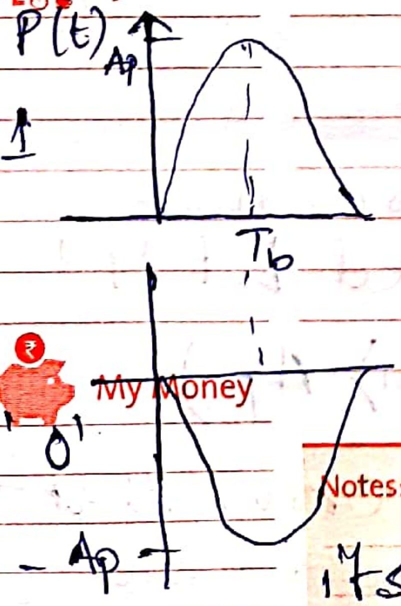
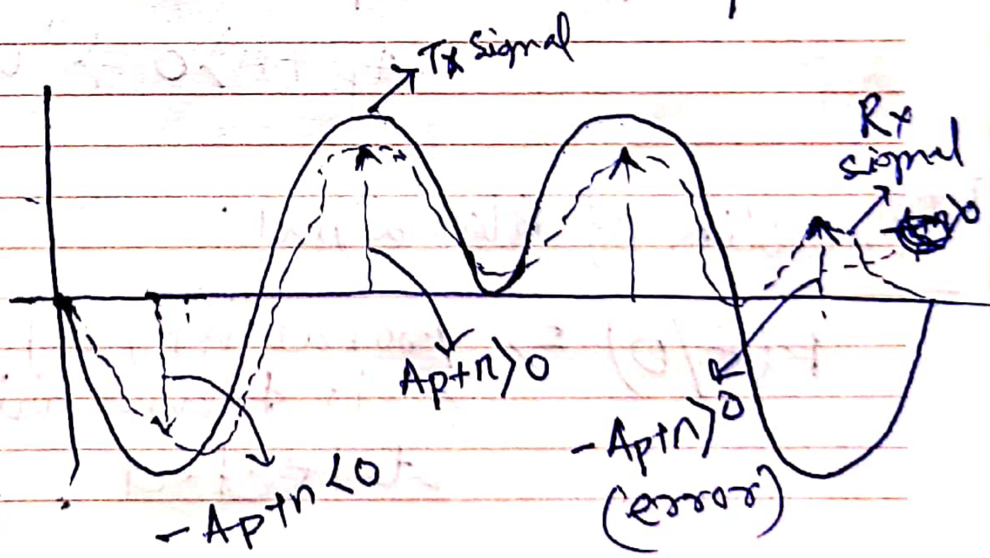
Error Probability: Let us consider a Polar Signal is transmitted over noisy channel. The Polar Signal $P(t)$ have Peak amplitude $\pm A_p$ for '1' and '0' respectively at sampling instant. Now due to noise at receiver peak amplitude of Pulse $\pm A_p + n$ at sampling instant where n is random noise amplitude $(-\alpha, \alpha)$.



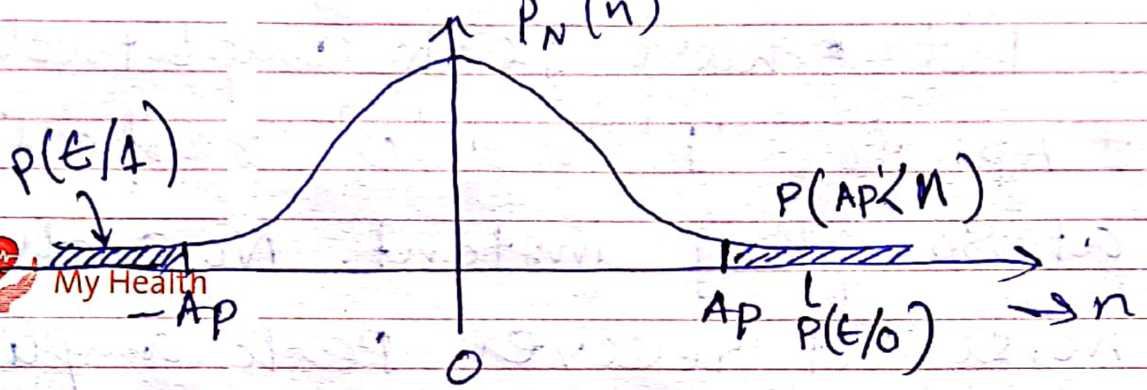


Notes: As noise is random variable its Probability density function PDF

$$P_N(n) = \frac{1}{\sqrt{2\pi\sigma_n^2}} e^{-\frac{(n-\mu)^2}{2\sigma_n^2}}$$

where N random variable
n = value taken of N
μ = mean
σ_n^2 = variance

Now if we consider $\mu = 0$ mean.
then PDF, of noise.



Now, the detection error will occur if,
$+Ap + n < 0 \Rightarrow$ '1' will detect as 0
$-Ap + n > 0 \Rightarrow$ '0' will detect as 1

Error Probablity of Polar Signal:

$P(\epsilon/0) \Rightarrow$ probability of error that 0 is transmitted but '1' detected $= P(n > Ap)$

$P(\epsilon/1) \Rightarrow$ Probability of error that '1' is transmitted but '0' detected $= P(n < -Ap)$

My Family

So, $P(\epsilon/0)$ is shaded area at $n > A_p$

$$= \int_{A_p}^{\alpha} \frac{1}{\sqrt{2\pi\sigma_n^2}} e^{-\frac{n^2}{2\sigma_n^2}} \, dn \quad — (1)$$

My Friends

and $P(\epsilon/1)$ is shaded area at $n < -A_p$

$$= \int_{-\alpha}^{-A_p} \frac{1}{\sqrt{2\pi\sigma_n^2}} e^{-\frac{n^2}{2\sigma_n^2}} \, dn \quad — (2)$$

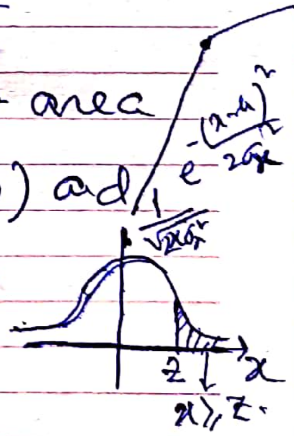
My Health

Now we define Q-function & Complementary error function

My Skills

Q-function is defined as area of +ve tail $(x \geq z)$ of zero mean ($\mu = 0$) and unit variance ($\sigma_n^2 = 1$)

$$Q(z) = \int_z^{\alpha} \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} \, dz$$



My Money

Notes:

March 2017

S	M	T	W	T	F	S
			1	2	3	4
5	6	7	8	9	10	11
12	13	14	15	16	17	18
19	20	21	22	23	24	25
26	27	28	29	30	31	

$$Q(z) = \frac{1}{\sqrt{2\pi}} \int_z^{\infty} e^{-\frac{x^2}{2}}\, dx \quad -(3)$$

Saturday

25

My Family

Complementary error function :

$$\operatorname{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_x^{\infty} e^{-u^2}\, du \quad (4)$$

My Friends

Relation between eq 3 & eq 4

at eq (4)

$$u = \frac{u_1}{\sqrt{2}}$$

$$du = \frac{1}{\sqrt{2}}\, du_1$$

$$\operatorname{erfc}(x) = \frac{\sqrt{2}}{\sqrt{\pi}} \int_{\sqrt{2}x}^{\infty} e^{-\frac{u_1^2}{2}}\, du_1$$

My Health

Sunday

26

$$= 2 \cdot \frac{1}{\sqrt{2\pi}} \int_{\sqrt{2}x}^{\infty} e^{-\frac{u_1^2}{2}}\, du_1$$

So, $\operatorname{erfc}(x) = 2 \cdot Q(\sqrt{2}\, x)$

My Skills

so, $\boxed{Q(\sqrt{2}\, x) = \frac{1}{2} \operatorname{erfc}(x)}$

Similarly or, $\boxed{Q(x) = \frac{1}{2} \operatorname{erfc}\left(\frac{x}{\sqrt{2}}\right)}$

My Money

February 2017

S	M	T	W	T	F	S
			1	2	3	4
5	6	7	8	9	10	11
12	13	14	15	16	17	18
19	20	21	22	23	24	25
26	27	28				

Notes:

$-\sigma_n\, dx$.

February

ACC

Week 10 / Day 58

My Family

Monday

27

So, from eq (1) & (2)

Let $x = \frac{n}{\sigma_n}$ and $x = -\frac{n}{\sigma_n}$ respectively

My Friends

$$P(\epsilon/0) = \frac{1}{\sqrt{2\pi}} \int_{\frac{A_p}{\sigma_n}}^{\infty} e^{-\frac{x^2}{2}}\, dx = Q\left(\frac{A_p}{\sigma_n}\right)$$

Similarly

$$P(\epsilon/1) = \frac{1}{\sqrt{2\pi}} \int_{\frac{A_p}{\sigma_n}}^{\infty} e^{-\frac{x^2}{2}}\, dx = Q\left(\frac{A_p}{\sigma_n}\right)$$

My Health

$$-\int_a^b = \int_b^a$$

∴ The average error Probability for Polar signal.

My Skills

$$P_e = P(1)\, P(\epsilon/1) + P(0)\, P(\epsilon/0)$$

$$= Q\left(\frac{A_p}{\sigma_n}\right) = \frac{1}{2}\operatorname{erfc}\left(\frac{A_p/\sigma_n}{\sqrt{2}}\right)$$

My Money

Notes:

March 2017						
S	M	T	W	T	F	S
			1	2	3	4
5	6	7	8	9	10	11
12	13	14	15	16	17	18
19	20	21	22	23	24	25
26	27	28	29	30	31	