Intersymbol Interference (ISI)

From Power spectral density of a rectangular Pulse, we can see that the ~~abs~~ absolute bandwidth is infinite, But the essential bandwidth of all Physical Communication channel are finite. So, If these rectangular Pulse is transmitted over a channel of bandwidth R_B Hz, a significant portion of its spectrum is transmitted, but a small portion of the spectrum is suppressed. As a result, such spectral distortion tends to spread the Pulse for each symbol. Spreading of a Pulse ~~bey~~ beyond T_b will cause it to interfere with neighboring Pulse. This is known as Intersymbol Interference (ISI), which can cause errors in the correct detection of Pulse.

Fig: transmitted Pulse   Fig: Spreading of Pulse (ISI)   Fig: received waveform.

Example of ISI or received Pulse in ~~a~~ binary comm system.

Causes of ISI :

1. Timing inaccuracies : This is more ~~lead~~ likely to occur in receiver.
2. Insufficient Bandwidth : Timing error are less likely occur if transmission rate is well below channel Bandwidth.
3. Amplitude distortion : Because of frequency selective nature of channel.
4. Phase distortion : Phase sensitivity of different frequency component present in the Pulse.

Nyquist Criterion for Zero ISI

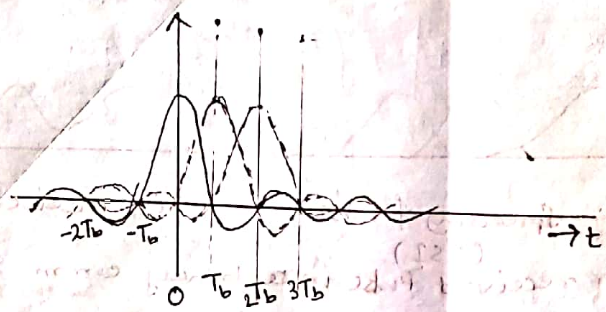
we need to transmit a Pulse at every T_b interval, the kth Pulse being $a_k P(t-kT_b)$. The channel has a finite bandwidth and we required to detect the Pulse amplitude a_k correctly (without ISI) if there is no ISI at the decision making instants. This can be accomplished by a Properly shaped band limited Pulse. To eliminate ISI, Nyquist Proposed different criteria for Pulse shaping.

Nyquist First criterior for zero ISI

zero ISI can be achived if the Pulse shape statisfies

$$P(t) = \begin{cases} 1 & t = 0 \\ 0 & t = \pm nT_b \ \left(T_b = \frac{1}{R_b}\right) \end{cases} \quad \text{--- (1)}$$

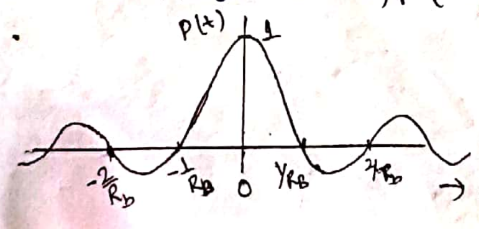
where R_b bit rate.

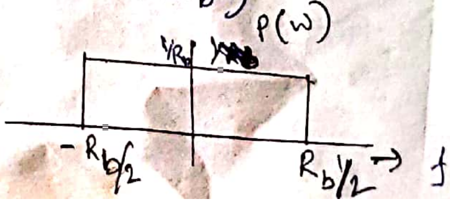


Here we show several successivel Pulse (dotted) centered at $0, \pm T_b, \pm 2T_b$ ---- . It is clear from this figure that the samples at, $0, T_b, 2T_b$ --- consist of the amplitude of only one Pulse (centred at the sampling instant) with no interference from the remaing Pulse. $P(t-kT_b)$

Now we know that transmission of R_b bit/s, we require a threoritical minimum bandwidth of $R_b/2$ Hz

From equation (1), the Pulse $P(t)$ can be represented by $P(t) = \text{sinc}(\pi R_b t)$. Moreover fourier transform of this Pulse is $= P(\omega) = \frac{1}{R_b}\text{rect}\left(\frac{\omega}{2\pi R_b}\right)$





which has a bandwidth $R_b/2$ Hz. using this Pulse, we can transmit at a rate of R_b Pulse Per second without ISI over a bandwidth of $R_b/2$.

- unfortunately this Pulse impractical because it starts at $-\alpha$. we have to wait an infinite time to generate it. Any attemp to truncate it would increase its bandwidth beyond $R_b/2$ Hz.
- It necessary that the amplitude characteristics of $P(\omega)$ be flat from $-R_b/2$ to $R_b/2$ and zero elsewhere. This is Physically unrealizable because of the abrupt transitions at $\pm R_b/2$
- The function $P(t)$ decrease at $1/|t|$ for large $|t|$, resulting in a slow rate of decay. now if sampling instants deviated slight at receiver, there is Practically no margin of error.

Due to this Practical Problem Nquist gives more Practical solution:

<u>Raised Cosine - Roll off Nyquist Filtering</u> : To avoid above difficulties we need a Pulse shape whose Properties is same as eq 1 but decay at a faster than $1/x$, for that we need the ~~increase~~ Bandwidth of the Pulse inbetween $R_b/2$ to R_b.

This can be Proved as follows. let $P(t)$ Pulse shape whose Spectrum be $P(\omega)$, bandwidth in the range $(R_b/2, R_b)$.

If we sample $P(t)$ every T_b second by multiplying $P(t)$ by $\delta_{T_b}(t)$ (Pulse train) we get, $\bar{p}(t) = P(t)\,\delta_{T_b}(t) = \delta(t)$ —(2)

Now, we know that the spectrum of sample signal $\bar{P}(t)$ is the spectrum of $P(t)$ repeating Periodically at intervals of the sampling frequency ω_b. Therefore FT of eq 2

$$\frac{1}{T_b}\sum_{n=-\alpha}^{\alpha} P(\omega - n\omega_b) = 1 \qquad \omega_b = \frac{2\pi}{T_b} = 2\pi R_b$$

$$\sum_{n=-\alpha}^{\alpha} P(\omega - n\omega_b) = T_b \qquad \text{—(3)}$$

The sum of spectra formed by repeating $P(\omega)$ every ω_b is a constant T_b.

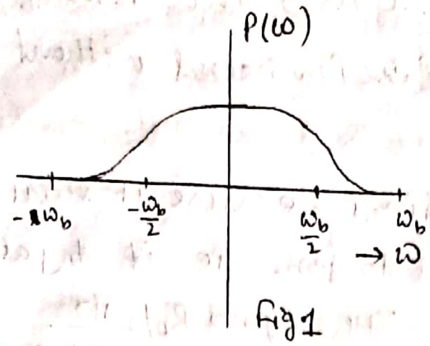


Fig 1

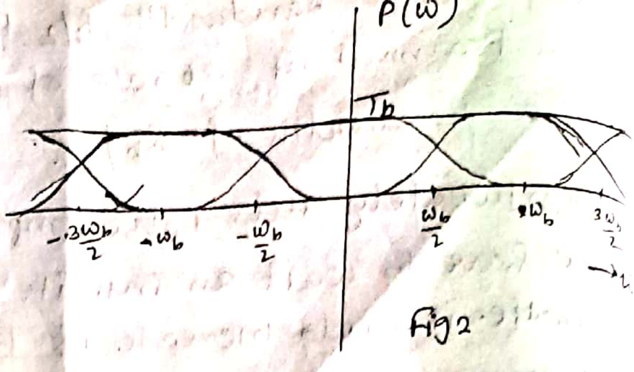


Fig 2

From fig 2 over the range $0 < \omega < \omega_b$, only two terms $P(\omega)$ and $P(\omega - \omega_b)$ in the summation eq 3, Hence

$$P(\omega) + P(\omega - \omega_b) = T_b$$

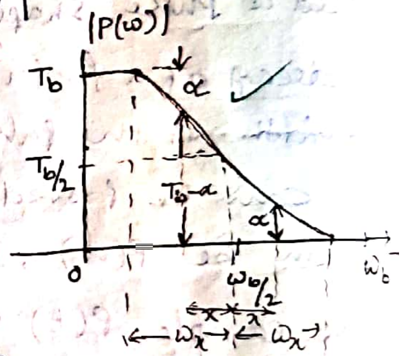
let $\omega = x + \omega_b/2$

$$P\left(x + \frac{\omega_b}{2}\right) + P\left(x - \frac{\omega_b}{2}\right) = T_b \qquad |x| < \frac{\omega_b}{2}$$

$$P\left(\frac{\omega_b}{2} + x\right) + P^*\left(\frac{\omega_b}{2} - x\right) = T_b \qquad \left[G(-\omega) = G^*(\omega) \right.$$

If we assume $P(\omega) = |P(\omega)| e^{-j\omega T_d}$, so for the real case only

$$\left| P\left(\frac{\omega_b}{2} + x\right) \right| + \left| P\left(\frac{\omega_b}{2} - x\right) \right| = T_b$$



The curve has an odd symmetry about the point $\omega = \omega_b/2$

The bandwidth of $P(\omega)$ is $\omega_b/2 + \omega_x$ where ω_x is the bandwidth in excess of the theoritical minimum bandwidth.

let r be the ratio of excess bandwidth ω_x to the theoritical minimum bandwidth $\omega_b/2$

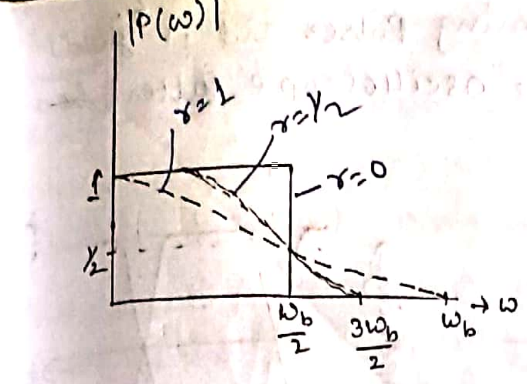
$$r = \frac{\omega_x}{\omega_b/2} = \frac{2\omega_x}{\omega_b}$$

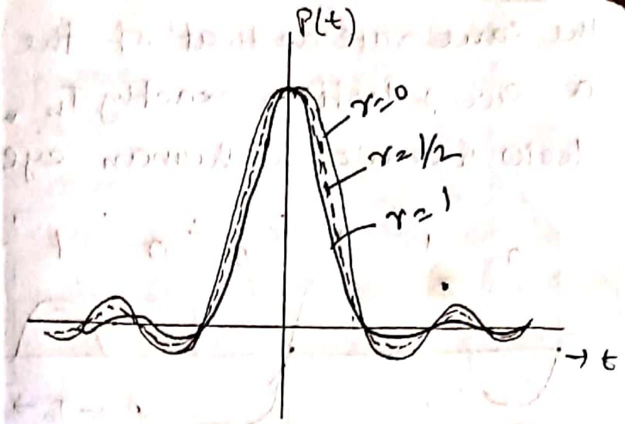
now $\omega_x \leq \omega_b/2 \Rightarrow 0 \leq r \leq 1$.

Therefore theoritical bandwidth is $R_b/2$ and excess bandwidth is $r\frac{R_b}{2}$. Therefore bandwidth $P(\omega) = \left(\frac{R_b}{2} + r\frac{R_b}{2}\right)$ Hz.

The constant r is called ~~roof~~ 'roll off factor'

Because of $0 \leq r \leq 1$, the bandwidth of $P(\omega)$ is restricted to the range of $R_b/2$ to R_b Hz. The Pulse ~~can~~ $P(t)$ can be generated as a unit impulse response of a filter with transfer function $P(\omega)$.





i from above equation we can seen that increasing ω_x or r improves $P(t)$; that is more gradual cutoff reduces the oscillatory nature of $P(t)$ and causes it to decay more rapidly. For the case of maximum value of $\omega_x = \frac{\omega_b}{2}$ $(r=1)$, $P(\omega) = \frac{1}{2}\left[1+\cos\frac{\omega}{2R_b}\right] \text{rect}\left(\frac{\omega}{4\pi R_b}\right)$

$$= \cos^2\left(\frac{\omega}{4R_b}\right) \text{rect}\left(\frac{\omega}{4\pi R_b}\right)$$

This characteristics is known as raised cosine.

The inverse fourier transform of this spectrum is

$$P(t) = R_b \frac{\cos \pi R_b t}{1-4R_b^2t^2} \text{sinc}(\pi R_b t)$$

The time response $P(t)$ consists of two factors.

① The factor $\text{sinc}(\pi R_b t)$ which is same as that of Nyquist 1st criterion. This ensures zero crossing of $P(t)$ at desired sampling instants.

② The factor $\frac{\cos(\pi R_b t)}{1-4R_b^2t^2}$ decreases as $1/t^2$. This reduces the tails of the pulse considerably below that obtained from 1st criterion. Therefore the transmission of binary waves using such pulses are relatively insensative to sampling time error.

EYE PATTERN :

Eye diagram is an improtant experimental display which visulizes the effect of channel Intersymbol interference and channel noise in digital transmission. A random binary pulse sequence is sent over the channel. The channel output is applied to the vertical input of an oscilloscope. The time base of the scope is triggered at the same rate as that of the incoming pulses and it yields a sweep lasting exactly T_b. The oscilloscope pattern looks like a human eye.

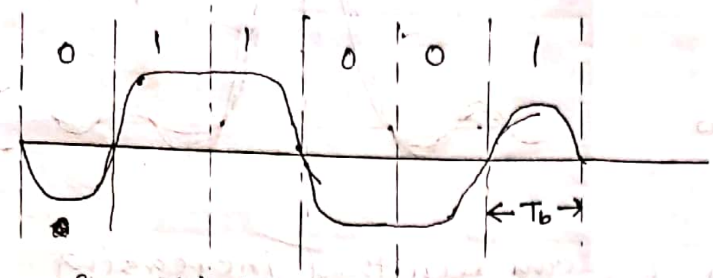


fig: Distorted binary wave

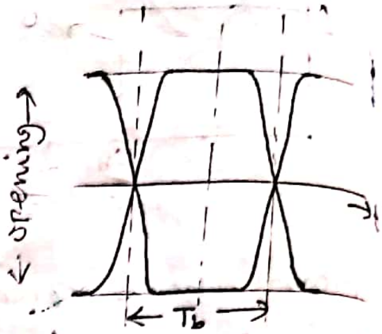


Fig: Eye Pattern

The eye pattern provides a great deal of information about the performance of the system.

① The width of eye opening defines the time interval over which the received wave can be sampled without error from ISI. (the instant when the vertical opening of eye is the longest)

② The sensitivity of the system to timing error is determined by the rate of closer of the eye as the sampling time is varied. how much ISI is there

③ The height of the eye opening, at a specified sampling time, defines the margin over noise. (set by signal to noise)

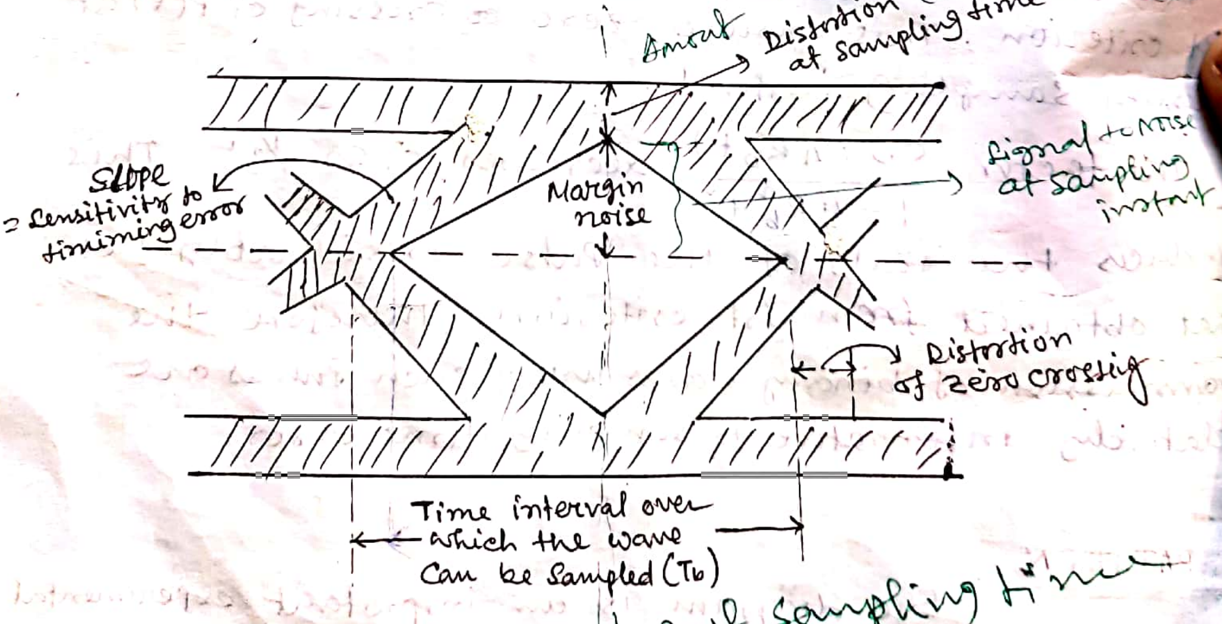


Fig: Interpretation of Eye Pattern