Pulse Code Modulation

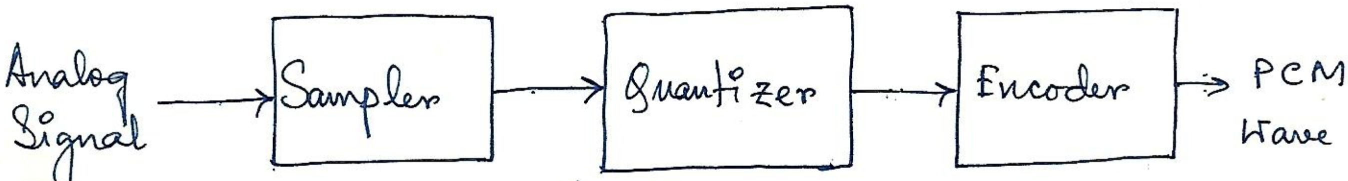
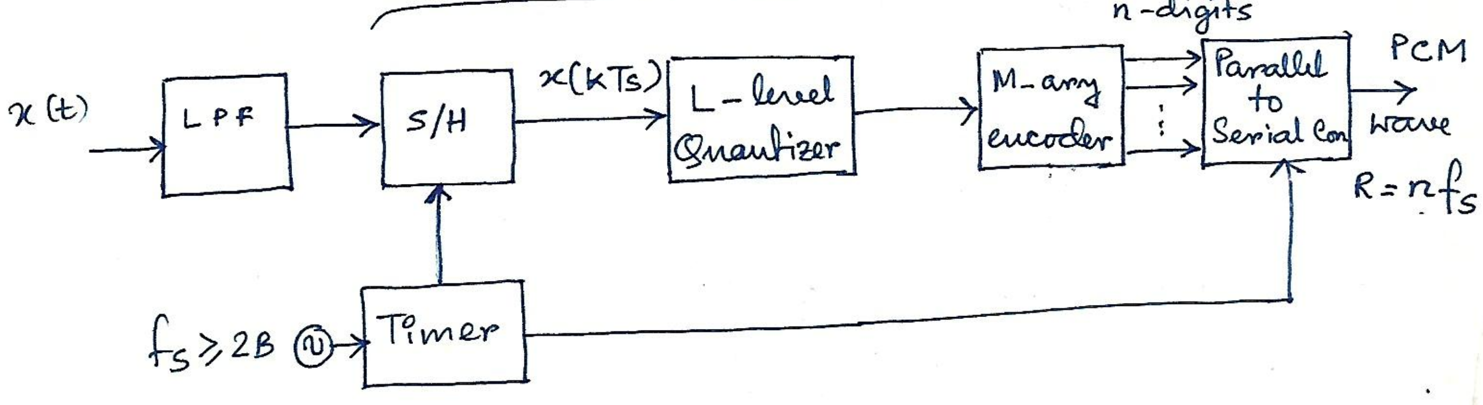


fig.(i)

In PCM the analog signal is necessarily passed through various blocks as shown in fig. The fundamental blocks are like this: (i) Sampler, (ii) Quantizer and (iii) Encoder.

I. **Sampler**: The incoming message wave is sampled with a train of narrow pulses so as to closely approximate the instantaneous sampling process. In order to ensure perfect reconstruction of message at the receiver, the sampling rate must be greater than twice the bandwidth or highest frequency (Sampling Theorem). Generally a LPF is pre-employed to eliminate higher unwanted frequency.

II. PCM Generation and Reconstruction:



PCM Generation System

fig.(ii)

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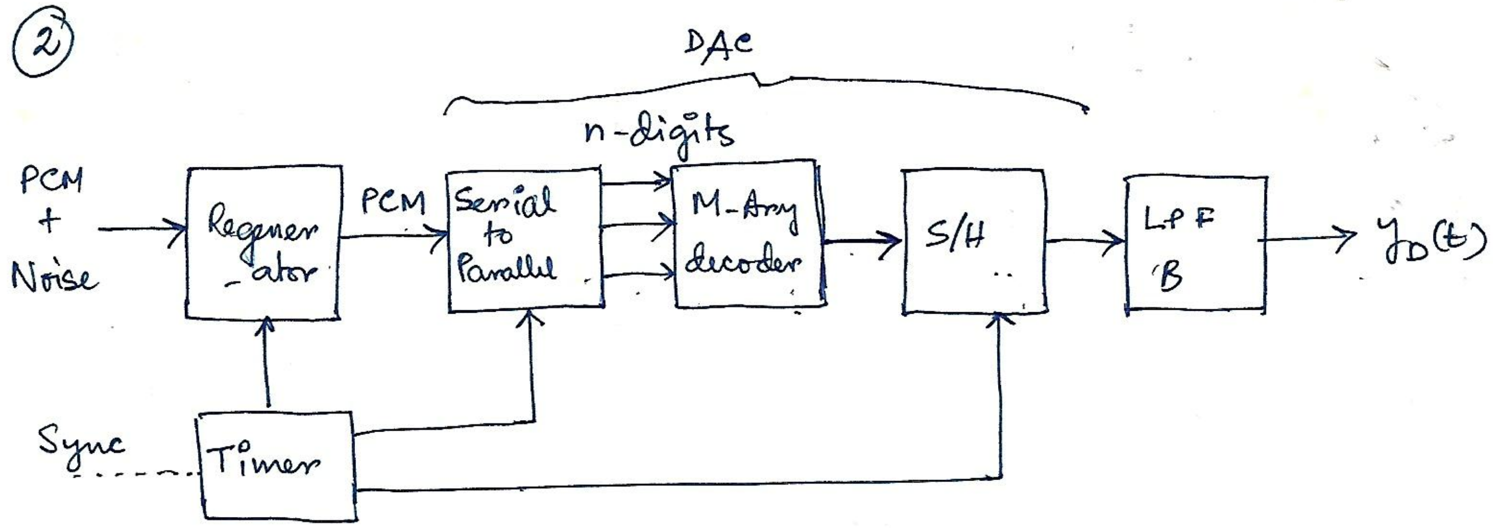


fig (iii) PCM receiver

Fig.(ii) is the functional blocks of a PCM generation system. The analog input waveform $x(t)$ is lowpass filtered and sampled to obtain $x(KT_s)$. A quantizer rounds off the sample values to the ~~...~~ nearest discrete value in a set of L quantum levels. The resulting quantized samples $x_q(KT_s)$ are discrete in time and discrete in amplitude.

An encoder translates the quantized samples into digital code words. The encoder works with M-ary digits and produces for each sample a codeword consisting of n-digits in parallel. Since there are M^n possible M-ary codewords with n-digits per word, unique encoding of the L different quantum levels require that $M^n \geq L$. The parameters M, n and L should be chosen to satisfy the equality. So that,

$$L = M^n \quad \text{or} \quad n = \log_M L$$

Thus, the number of quantum levels for binary PCM equals some power of 2.

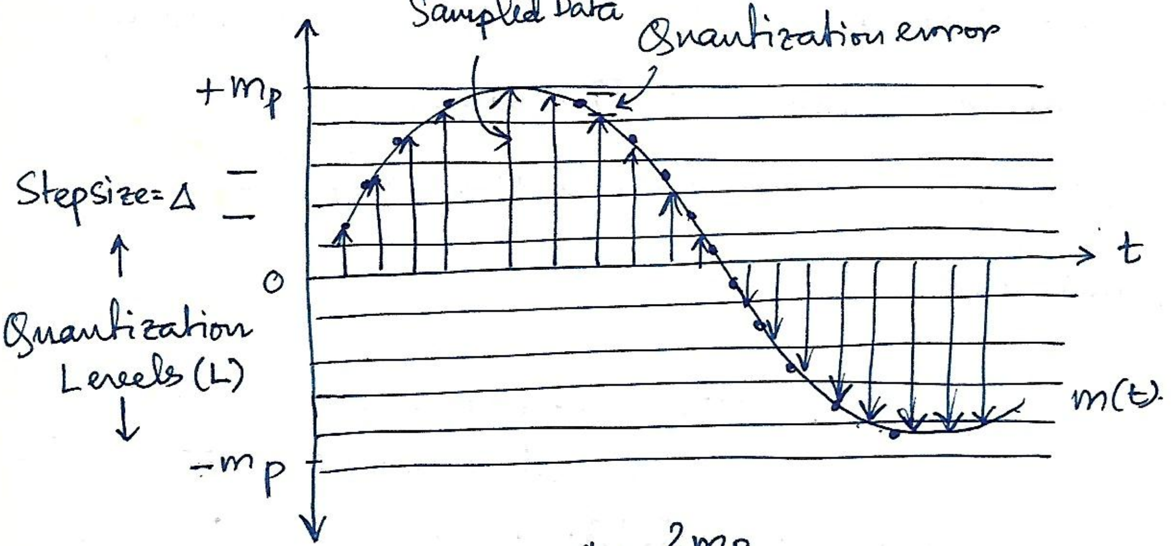
$$\therefore \boxed{L = 2^n}$$

Finally, successive codewords are read out serially to constitute the PCM waveform, an M-ary digital signal. The PCM generator thereby acts as an ADC, performing analog-to-digital conversions at the sampling rate $f_s = 1/T_s$. A timing circuit coordinates the sampling and parallel-to-serial readout.

A PCM receiver with the reconstruction system is shown in fig (iii). The received signal may be contaminated by noise, but regeneration yields a clean and nearly errorless waveform if SNR is sufficiently large. The DAC operations of serial-to-parallel conversion, M-ary decoding and sample-and-hold generate the analog waveform $x_q(t)$. The waveform is a "staircase" approximation of $x(t)$. Lowpass filtering then produces the smoothed output signal $y_D(t)$, which differs from the message $x(t)$ to the extent that the quantized samples differ from the exact sample values $x(kT_s)$.

Perfect message reconstruction is therefore impossible in PCM, even when random noise has no effect. The ADC operation at the transmitter introduces permanent errors that appear at the receiver as quantization noise in the reconstructed signal.

III. Quantizing :



$$\Delta = \frac{2m_p}{L}$$

An analog signal can be converted into a digital signal by means of sampling and quantization, which means rounding off or approximation of its sampled value to one of the closest permissible values or numbers which is known as Allowed Quantized levels.

If we consider an analog signal m(t) whose amplitude varies from (m_p to $-m_p$). This amplitude ~~level~~ range can be divided into L uniformly spaced intervals and having width,

$$\Delta = \frac{2m_p}{L}, \text{ where, } L = 2^n.$$

A sample value is approximated by the mid-point of the interval in which it lies. The various quantized levels have seperate binary codes. hence, corresponding to that sampled value a digital data is sent.

In doing so, an error is introduced, which is known as Quantization Noise or error. This is introduced due to approximation of sampled data and equals to diffⁿ betⁿ I/P and O/P data of the quantizer.

Quantization Noise :

The difference between the input and output signals of the quantizer becomes the Quantization Error. It is apparent that with a random input signal, the quantizing error q_e varies randomly within the interval

$$-\frac{\Delta}{2} \leq q_e \leq \frac{\Delta}{2}$$

Assuming that the error is equally likely to lie anywhere in the range $(-\Delta/2, \Delta/2)$, the mean quantizing error $\overline{q_e^2}$ is given by

$$\overline{q_e^2} = \frac{1}{\Delta}\int_{-\Delta/2}^{\Delta/2} q_e^2 \, dq_e = \frac{\Delta^2}{12}$$

$$\because \Delta = \frac{2m_p}{L} \qquad \therefore \boxed{\overline{q_e^2} = \frac{m_p^2}{3L^2}}$$

Bandwidth Requirements of PCM :

Suppose that in a binary PCM, L quantizing levels are used, satisfying,

$L = 2^n$ or $n = \log_2 L$, where n is an integer. For this case, $n = \log_2 L$ binary pulses must be transmitted for each sample of the message signal.

If the message bandwidth is f_m and the sampling rate is $f_s (\geq 2f_m)$, then nf_s binary pulses must be transmitted per second.

Assuming the PCM signal is a low-pass signal of bandwidth B Hz, the required minimum sampling rate is 2B. Thus,

$$2B = nf_s$$

$$\text{or,} \quad \boxed{B = \frac{n}{2} f_s = n f_m \text{ Hz.}}$$

The above equation shows that minimum bandwidth for PCM is proportional to the message signal bandwidth and the number of bits per symbol.

SNR in a PCM system for a Sinusoidal Signal:

SNR in a PCM system is defined by the ratio of average signal power to average quatization noise power. Let us consider a sinusoidal signal with amplitude A volts.

If L is no. of quantizing levels, then quantizer step size

$$\Delta = \frac{2A}{L}$$

As we have already known that, average quantizing noise power is $N_q = \frac{\Delta^2}{12} = \frac{A^2}{3L^2}$

Therefore, output SNR $= \left(\frac{S}{N_q}\right)_o = \frac{A^2/2}{A^2/3L^2} = \frac{3L^2}{2}$

Expressing in decibels, $\left(\frac{S}{N_q}\right)_{o\,dB} = 10 \log \left(\frac{S}{N_q}\right)_o = 1.76 + 20 \log L$

As, $L = 2^n$, $\therefore \left(\frac{S}{N_q}\right)_{dB} = 1.76 + 6.02n$ dB $\rightarrow$ [6-dB Law].

Non-uniform Quantization :

As we have already derived that the expression for SNR of a uniform quantizer is given by,

$$\mathrm{SNR} = \frac{\sigma_x^2}{\Delta^2/12} = \left(\frac{\sigma_x^2}{\sigma_Q^2}\right), \quad N_Q = \sigma_Q^2 = \frac{\Delta^2}{12}$$

Here, the value of SNR is directly proportional to signal power σ_x^2 or $\overline{m^2(t)}$.

More the signal power higher the value of SNR, lower the signal strength less the value of SNR. So, for a widely varying signal (ex. loud or weak voice messages) SNR varies significantly. In most of the time SNR will be low as larger amplitudes are less frequent.

Now, for above type of system the step-sizes of the quantization level, Δ, is fixed, that means quantization noise $N_Q = \sigma_Q^2 = \Delta^2/12$ is fixed and if signal power is less, SNR falls significantly.

The above problem can be solved efficiently if it is possible to vary Δ (step-size) according to strength of the signal. Generally, Δ is made smaller for smaller signal strength and vice-versa. This process is known as Compression. Here, quantization noise is smaller for low signal power and larger for high signal power, as a result SNR becomes practically independent of signal power. An approximate logarithmic ~~char~~ compression characteristics yields a quantization noise proportional to signal power. ~~which~~

This is often known as Robust Quantizer.

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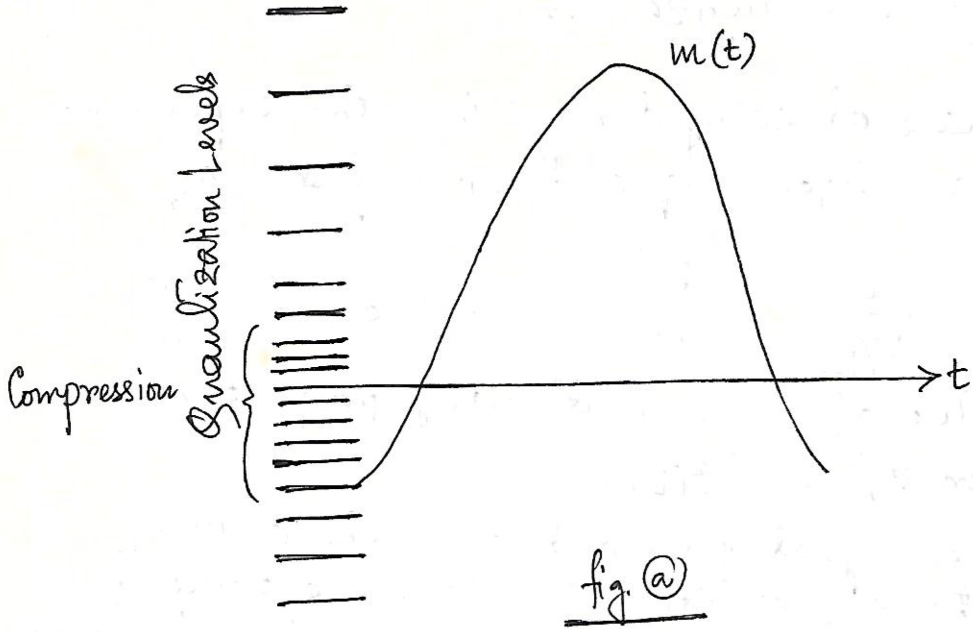


fig. (a)

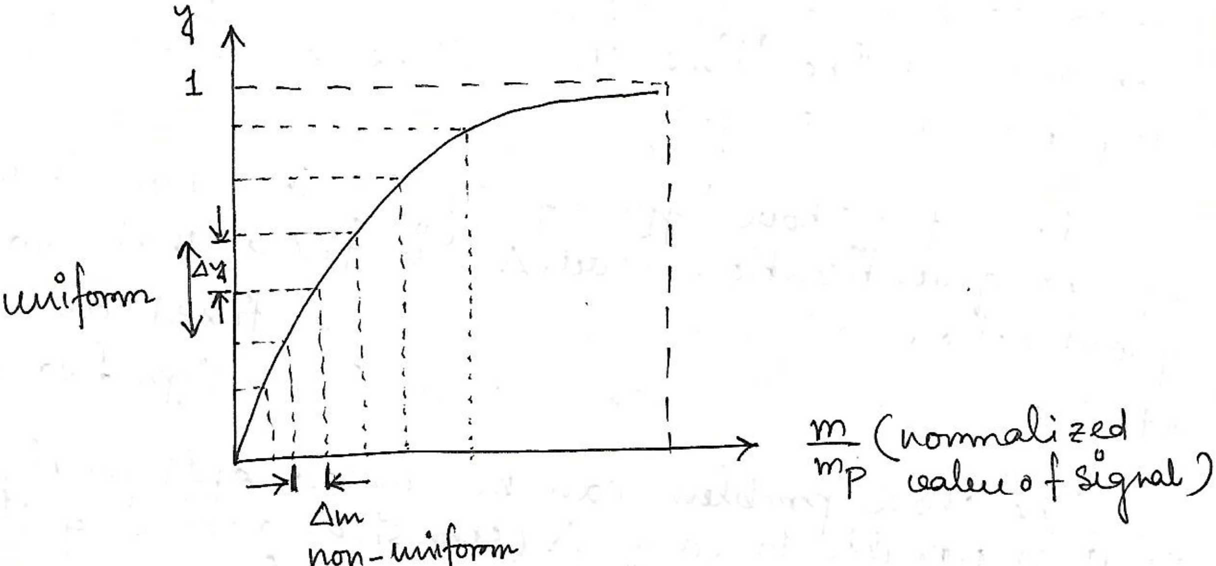


fig. (b)

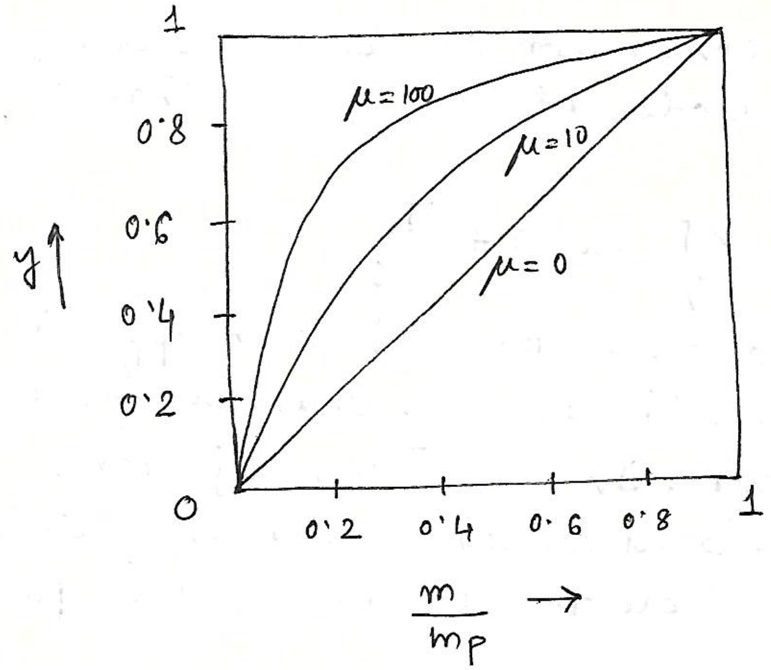
There are several compression methods, among which two laws have been widely accepted, those are known as μ-law and A-law.

<u>μ-law</u>:

$$y = \frac{1}{\ln(1+\mu)} \cdot \ln\left(1 + \frac{\mu m}{m_p}\right) \qquad 0 \leq \frac{m}{m_p} \leq 1$$

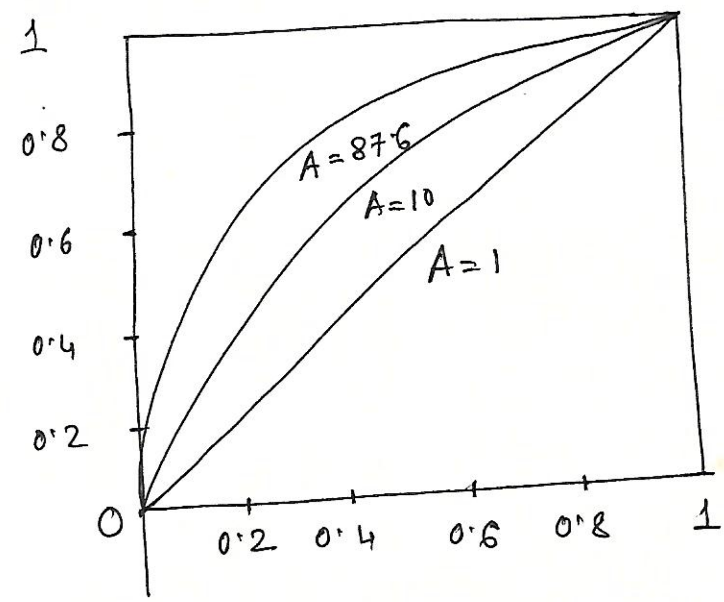
<u>A-law</u>:

$$y = \begin{cases} \frac{A}{1+\ln A} \cdot \left(\frac{m}{m_p}\right) & 0 \leq \frac{m}{m_p} \leq \frac{1}{A} \\ \frac{1}{1+\ln A}\left(1 + \ln \frac{Am}{m_p}\right) & \frac{1}{A} \leq \frac{m}{m_p} \leq 1 \end{cases}$$



$\boxed{\mu\text{-law}}$

$\underline{\mu = 255}$ (8 bit)
– 256 level



$\boxed{A\text{-law}}$

$\underline{A = 87.56}$ (8 bit)
– 256 level

The compressed sampled signal must be restored to their original value at the receiver by using an expander with a characteristics complementary to that of compressor. The Compressor and Expander together are called the <u>Compander</u>.

If $c(x)$ denotes compressor and $c^{-1}(x)$ is expander hence, we can write $\boxed{c(x)\, c^{-1}(x) = 1.}$

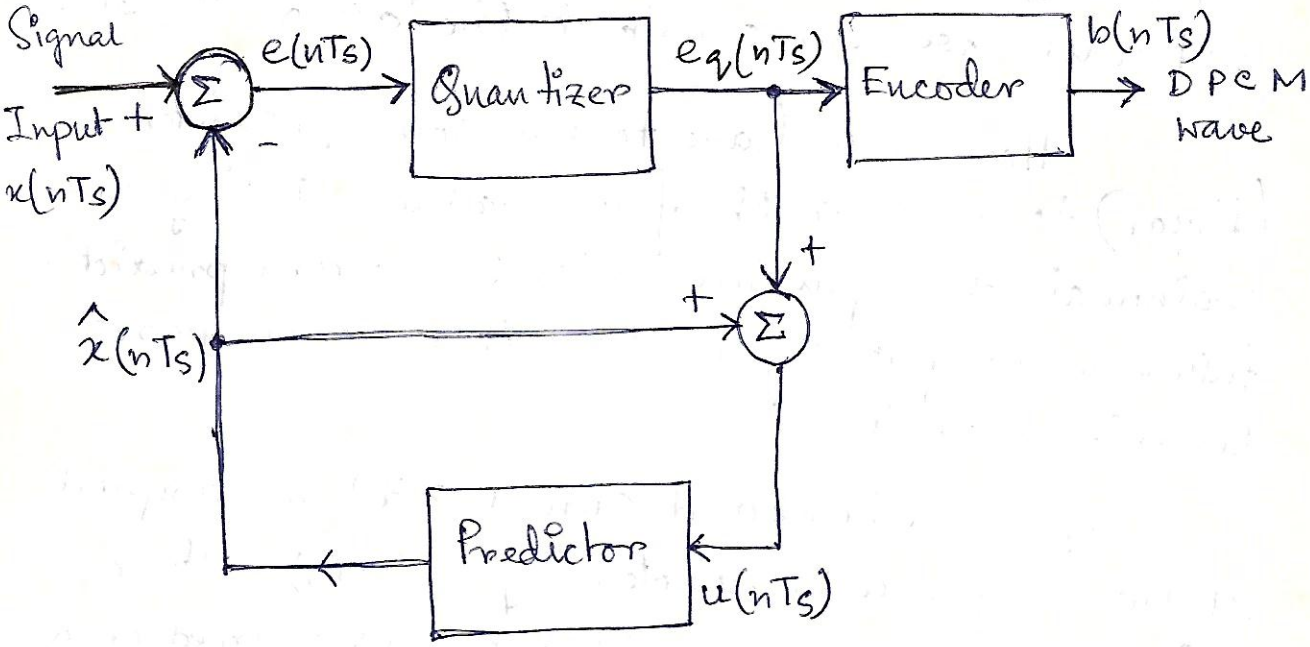
(10)

A logarithmic compressor can be realized by a semiconductor diode, because of V-I char as is given by;

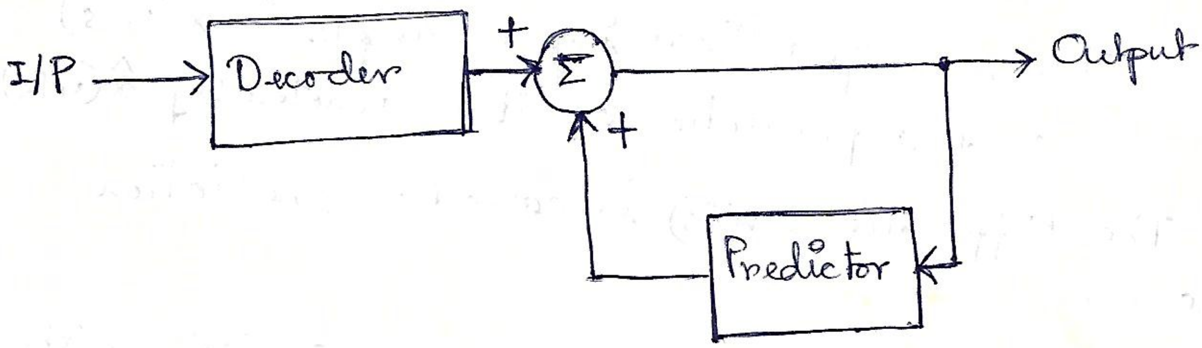
$$V = \frac{KT}{q} \ln\left(1 + \frac{I}{I_s}\right)$$

Two matched diodes in parallel with opposite polarity provide the approx. char. in first and third quadrant. In practice, adjustable resistors are placed in series with each diode and a third variable resistor is added in parallel.

Differential Pulse Code Modulation :



(a) DPCM Transmitter



(b) DPCM Receiver

In case of PCM transmitter we generally sample the signal and quantize it and finally encoding the quantized ~~so~~ message we transmit it. But, sending a quantized sample required lots of bits hence lots of bandwith. But, instead of sending a ordinary coded signal if we send the difference of the present signal compared with its past values, then it will be simpler to transmitt and detect the original signal. Here, we have to send only the error difference and just adding the error with past values we can construct the actual signal.

Here, we have to use some predictor (linear) to estimate the future values. Using various mathematical expansion methods we can predict a future value of a signal even if we know past behaviour of it.

Suppose a baseband signal $x(t)$ is sampled at the sequence be denoted by $x(nT_s)$, where n is the nth sample. So, if it possible to predict a future value then, the input to the Quantizer will be

$$e(nT_s) = x(nT_s) - \hat{x}(nT_s) \quad \text{----} (1)$$

where, unquantized input sample is $x(nT_s)$ and a prediction of it is denoted by $\hat{x}(nT_s)$.

The difference $e(nT_s)$ is called as prediction error.

Let, the non-linear function Q defines the behaviour of input-output characteristics of the Quantizer.

Hence, quantizer output may be represented by,

$$v(nT_s) = Q[e(nT_s)]$$

$$= e(nT_s) + q(nT_s) \quad \text{----} (2)$$

where, $q(nT_s)$ is quantization error.

Now, quantizer output is added to the predicted value $\hat{x}(nT_s)$ to produce the predictor ~~output~~ input

$$u(nT_s) = \hat{x}(nT_s) + v(nT_s) \quad \text{------} \quad (3)$$

Hence, substituting from eqn. (2), we get

$$u(nT_s) = \underbrace{\hat{x}(nT_s) + e(nT_s)} + q(nT_s)$$

$$u(nT_s) = x(nT_s) + q(nT_s) \quad \text{----} \quad (4)$$

Which simply represents quantized version of input signal $x(nT_s)$.

It also shows ~~that~~ irrespective of properties of predictor, the input to it differs from the original signal by quantization error only.

If the prediction is good less will be the prediction error, hence quantizer with a given no. of representation level can be adjusted to produce a quantizing error instead of quantize it directly.

<u>SNR</u> :

$$(SNR)_o = \frac{\sigma_x^2}{\sigma_Q^2} = \frac{\sigma_x^2}{\sigma_E^2} \cdot \frac{\sigma_E^2}{\sigma_Q^2}$$

$$= G_p \cdot (SNR)_p .$$

Where, $(SNR)_p$ = prediction error to - quantization noise ratio.

G_p = Prediction Gain.

Hence, G_p is maximised by minimizing the variance σ_E^2 of prediction error $e(nT_s)$.

Delta Modulation : (DM)

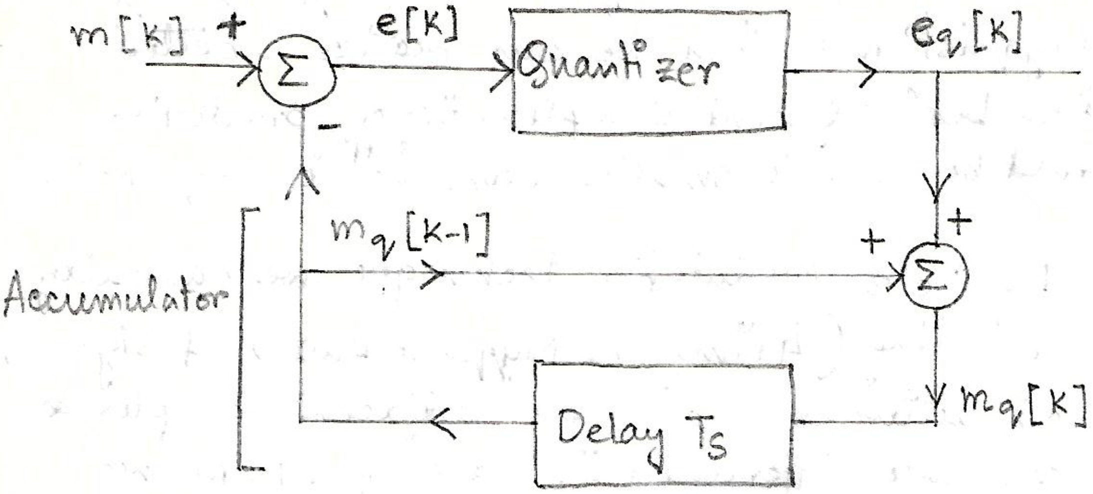


fig. I (a) Transmitter

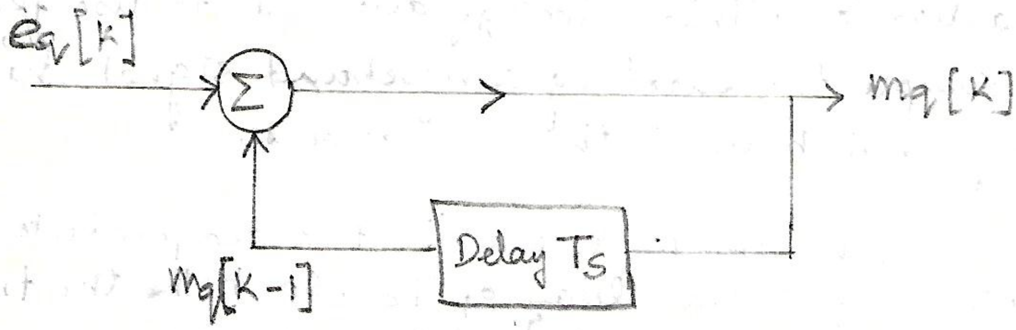


fig. II (b) Receiver.

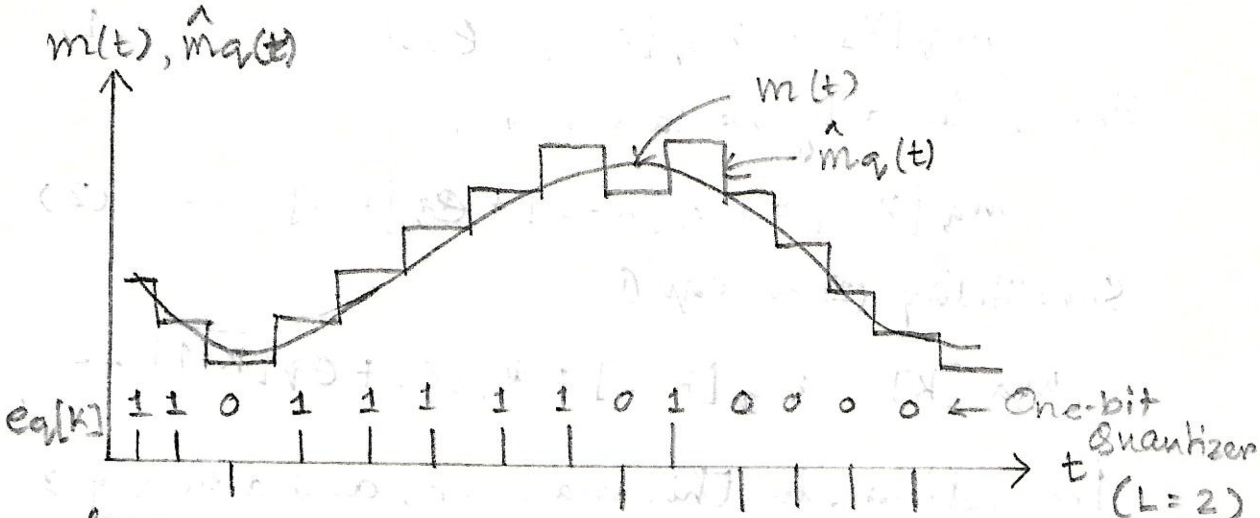
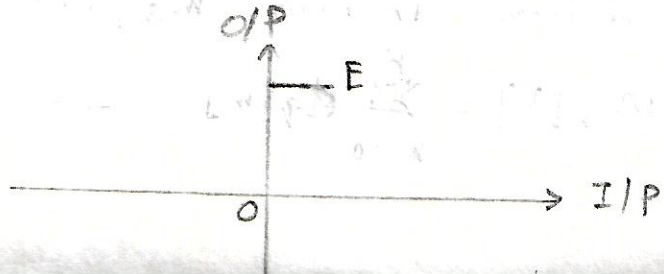


fig. III



Delta Modulation is an improved version of DPCM. For, DPCM if the sampling rate is slightly higher than Nyquist rate, then we can realize a better correlation betn. adjacent samples. Hence, prediction error could be less. So smaller variance diffn. is required.

By Delta Modulation technique we basically use over sampling (4 times the Nyquist Rate) of signal, hence much better correlation betn. adjacent samples & smaller variance difference. In case of DPCM we used more no. of bits to quantize the error. But for DM we use only 1-bit quantization i.e. 2-level Quantization. This strategy allows us to use fewer bits per sample for encoding a baseband signal, hence very small bandwidth requirement.

In DM we use a first-order predictor which is basically a time delay of T_s. So, both the transmitter and receiver use a time delay ckt. Hence, we can write,

$$m_q[K] = m_q[K-1] + e_q[K] \quad \text{-----}(1)$$

Hence, similarly we can write,

$$m_q[K-1] = m_q[K-2] + e_q[K-1] \quad \text{-----}(2)$$

Substituting 2 in eqn 1, ...

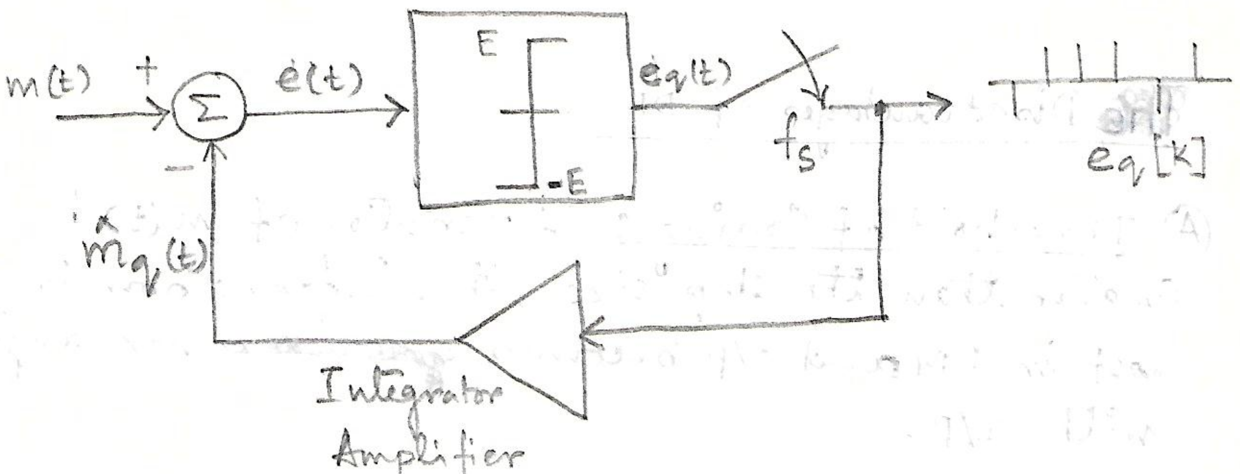
$$m_q[K] = m_q[K-2] + e_q[K] + e_q[K-1] \quad \text{----}(3)$$

Proceeding, in this manner, and assuming zero initial condition, that is, $m_q[0] = 0$, we get,

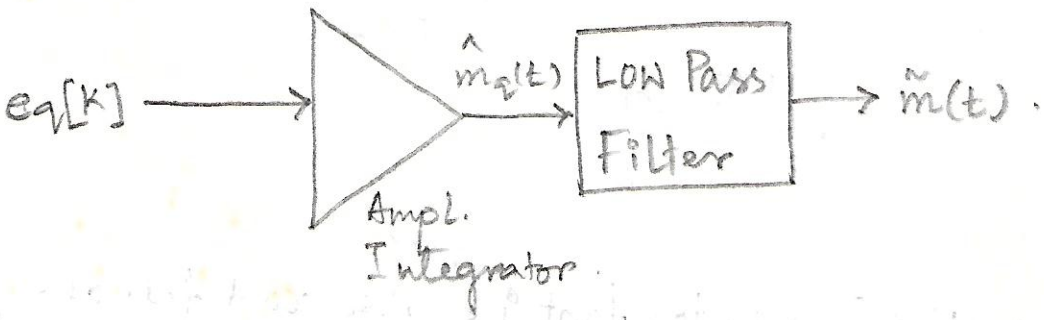
$$m_q[K] = \sum_{m=0}^{K} e_q[m] \quad \text{-----} (4)$$

~~This shows receiver is just an accumulator (Adder).~~

The analog signal m(t) is compared with feedback signal $\hat{m}_q(t)$. The error signal is
$e(t) = m(t) - \hat{m}_q(t)$, which is applied to a comparator. The O/P of comparator is a Signum function, i.e. $e_q(K) = E\,\mathrm{Sgn}[e(K)]$.



Delta Modulator



Delta De-Modulator

If e(t) is +ve, the comparator output is a constant Signal +E, if e(t) is −ve, the O/P is −E constant. Thus the difference is a binary signal (L = 2) that is needed to generate 1-bit DPCM. The sampler thus produces a +ve pulse, if $m(t) > \hat{m}_q(t)$ and −ve pulse, when $m(t) < \hat{m}_q(t)$. The pulse-train

~~28~~ 18

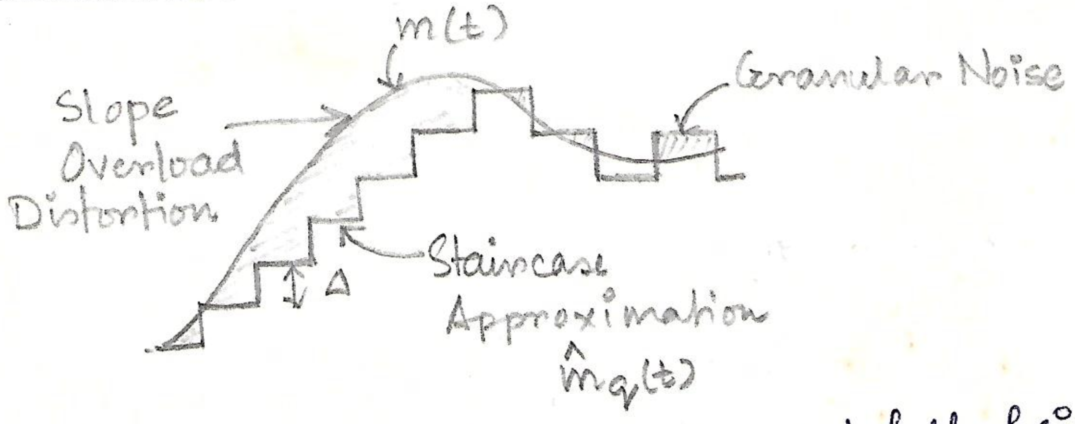
$\hat{m}_q(t)$ is a stair-case approximation of $m(t)$ which is produced by the Integrator-Amplifier.

And conceptually we can see that receiver is nothing but an accumulator which may be realized by an Integrator similar to that in modulator. The output of which when passed through LPF, the desired signal is reconstructed from it.

Disadvantages of DM :

(A) **Threshold of Coding** : If variation of $m(t)$ is smaller than the step size, then information is lost in DM and O/P becomes nonvarying with I/P.

(B) **Slope Overload** :



If $m(t)$ changes too fast i.e. rise and fall of signal is too sharp, then the staircase approximation $\hat{m}_q(t)$ of signal $m(t)$ cannot follow it. Hence overloading occurs, which is known as Slope Overload.

Hence, from the above two points we can conclude that, step size (Δ) neither can't be too small nor large. So, an optimum value of step-size (Δ) is to be decided. The optimum value depends on sampling freq. f_s and nature of signal.

Now, during the sampling interval T_s, $\hat{m}_q(t)$ is able to change by $\pm\Delta$, hence, maximum rate of change of $\hat{m}_q(t)$ which is slope of it, is given by $\frac{\Delta}{T_s}$ or Δf_s. So, no slope overload occurs if

$$\boxed{\frac{d}{dt}[m(t)] < \Delta f_s.}$$

ⓒ Granular Noise :

If step-size (Δ) is too large compared to slope characteristics of input signal, ~~then the staircase~~ approximation haunts the input waveform, as a result the o/p becomes granular, that means smoth waveform is not obtained, which is analogous to quantization noise.

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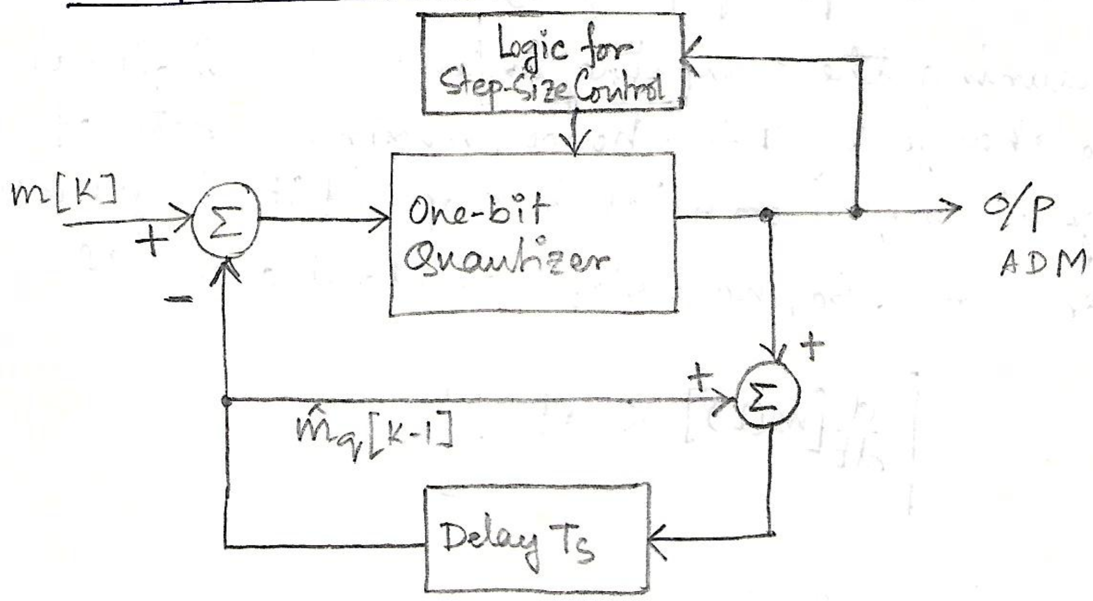
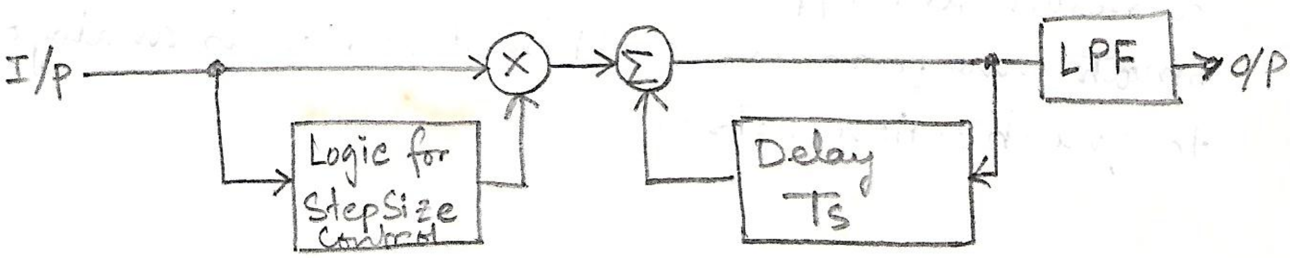
Adaptive Delta Modulation (ADM)



ⓐ Transmitter



ⓑ Receiver

The problem of slope-overload and threshold effect can be reduced by ADM technique. Here, we vary the step-size of the modulator according to slop of input signal. That means, for steep segment the step size is increased, conversely, when the input signal is varying slowly, it is reduced. In this way, the step-size is adapted to the level of input signal.

There are several types of ADM available, depending on the type of scheme used for adjusting

Among various types, mainly two schemes are used. In one type, the step-size is varied continuously.

On other type, the step size is constrained to lie between max. and min value. The upper-limit

$$\Delta_{min} \leq \Delta[K] \leq \Delta_{max}$$

controls slope-overload, and lower limit controls threshold problem. A time varying multiplier is generally incorporated to vary step size, depending on binary output of modulator.