

Frequency Shift Keying (FSK)

In binary FSK, the frequency of the carrier is shifted according to binary symbol. The symbols 1 and 0 are distinguished from each other by transmitting one of two sinusoidal waves that differ in frequency by a fixed amount, with phase is constant.

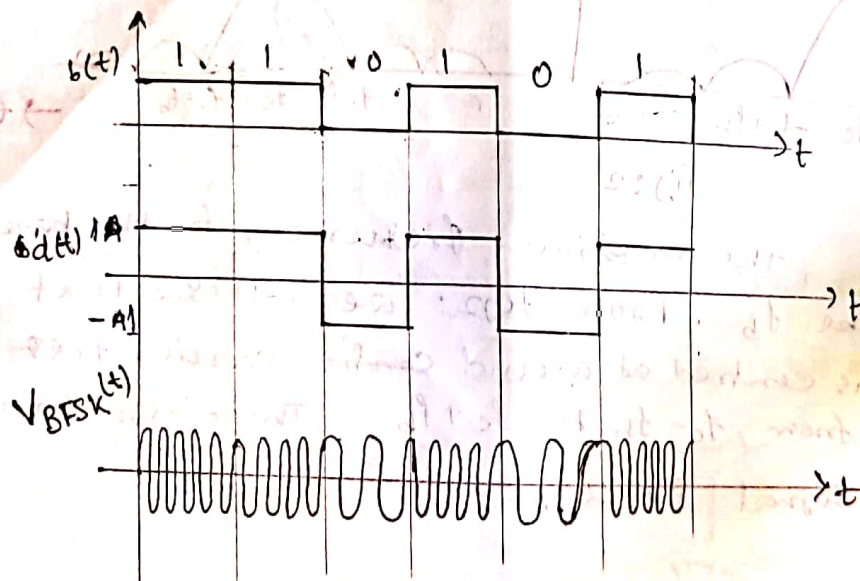
In binary FSK the binary data waveform $d(t)$ generates a binary signal

$$V_{\text{BFSK}}(t) = \sqrt{2P_s} \cos(\omega_c t + d(t)\Omega t) \text{ here}$$

$d(t) = +1$ or -1 corresponding to the logic levels 1 and 0 of the data waveform.

$$\begin{aligned} \text{Thus } S_H(t) &= \sqrt{2P_s} \cos(\omega_c + \Omega)t = \sqrt{P_s T_b} \sqrt{\frac{2}{T_b}} \cos 2\pi f_H t \\ S_L(t) &= \sqrt{2P_s} \cos(\omega_c - \Omega)t = \sqrt{P_s T_b} \sqrt{\frac{2}{T_b}} \cos 2\pi f_L t \end{aligned}$$

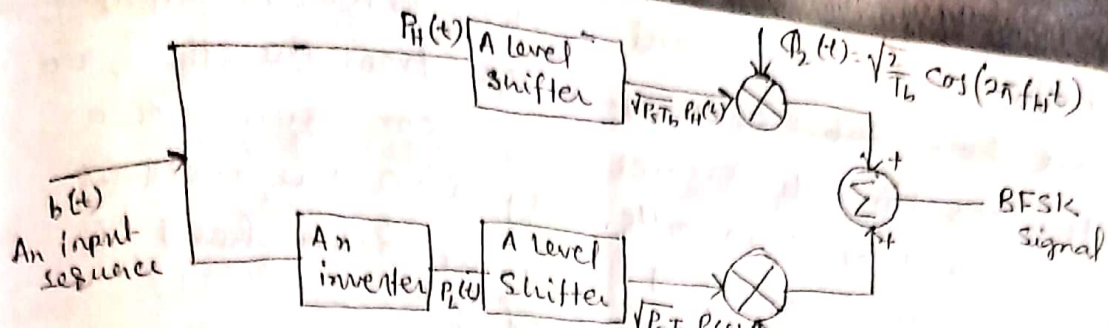
Hence if symbol '1' is to be transmitted, the carrier freq will be: $f_c + \frac{\Omega}{2\pi}$ and if '0' is to be transmitted the carrier freq will be $f_c - \frac{\Omega}{2\pi}$



Generation of BFSK Signal

So for the generation BFSK signal, two balanced modulators are used, one with carrier ω_H and one with carrier ω_L . The voltage values of $P_H(t)$ and $P_L(t)$ are related to the voltage values of $d(t)$ in the following manner.

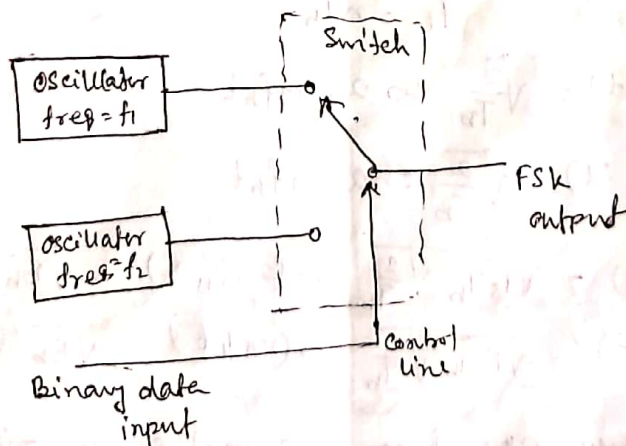
$b(t)$ input	$d(t)$	$P_H(t)$	$P_L(t)$
1	+1 V	+1 V	0 V
0	-1 V	0 V	+1 V



From table we see that the input sequence $b(t)$ is same as $P_H(t)$. An inverter is added after $b(t)$ to get $P_L(t)$. The level shifter converts the '+1' level to $\sqrt{P_s T_b}$ and 0 or is unaffected. Then two ^{orthogonal} carrier $\Phi_1(t)$ and $\Phi_2(t)$ signal are used. Thus the modulated signal having continuous phase. The adder then adds the two signals. The two frequencies f_L and f_H are chosen to equal integer multiples of the bit rate $1/T_b$.

Reception of BPSK [Switching technique the transmitter output line between two adjacent oscillator. It is called discontinuous phase FSK, because $\theta(t)$ phase is discontinuous at the switching time.]

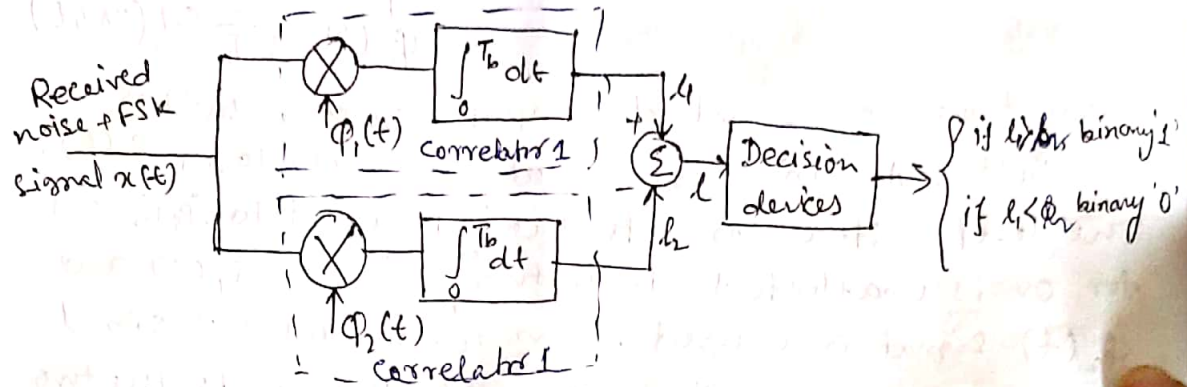
$$S(t) = A_c \cos(\omega_1 t + \theta_1) \quad 0 \leq t \leq T_b \text{ for binary 1} \\ = A_c \cos(\omega_2 t + \theta_2) \quad 0 \leq t \leq T_b \text{ for binary 0}$$



FSK Demodulation:

In order to detect the original binary sequence given the noisy received wave $x(t)$, we may use the following receiver. The detector consists of two correlators that are tuned to two different carrier frequencies with a common input, which are supplied with locally generated coherent reference signal $\Phi_1(t)$ and $\Phi_2(t)$.

with a common input. The correlator outputs are then subtracted one from the other, and the resulting difference l is compared with a threshold of zero volts. If $l > 0$ the receiver decides in favor of 1. on the other hand, if $l < 0$ it decides in favor of 0.



Geometrical Representation of orthogonal BFSK

In BFSK we use two ϕ orthogonal carrier $\Phi_1(t)$ and $\Phi_2(t)$ of two different frequencies f_H and f_L are used for modulation. To make $\Phi_1(t)$ and $\Phi_2(t)$ orthogonal the frequencies f_H and f_L must be some integer multiple of baseband frequency f_b .

$$\text{Thus } f_H = m f_b$$

$$f_L = n f_b$$

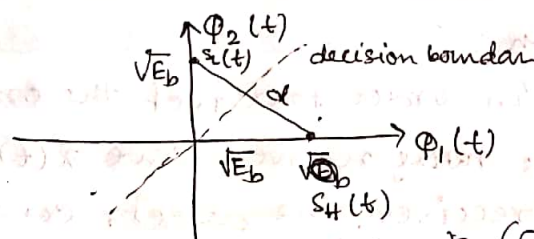
$$\therefore \Phi_1(t) = \sqrt{\frac{2}{T_b}} \cos 2\pi m f_b t$$

$$\Phi_2(t) = \sqrt{\frac{2}{T_b}} \cos 2\pi n f_b t$$

$$\therefore S_H(t) = \sqrt{P_s T_b} \sqrt{\frac{2}{T_b}} \cos(2\pi f_H t) = \sqrt{E_b} \Phi_1(t)$$

$$S_L(t) = \sqrt{P_s T_b} \sqrt{\frac{2}{T_b}} \cos(2\pi f_L t) = \sqrt{E_b} \Phi_2(t)$$

based on above equation signal space diagram is



distance b/w signal points, $d^2 = (\sqrt{E_b})^2 + (\sqrt{E_b})^2 = 2E_b$

$$\therefore d = \sqrt{2E_b}$$

Spectrum of BFSK

The BFSK signal $V_{BFSK}(t)$ may be written as

$$V_{BFSK}(t) = \sqrt{2P_s} P_H(t) \cos(2\pi f_H t) + \sqrt{2P_s} P_L(t) \cos(2\pi f_L t)$$

now we know the BPSK signal is $V_{BPSK}(t) = b(t) \sqrt{2P_s} \cos(2\pi f_c t)$ where $b(t)$ is a bipolar signal, while in BFSK signal $P_H(t)$ or $P_L(t)$ are unipolar. now we convert these value in bipolar variable, that is

$$P_H(t) = \frac{1}{2} + \frac{1}{2} P_H'(t)$$

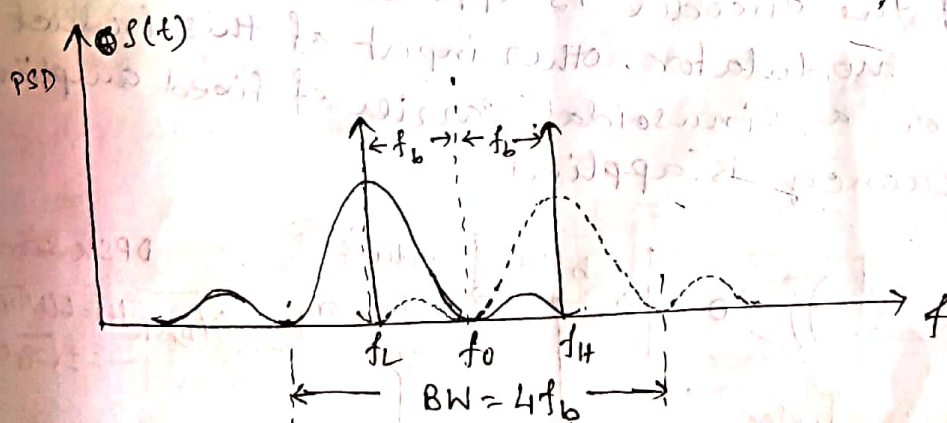
$$P_L(t) = \frac{1}{2} - \frac{1}{2} P_L'(t) \text{ where } P_H'(t) \text{ and } P_L'(t)$$

are bipolar

$$\text{So, } V_{BFSK}(t) = \sqrt{\frac{P_s}{2}} \cos(2\pi f_H t) + \sqrt{\frac{P_s}{2}} P_H'(t) \cos(2\pi f_H t) + \sqrt{\frac{P_s}{2}} \cos(2\pi f_L t) - \sqrt{\frac{P_s}{2}} P_L'(t) \cos(2\pi f_L t)$$

The first term and third term produce a Power Spectral density which of two impulses one at f_H and one at f_L . The second and last term produce the spectrum of two binary PSK signal at centered about f_H and f_L .

$$S(f) = \sqrt{\frac{P_s}{2}} \left[\delta(f - f_H) + \delta(f - f_L) \right] + \frac{P_s T_b}{2} \left(\frac{\sin \pi f_H T_b}{\pi f_H T_b} \right)^2 + \frac{P_s T_b}{2} \left(\frac{\sin \pi f_L T_b}{\pi f_L T_b} \right)^2$$



∴ The Bandwidth of BFSK signal is $= 4f_b$ which is twice the bandwidth of BPSK

$$P_e = \frac{1}{2} \exp\left(-\frac{E_b}{2N_0}\right)$$