Sajid Ibna Mahbub

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Assignment-1

x	-1	0	2
$f(x)$	-12	-7	9

We know that for (x_0, y_0), (x_1, y_1) and (x_2, y_2)

$$L_2(x) = \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)} y_0 + \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)} y_1$$

$$+ \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)} y_2$$

Now,

$$f(x) = \frac{x\ (x-2)}{(-1)\ (-1-2)} (-12) + \frac{(x+1)\ (x-2)}{(1)(0-2)} (-7)$$

$$+ \frac{(x+1)\ (x-0)}{(2+1)\ (2-0)} (9)$$

$$= \frac{x(x-2)}{3}(-12) + \frac{(x+1)(x-2)}{-2}(-7)$$

$$+ \frac{x(x+1)}{6}(9)$$

$$= -4x(x-2) + \frac{7}{2}(x+1)(x-2) + \frac{3}{2}x(x+1)$$

$$\therefore f(1) = -4 \cdot 1(1-2) + \frac{7}{2}(1+1)(1-2) + \frac{3}{2} \cdot 1(1+1)$$

$$= 0$$

(Ans)

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Answer no-2

x	$f(x)$	$f^1[\ ]$	$f^2[\ ]$
-1	-12		
0	-7	5	
2	9	8	1

Applying Newton's divided difference formula, we get

$$f(x) = -12 + 5(x+1) + 1(x+1)(x-0)$$

Here,

$$f(x) = 0$$

$$\Rightarrow -12 + 5(x+1) + 1(x+1)(x-0) = 0$$

$\Rightarrow -12 + 5x + 5 + x^2 + x = 0$

$\Rightarrow -12 + 5x + 5 + x^2 + x = 0$

$\Rightarrow x^2 + 6x - 7 = 0$

$\therefore x = -7, 1$ (Ans)

$\therefore$ the root is $(-7, 1)$

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Ans to que no-3

Using central difference formula,

we get,

$$f''(1.2,\ 0.4) = \frac{1}{(0.4)^2}\left(f(0.8) - 2\times f(1.2) + f(1.6)\right)$$

$$= \frac{1}{(0.4)^2}\left(0.754 - 2\times 2.623 + 5.677\right)$$

$$= 8.781$$

$\therefore f''(1.2) = 8.781$ (Ans)

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Ans to Qno-4

Using trapezoidal rule, we have

$h = 1$

$h = (1.2 - 0.8) = 0.4$

Now,

$$I^{(0)}(1, 0.4) = \frac{0.4}{2}\left[f(0.8) + f(1.2)\right]$$

$$= 0.7154$$

$$I^{(1)}(2, 0.2) = \frac{0.2}{2}\left[f(0.8) + 2 \times f(1) + f(1.2)\right]$$

$$= 0.1\left[0.754 + 2 \times 1.648 + 2.623\right]$$

$$= 0.6873$$

First order extrapolated values are,

$$I^{(2)}(2,0.2) = I^{(1)}(2,0.2) + \frac{I^{(1)}(2,0.2) - I^{(0)}(1,0.4)}{2^2 - 1}$$

$$= 0.6873 + \frac{0.6873 - 0.7154}{3}$$

$$= 0.6777 \approx 0.678$$

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Ans to que-5

$$y' = y^2 + x + x^2 \qquad y(1) = -1$$

We know,

$$K_1 = h f(x_0, y_0)$$

$$K_2 = h f(x_0 + h, y_0 + K_1)$$

Now,

$$K_1 = h f(x_0, y_0)$$
$$= 0.2 f(1, -1)$$
$$= 0.2 \times 3 = 0.6$$

$$K_2 = h f(x_0 + h, y_0 + K_1)$$
$$= 0.2 f(1 + 0.2, -1 + 0.6)$$
$$= 0.2 f(1.2, -0.4)$$

$= 0.2 \times 2.8 = 0.56$

Now,

$$y_1 = y_0 + \frac{K_1 + K_2}{2}$$

$$\therefore \; y(1.2) = -1 + \frac{0.6 + 0.56}{2}$$

$$= -0.42$$

$$\therefore \; y(1.2) = -0.42$$