Figure 3: An open chain with a screw joint.

Notes for Questions 1, 2:

- Your report should include a brief description of your results with supporting figures which are usually output of your code. You do not have to include a screenshot of your code in the report.
- Add proper comments to your code, which detail what each part of the code is doing.
- Submit your report and code files in a single Zip file on Brightspace. Name the Zip file as `HW#N_FullName`, where `N` is the homework number and `FullName` is your full name. A report without its supporting code files and code files without a supporting report are NOT acceptable.
- Make sure to submit all the files/functions required to properly execute your code.