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Optimum Filter : As we know

error Probability $P_e = Q\left(\frac{A_P/\sigma_n}{\sqrt{2}}\right)$ where σ_n^2 is variance of noise. To reduce the P_e, we need to maximize A_P/σ_n, it is Possible when received signal passed through filter, that enhance the amplitude as well as reduce the noise factor.

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Optimum filter is linear time system which minimize the P_e at receiver.

Let $S_1(t)$ and $S_2(t)$ transmitted symbol of bit '1' & '0' for time duration $[0, T_b]$. Now due to Presence of Gaussian noise inside channel, the received signal at receiver $S_1(t) + n(t)$ and $S_2(t) + n(t)$. Now this two signal filter and then sample every T sec, we sample signal $V_0(T) = S_{01}(T) + n(T)$ or $S_{02}(T) + n(T)$ then Passing it through a decision device with threshold voltage V_T.

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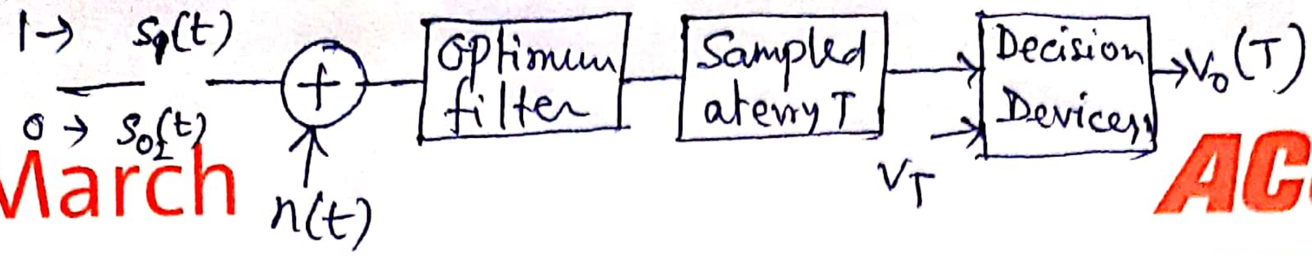
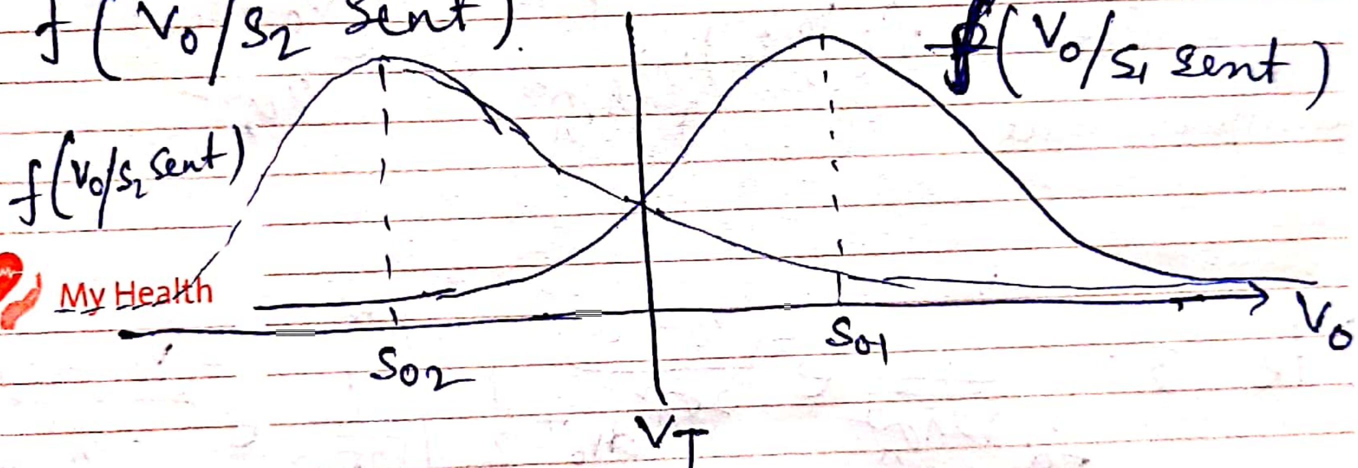


if $V_0(T) > V_T \rightarrow$ detect as '1'

$V_0(T) < V_T \rightarrow$ detect as '0'

As the received signal is random variable, we get two conditional PDF ~~probability~~, i.e. $f(V_0/S_1 \text{ sent})$ and $f(V_0/S_2 \text{ sent})$



NOW due to noise, there are two error can occur when '$V_0 < V_T$' if S_1 send and $V_0 > V_T$ if S_2 send.

$$\left.\begin{aligned} \therefore\ P(E/S_1 \text{ sent}) &= \int_{-\infty}^{V_T} f(V_0/S_1)\, dV_0 \\ \text{and } P(E/S_2 \text{ send}) &= \int_{V_T}^{\infty} f(V_0/S_2)\, dV_0 \end{aligned}\right\} \quad (1)$$

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Here we consider the mean value of $f(v_0/s_1)$ and $f(v_0/s_2)$ are S_{01} and S_{02} respectively.

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Now, $f(v_0/s_1) = \frac{1}{\sqrt{2\pi\sigma_n^2}} e^{-\frac{(v_0 - S_{01})^2}{2\sigma_n^2}}$

where $\sigma_n^2 = n^2(t)$ meansquare value

$$f(v_0/s_2) = \frac{1}{\sqrt{2\pi\sigma_n^2}} e^{-\frac{(v_0 - S_{02})^2}{2\sigma_n^2}}$$

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$$\therefore P_e = \frac{1}{2} \int_{-\infty}^{V_T} \frac{1}{\sqrt{2\pi\sigma_n^2}} e^{-\frac{(v_0 - S_{01})^2}{2\sigma_n^2}} dv_0 + \frac{1}{2} \int_{V_T}^{\infty} \frac{1}{\sqrt{2\pi\sigma_n^2}} e^{-\frac{(v_0 - S_{02})^2}{2\sigma_n^2}} dv_0 \quad \text{— (2)}$$

Let $x = -\frac{(v_0 - S_{01})}{\sigma_n}$ in first integral component

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and $x = \frac{v_0 - S_{02}}{\sigma_n}$ in 2nd " "

$$P_e = \frac{1}{2} \int_{-\frac{(V_T - S_{01})}{\sigma_n}}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} dx + \frac{1}{2} \int_{\frac{V_T - S_{02}}{\sigma_n}}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} dx$$

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$$= \frac{1}{2} Q\left(\frac{-V_T + S_{01}}{\sigma_n}\right) + \frac{1}{2} Q\left(\frac{V_T - S_{02}}{\sigma_n}\right) \quad \text{— (3)}$$

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S	M	T	W	T		
			1	2	3	4
5	6	7	8	9	10	11
12	13	14	15	16	17	18
19	20	21	22	23	24	25
26	27	28	29	30	31	

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Now for appropriate value of V_T Pe can be minimizied,

So, $$\frac{dP_e}{dV_T} = 0.$$

$$\frac{dP_e}{dV_T} = \frac{1}{2}\frac{1}{\sqrt{2\pi}\sigma_n} e^{-\frac{(V_T - S_{01})^2}{2\sigma_n^2}} + \frac{1}{2}\frac{1}{\sqrt{2\pi}\sigma_0} e^{-\frac{(V_T-S_{02})^2}{2\sigma_n^2}} = 1$$

$$\Rightarrow e^{-\frac{(V_T - S_{01})^2}{2\sigma_n^2}} = e^{-\frac{(V_T - S_{02})^2}{2\sigma_n^2}}$$

$$\Rightarrow (V_T - S_{01})^2 = (V_T - S_{02})^2$$

$$\Rightarrow \boxed{V_T = \frac{S_{01} + S_{02}}{2}}$$

Now Put V_T in eq(3)

∴ Probability of error

$$\boxed{P_e = Q\left(\frac{S_{01} - S_{02}}{2\sigma_n}\right) = \frac{1}{2}\operatorname{erfc}\left(\frac{S_{01} - S_{02}}{2\sqrt{2}\sigma_n}\right)}$$

So, Minimization of Pe is Possible if we maximize $\frac{S_{01} - S_{02}}{\sigma_n}$, from here we also calculate maximum SNR = $\frac{(S_{01} - S_{02})^2}{\sigma_n^2}$

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S	M	T	W	T	F	S
30						1
2	3	4	5	6	7	8
9	10	11	12	13	14	15
16	17	18	19	20	21	22
23	24	25	26	27	28	29

For PSD → Power per unit area.
PSD → square of signal in freq domain.

Transfer function and SNR of optimum filter

By maximize $\frac{s_{01}-s_{02}}{\sigma_n}$ we can reduce P_e.

From ~~analog~~ communication point of view, we know SNR is ratio of power signal to noise power.

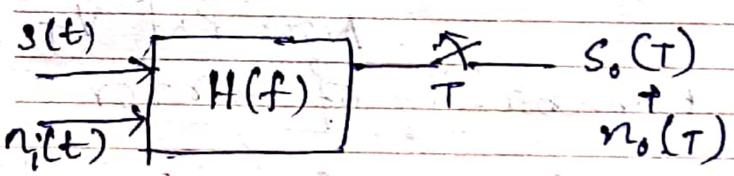
$$(SNR)_o = \frac{s_o^2(T)}{\sigma_n^2}$$

where $s_o(T) = s_{01}(T) - s_{02}(T)$.

here we assume then input the filter

$$s(t) = s_{01}(t) - s_{02}(t)$$

$s(t) \leftrightarrow S(f)$
$s_o(T) \leftrightarrow S_o(f)$



As we know, $S_o(f) = H(f)S(f)$.

Now, $s_o(T) = \int_{-\infty}^{\infty} S_o(f)\, H(f)\, e^{j\omega T}\, df$ — (4)
IFT.,

Let PSD of $n_i(t) = S_{ni}(f)$
" " $n_o(t)$ $S_{no}(f)$

$S_o^2(f)$ called PSD

$$\therefore S_{no}(f) = H^2(f)\, S_{ni}(f)$$

Power of o/p noise, $\sigma_n^2 = \int_{-\infty}^{\infty} H^2(f)\, S_{ni}(f)\, df$ — (5)

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So,
$$\frac{S_0^2(T)}{\sigma_n^2} = \frac{\left|\int_{-\alpha}^{\alpha} H(f)\, S_o(f)\, e^{j\omega T} df\right|^2}{\int_{-\alpha}^{\alpha} H^2(f)\, S_{ni}(f)\, df}$$

Apply Schwarz inequity.

$$\boxed{\begin{aligned}&\left|\int_{-\alpha}^{\alpha} A(\omega)\, B(\omega)\right|^2 \leq \int_{-\alpha}^{\alpha} |A(\omega)|^2 d\omega \int_{-\alpha}^{\alpha} |B(\omega)|^2 d\omega\\ &\text{this will obtain only when } A(\omega) = k\, B^*(\omega).\\ &\qquad k \text{ arbitary real non zero.}\end{aligned}}$$

then,
$$\frac{S_0^2(T)}{\sigma_n^2} \leq \frac{\int_{-\alpha}^{\alpha} |H(f)|^2 S_{ni}(f)\, df \int_{-\alpha}^{\alpha} \frac{|S_o(f)|^2}{S_{ni}(f)} df}{\int_{-\alpha}^{\alpha} |H(f)|^2 S_{ni}(f)\, df}$$

where $A(f) = H(f)\sqrt{S_{ni}(f)}$

$B(f) = \frac{S_o(f)}{\sqrt{S_{ni}(f)}} e^{j\omega T}$

$$\boxed{SNR = \frac{S_0^2(T)}{\sigma_n^2} \leq \int_{-\alpha}^{\alpha} \frac{|S_o(f)|^2}{S_{ni}(f)} df.} \quad (6)$$

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Now to get the maximum value of SNR, we have to choose H(f), in such ~~wave~~ way, where inequallity remain same,

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$$H(f)\sqrt{S_{ni}(f)} = K\frac{S^*(f)}{\sqrt{S_{ni}(f)}}e^{-j\omega T}$$

$$\Rightarrow \boxed{H(f) = K.\frac{S^*(f)}{S_{ni}(f)}e^{-j\omega T}} \quad (7)$$

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