Q Orthogonality b/w Signal

Let us define concept of orthogonality of two signal $f_1(t)$ and $f_2(t)$. We can approximate $f_1(t)$ in terms of $f_2(t)$ as

$$f_1(t) = C_{12} f_2(t) + f_e(t) \quad t_1 < t \leq t_2$$

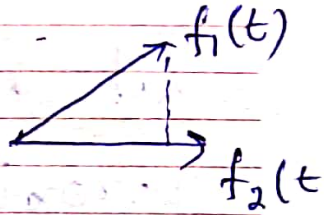
where error $\Rightarrow$ $f_e(t) = f_1(t) - C_{12} f_2(t)$

to minimize the error $f_e(t)$ is integrating over t_1 to t_2, of mean square error.

$$\mathcal{E} = \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} [f_e(t)]^2 \, dt.$$

where C_{12} is the value which minimize the error.

$$\frac{d\mathcal{E}}{dC_{12}} = 0$$



$$\Rightarrow C_{12} = \frac{\int_{t_1}^{t_2} f_1(t) f_2(t) \, dt}{\int_{t_1}^{t_2} f_2^2(t) \, dt}$$

if $C_{12} = 0$ then two signal are orthogonal,

$$\int_{t_1}^{t_2} f_1(t) f_2(t) \, dt = 0$$

~~energy of signal vector and signal~~.

Orthogonal signal space

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Let, $x_1(t), x_2(t) \ldots\ldots x_n(t)$ mutually orthogonal signal..

$$\int_{t_1}^{t_2} x_J(t)\, x_K(t)\, dt = 0 \text{ when } J \neq K$$

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and

$$\int_{t_1}^{t_2} x_K^2(t) = K_K \quad J = K$$

Let a function $f(t)$ can be approximated by ~~the~~ this orthogonal signal space by adding the components along mutually orthogonal signals.

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$$f(t) = C_1 x_1(t) + C_2 x_2(t) + \ldots\ldots + C_n x_n(t) + f_e(t)$$

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$$\therefore f_e(t) = f(t) - \sum_{r=1}^{n} C_r x_r(t)$$

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Notes:

Mean square error $\varepsilon = \dfrac{1}{t_2 - t_1} \displaystyle\int_{t_1}^{t_2} [f_e(t)]^2\, dt$

*

* The component which minimize the mean square error can be fond by,

$$\frac{d\epsilon}{dc_1} = \frac{d\epsilon}{dc_2} = \cdots\cdots = \frac{d\epsilon}{dc_k} = 0$$

Let us conside $\frac{d\epsilon}{dc_k} = 0$.

as all term do no contain c_k is zero.

$$\Rightarrow c_k = \frac{\int_{t_1}^{t_2} f(t)\, x_k(t)}{\int_{t_1}^{t_2} x_k^2(t)}$$

$\therefore$ if $c_k = 0$, $\int_{t_1}^{t_2} f(t)\, x_k(t) = 0$

$K = 1, 2, \ldots$

* $f(t)$ is should be orthogonal to every $x_k(t)$, then it called Orthogonal space

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Monday

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orthogonal signal $= \int_0^t \varphi_1(t)\,\varphi_2(t) = 0$

orthonormal signal $= \int_0^t \varphi_1^2(t)\,dt = 1$

Basically energy of a signal unity.

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Geometrical representation of Signal.

Consider M no. of Possible Tx signals
↳ $x_1(t), x_2(t) \text{ - - - - } x_M(t)$

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N no of orthonormal basis function.
$\varphi_j(t)$

$N \leq M$.

We can represent any signal $x_i(t)$

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as linear combination of orthonormal basis fn:

$\therefore\; x_i(t) = x_{i1}\varphi_1(t) + x_{i2}\varphi_2(t) + \cdots + x_{iM}\varphi_n(t)$

$\therefore\; x_i(t) = \sum_{j=1}^{N} x_{ij}\,\varphi_j(t) \quad i = 1, 2, \cdots M$

↓
Transmitted Signal.

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Notes:

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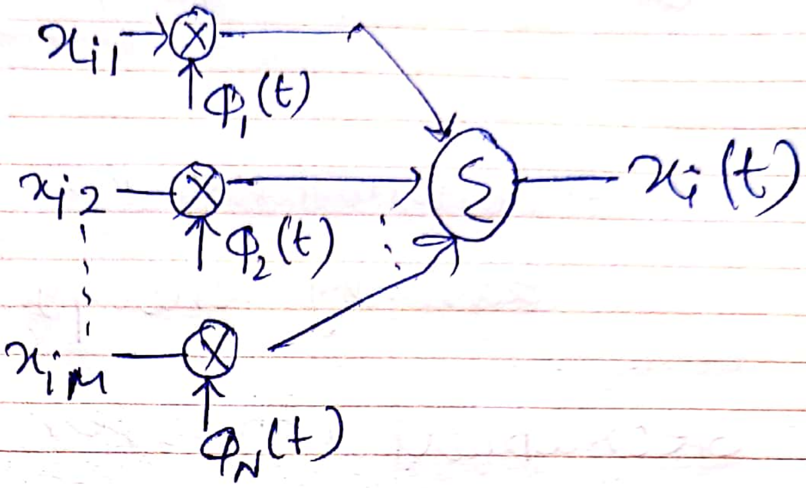
Transmitter



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Receiver:

Now the received $x_i(t)$ multiply $\phi_k(t)$ and pass this through integrator

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$$\int_0^T x_i(t)\,\phi_k(t)$$

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$$\sum_{j=1} x_{ij} \int_0^T \phi_k(t)\,\phi_j(t)$$

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Now if $j = k$, otherwise $= 0$

this become x_{ik}

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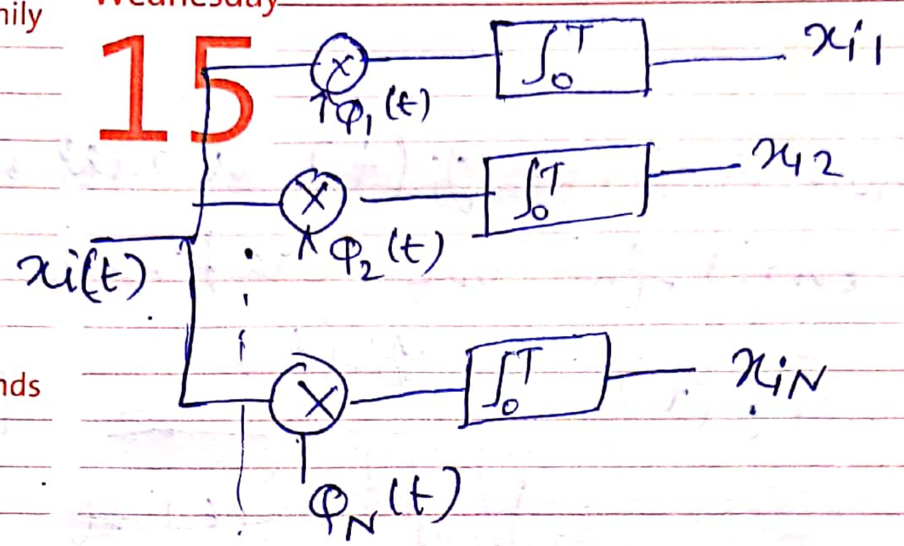
So,

$$\boxed{x_{ik} = \int_0^T x_i(t)\,\phi_k(t)\,dt}$$

Notes:



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Gram - schmidt Orthogonalization Procedure

To find how many no. ~~orthogonal~~ orthonormal basis fun. is required.

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Consider set of M energy signal $x_1(t)$; $x_2(t)$ ----- $x_M(t)$. all real.

E = find energy of $x_1(t) = \int_0^T x_1^2(t)\,dt$

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~~we assume~~ The first basis fun: $\varphi_1(t) = \dfrac{x_1(t)}{\sqrt{E}}$

normalised energy signal $x_1(t)$ as orthonormal sign energy 1

For defining othe basis fn., a new intermideate function

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is ~~introduced~~,

Notes:

$$g_i(t) = x_i(t) - \sum_{J=1}^{i-1} x_{iJ}\,\varphi_J(t)$$

$i = 1, \ldots M$

where $x_{iJ} = \int_0^T x_i(t)\,\varphi_J(t)\,dt$.

J is no. basis fun

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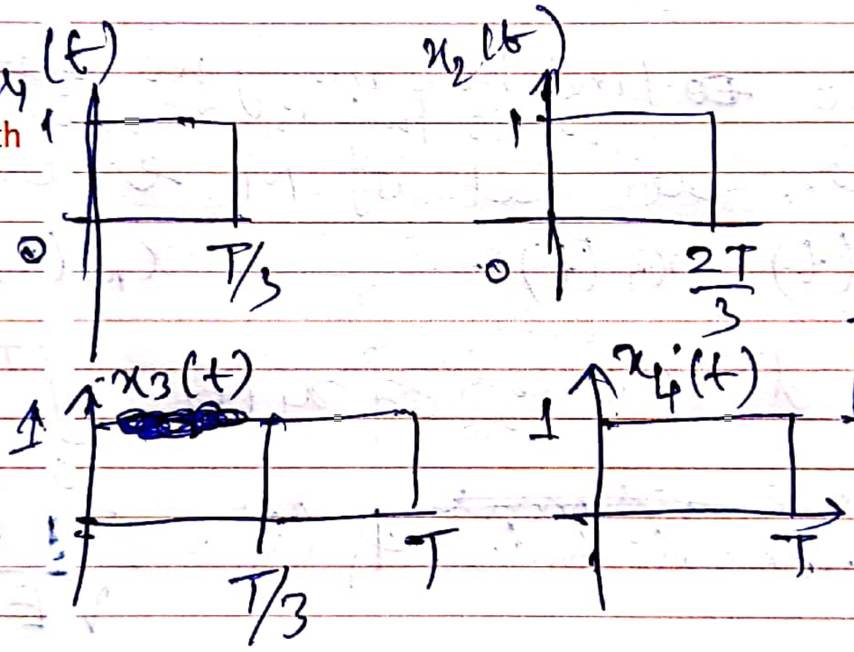
from a given $g_i(t)$ a real set of basis fn. are defined

$$\boxed{\phi_i(t) = \frac{g_i(t)}{\sqrt{\int_0^T g_i^2(t)\,dt}}} \qquad i = 1, 2 \cdots N$$

Prob

find orthonormal basis fn.



$M = 4$

$\boxed{N \leq M}$

$N = 3$ or 4

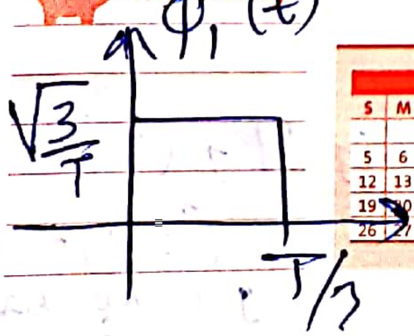
Sol :

$$\phi_1(t) = \frac{x_1(t)}{\sqrt{E_1}} \qquad E_1 = \int_0^{T/3},$$

$$= \frac{T}{3}$$

$$= \frac{x_1(t)}{\sqrt{T/3}}$$

$$= \sqrt{\frac{3}{T}}\, x_1(t)$$



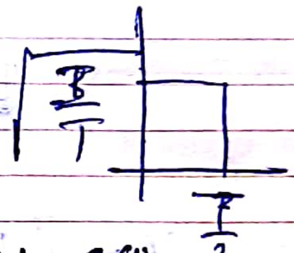
we know

$$g_i(t) = x_i(t) - \sum_{j=1}^{i-1} x_{ij}\,\phi_j(t)$$

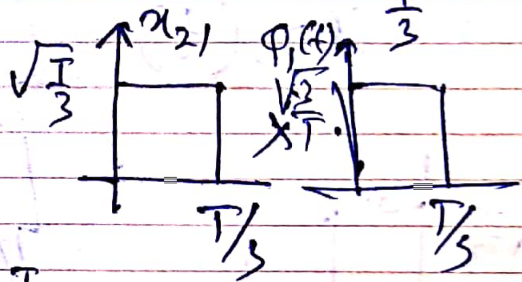
for, $i=2$, $g_2(t) = x_2(t) - x_{21}\,\phi_1(t)$

$$x_{21} = \int_0^T x_2(t)\,\phi_1(t)\,dt$$

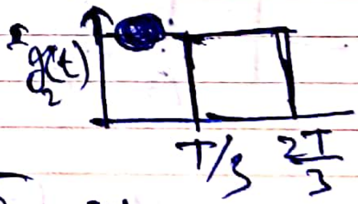
$$= \int_0^{T/3} \sqrt{\frac{3}{T}}\,dt$$



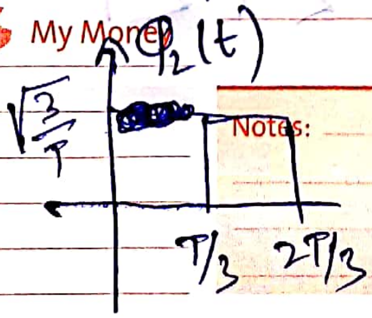
$$= \sqrt{\frac{T}{3}}$$



$g_2(t) = 1 \quad 0 \le t < \frac{T}{3}$



$$\phi_2(t) = \frac{g_2(t)}{\sqrt{\int_{T/3}^{2T/3} 1\,dt}} = \sqrt{\frac{3}{T}}\,g_2(t) \quad \frac{T}{3} \le t \le \frac{2T}{3}$$



Notes:

From here $\phi_1(t)$ & $\phi_2(t)$ are orthogonal

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$i = 3$

$$g_3(t) = x_3(t) - \sum_{J=1}^{2} x_{3J}\,\varphi_J(t)$$

$$= g_3(t) = x_3(t) - x_{31}\varphi_1(t) - x_{32}\varphi_2(t)$$

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$$x_{31} = \int_0^T x_3(t)\,\varphi_1(t) = 0$$

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$$x_{32}(t) = \int_0^T x_3(t)\,\varphi_2(t)$$

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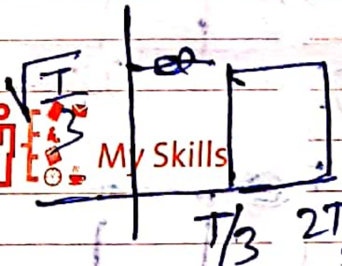

$$= \int_{T/3}^{2T/3} \sqrt{\frac{3}{T}}\,dt$$

$$= \sqrt{\frac{T}{3}} \qquad T/3 \le t < \frac{2T}{3}$$

$$x_{32}(t)\,\varphi_2(t) = 1 \qquad T/3 \le t < \frac{2T}{3}$$

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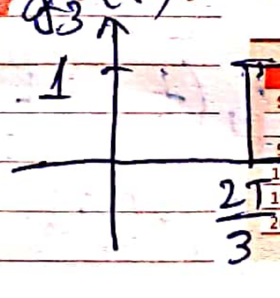
$$g_3(t) = 1 \qquad \frac{2T}{3} < t < T$$



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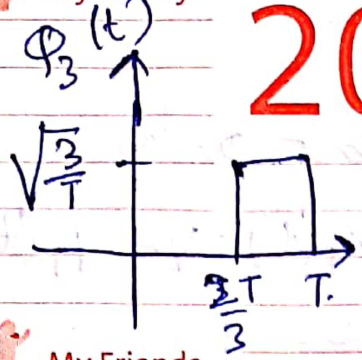

$$\varphi_3(t) = \frac{g_3(t)}{\sqrt{\int_{\frac{2T}{3}}^{T} \varphi_3 \, dt}} = \sqrt{\frac{3}{T}}\, g_3(t)$$

Now for, $i = 4$, $g_4(t) = x_4(t) - \sum_{J=1}^{3} x_{4J}\varphi_J(t)$

$\Rightarrow g_4(t) = x_4(t) - x_{41}\varphi_1(t) - x_{42}\varphi_2(t) - x_{43}\varphi_3(t)$

$0 < t < T_3$ $\quad x_{41} = 0 = \int_0^{T/3} \cancel{x_4(t)\varphi_1(t)}\, dt = \sqrt{\frac{T}{3}}$

$T/3 < t < 2T/3$ $\quad x_{42} = 0 = \int_{T/3}^{2T/3} x_4(t)\,\varphi_2(t)\, dt = \sqrt{\frac{T}{3}}$

$\frac{2T}{3} < t < T$ $\quad x_{43} = \int_{2T/3}^{T/3} x_4(t)\,\varphi_3(t) = \sqrt{\frac{T}{3}}$

Now, $x_{41}\varphi_1(t) = 1 \quad 0 < t < T/3$

$x_{42}\varphi_2(t) = 1 \quad T/3 < t < 2T/3$

$x_{43}\varphi_3(t) \; \cancel{x_{43}\varphi_3(t)} = 1 \quad \frac{2T}{3} < t < T$

$g_4(t) = 0 \quad 0 < t < T.$

$\varphi_4(t) = 0$ no orthonormal fn!

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Tuesday

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So, 1st signal may represent only one basis function, $x_1(t) = \sqrt{\frac{T}{3}}\,\phi_1(t)$

2nd signal may represen two basis fun. and so on.

$$x_2(t) = \sqrt{\frac{T}{3}}\,\phi_1(t) + \sqrt{\frac{T}{3}}\,\phi_2(t)$$

$$x_3(t) = \sqrt{\frac{T}{3}}\,\phi_2(t) + \sqrt{\frac{T}{3}}\,\phi_3(t)$$

$$x_4(t) = \sqrt{\frac{T}{3}}\,\phi_1(t) + \sqrt{\frac{T}{3}}\,\phi_2(t) + \sqrt{\frac{T}{3}}\,\phi_3(t)$$