# Binary Phase Shift Keying (BPSK)

In a coherent binary PSK system, the Pair of signals $\phi_1(t)$ and $\phi_2(t)$, has fixed amplitude, it has one fixed phase when the data is binary symbols 1 and when the data is binary symbol '0' ~~signal~~ other signal has the phase is shifted by 180°. Let assume that the, A is the peak amplitude of carrier signal $S_1(t) = A\cos(2\pi f_c t)$

the Power of the signal $P_s = \frac{1}{2}A^2$, so that, $A = \sqrt{2P_s}$.

$E_s = \frac{1}{2}A^2T$

Let us consider transmitted signal for symbol '1'

$$\phi_1(t) = \sqrt{2P_s}\cos(2\pi f_c t)$$

and for symbol '0'

$$\phi_2(t) = \sqrt{2P_s}\cos(2\pi f_c t + \pi)$$

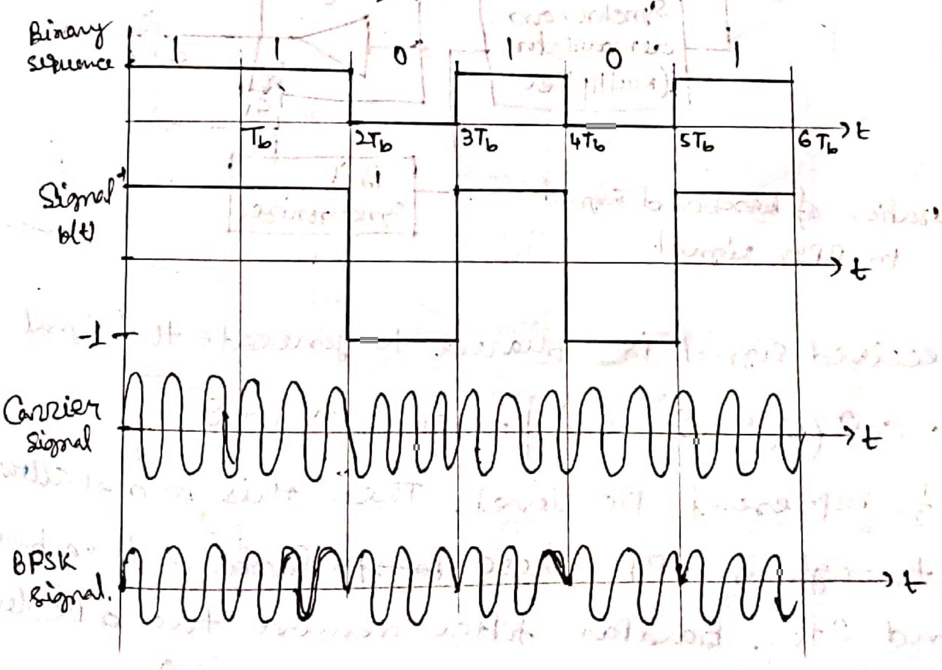
Now we know that $\cos(\theta + \pi) = -\cos\theta$

$$\therefore \phi_2(t) = -\sqrt{2P_s}\cos(2\pi f_c t)$$

Hence, from above equations we can define BPSK signal

$$V_{BPSK}(t) = b(t)\sqrt{2P_s}\cos(2\pi f_c t)$$

where $b(t) = +1$ when binary '1' is to be transmitted.
$= -1$ when binary '0' is to be transmitted.




**Generation of BPSK Signal**: In Practice, The BPSK signal may be generated by applying the carrier signal to a balanced modulator and applying the baseband signal b(t) as the Modulating waveform.

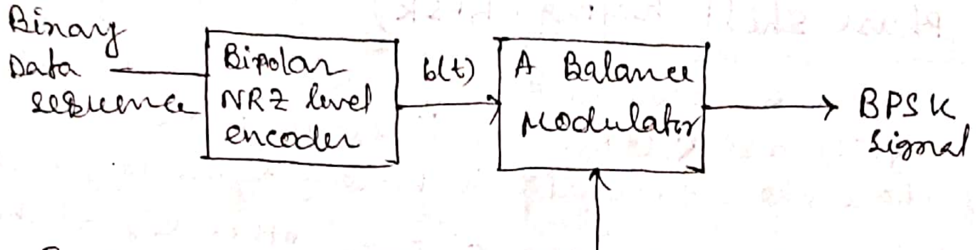



Fig: Generation of BPSK.

## Reception of BPSK :

The received signal has the form $V_{BPSK} = b(t)\sqrt{2P_s}\cos(\omega_c t + \theta)$, $\theta$ is the fixed phase shift depending upon the time delay from transmitter end to receiver end. The demodulation technique usually employed is called synchronous demodulation and hence a synchronous local carrier is necessary which is generated in synchronizing circuit.

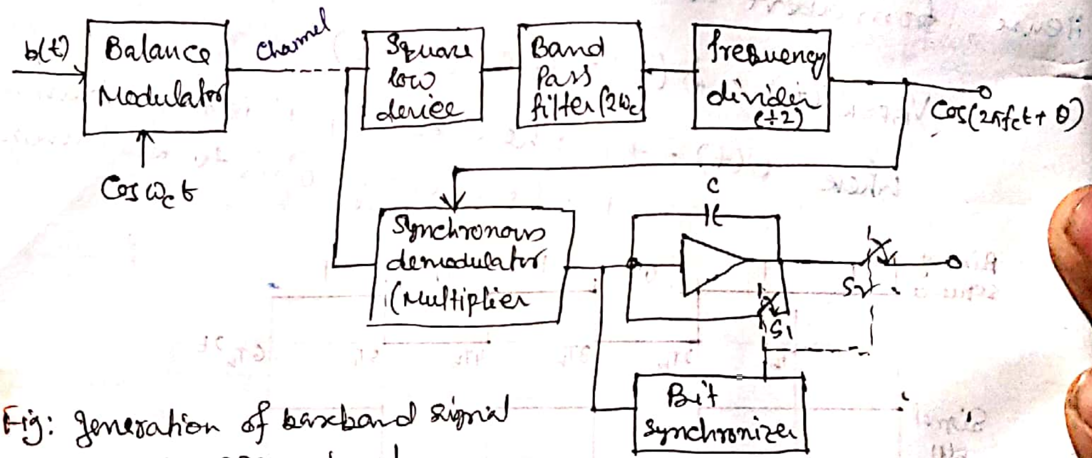



Fig: generation of baseband signal in BPSK signal.

The received signal is squared to generate the signal

$$\cos^2(\omega_c t + \theta) = \frac{1}{2}\left[1 + \cos 2(\omega_c t + \theta)\right].$$

Here $\frac{1}{2}$ represent DC level. Then this signal allowed to pass through a BPF whose ~~passba~~ passband is centred around $2f_c$.. Bandpass filter remove the DC level and at the output, we get $\frac{1}{2}\cos 2(\omega_c t + \theta)$.

This signal having frequency equal to $2f_c$. Hence it is passed through a frequency divider by two. Thus at the output of frequency divider, we get a carrier signal whose frequency $f_c$ i;e, $\cos(2\pi f_c t + \theta)$.

The synchronous demodulator is multiplied the input signal and the recovered carrier, at the output

$$b(t)\sqrt{2P_s}\cos^2(\omega_c t+\theta) = b(t)\sqrt{2P_s}\,\frac{1}{2}\left[1+\cos 2(2\pi f_c t+\theta)\right]$$

$$= b(t)\sqrt{\frac{P_s}{2}}\left[1+\cos 2(2\pi f_c t+\theta)\right]$$

This signal is then applied to the bit synchronizer and integrator. The bit synchronized takes care of starting and ending times of a bit. The integrator integrates the signal over one bit period. At the end of bit duration $T_b$, the bit synchronizer close $S_2$ temprarily. This connects the output of an integrator to the decision device.

~~At the~~ At the $k^{th}$ bit interval, we can write output signal.

$$V_o(kT_b) = b(kT_b)\sqrt{\frac{P_s}{2}}\int_{(k-1)T_b}^{kT_b}\left[1+\cos 2(2\pi f_c t+\theta)\right]dt$$

$$= b(kT_b)\sqrt{\frac{P_s}{2}}\int_{(k-1)T_b}^{kT_b} 1\,dt + b(kT_b)\sqrt{\frac{P_s}{2}}\int_{(k-1)T_b}^{kT_b}\cos 2(2\pi f_c t+\theta)\,dt$$

$$= b(kT_b)\sqrt{\frac{P_s}{2}}\left[kT_b-(k-1)T_b\right]$$

$$= b(kT_b)\sqrt{\frac{P_s}{2}}\,T_b.$$

Since the integral of a sinusoid over a whole number of complete cycle has the value zero

∴ from here we can write

$$v_o(kT_b) \propto b(kT_b)$$

This signal is then pass through decision device to determine whether transmitted ~~signal~~ symbol was '1' or '0'.

## Geometrical Representation of BPSK signals

We know that BPSK signal is expressed as

$$v_{BPSK}(t) = b(t)\sqrt{2P_s}\cos(2\pi f_c t)$$

$$= b(t)\sqrt{P_s T_b}\sqrt{\frac{2}{T_b}}\cos(2\pi f_c t)$$

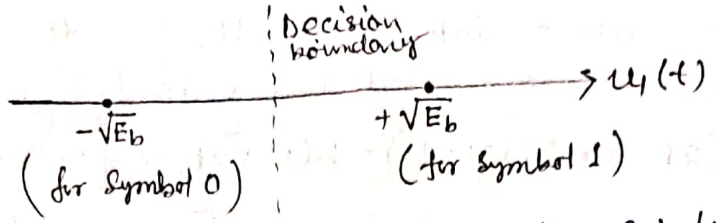
Let $u_1(t) = \frac{\sqrt{2}}{\sqrt{T_b}}\cos(2\pi f_c t)$ represent an orthonormal carrier signal.

$$V_{BPSK}(t) = b(t)\sqrt{P_s T_b}\,u_1(t)$$

The bit energy is $E_b$ is defined by $= P_s T_b$.

$$V_{BPSK} = \pm\sqrt{E_b}\,u_1(t) \qquad \text{when } b(t)=\pm 1$$

Thus one the single axis of $u_1(t)$, there will be two points at $+\sqrt{E_b}$ & at $-\sqrt{E_b}$.




The distance between these two Points is

$$d = 2\sqrt{E_b}$$

We know that the distance $d$ is inversely Proportional to the Probability that we make an error when, in the Presence of noise.

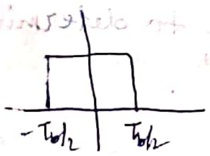
$$\left[ P_e = \frac{1}{2}\,\mathrm{erfc}\sqrt{\frac{E_b}{N_0}} = \frac{1}{2}\,\mathrm{erfc}\sqrt{\frac{d^2}{4N_0}} \right.$$

## The spectrum of BPSK Signal :

We know that $b(t)$ is a NRZ binary waveform there are rectangular Pulse of amplitude $\pm V_b$. If we assume that each Pulse is $\pm T_b/2$ ~~around~~ around its centre, then it is easier to find fourier transform of such Pulse. The Fourier transform of this of Pulse is given by

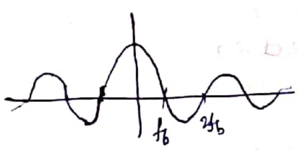
$$X(f) = V_b T_b \frac{\sin(\pi f T_b)}{(\pi f T_b)}$$

The Power spectral density of this Pulse



$$S(f) = \frac{\overline{|X(f)|^2}}{T_s} \quad T_s \text{ is bit duration.}$$

$$= \frac{V_b^2 T_b^2}{T_s}\left[\frac{\sin(\pi f T_b)}{\pi f T_b}\right]^2$$



$$= V_b^2 T_b \left[\frac{\sin(\pi f T_b)}{\pi f T_b}\right]^2 \quad \text{when } T_s = T_b$$

The BPSK signal generated by by modulating a carrier by the baseband signal $b(t)$. Due to modulation the carrier of frequency $f_c$, the spectral components are translated from $f_0$ to $f_c - f$ and $f_c + f$ and the magnitude of these components is divided by half.

therefore

$$S_{BPSK}(f) = V_b^2 T_b \left\{ \frac{1}{2}\left[\frac{\sin\pi(f_c-f)T_b}{\pi(f_c-f)T_b}\right]^2 + \frac{1}{2}\left[\frac{\sin\pi(f_c+f)T_b}{\pi(f_c+f)T_b}\right]^2 \right\}$$

we know that $V_b = \sqrt{P_s 2}$

for simplicity $= \sqrt{P_s}$

$$\therefore \quad S_{BPSK}(f) = \frac{P_s T_b}{2}\left[\left(\frac{\sin\pi(f_c-f_0)T_b}{\pi(f_c-f_0)T_b}\right)^2 + \left(\frac{\sin\pi(f_c+f)T_b}{\pi(f_c+f)T_b}\right)^2\right]$$

where as $S(f) = P_s T_b \left[\frac{\sin \pi f T_b}{\pi f T_b}\right]^2$

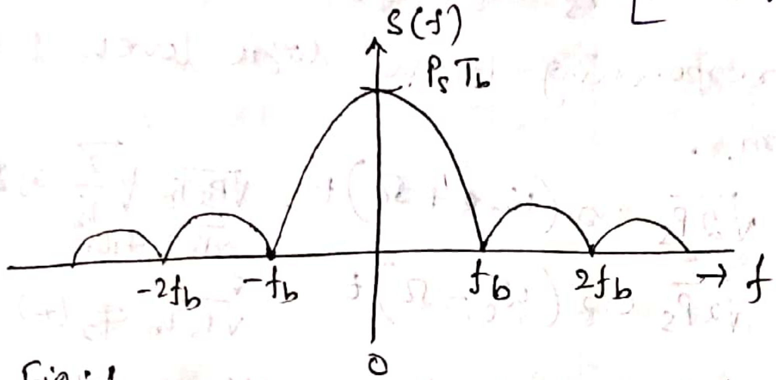



Fig:1

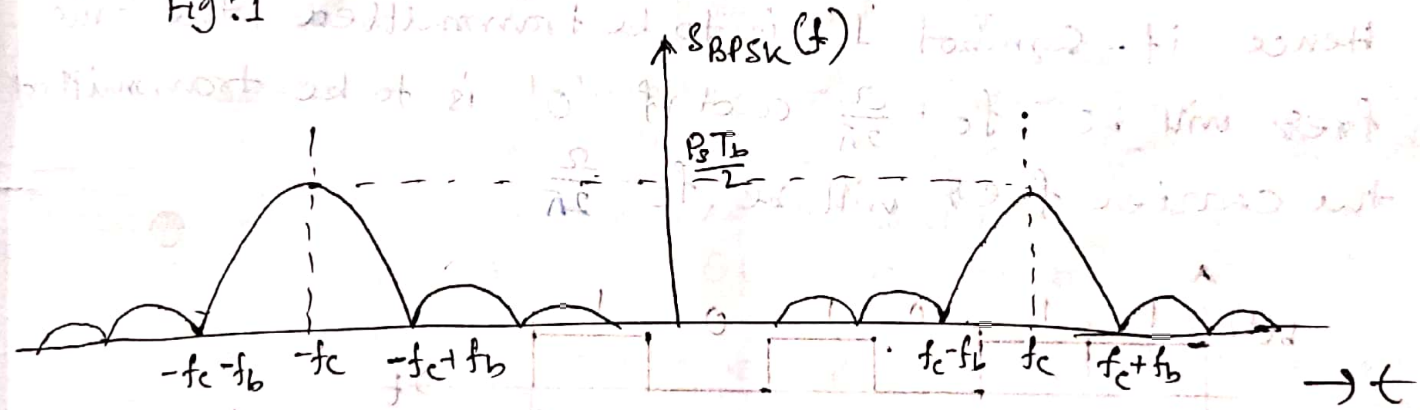



fig:2

From Fig 1, the ~~maximum frequency in the~~ baseband signal will be $f_b$. From fig2: we will see that centre lobe is centred at around ~~centre~~ carrier freq $f_c$ and extends from $f_c - f_b$ to $f_c + f_b$. Therefore bandwidth of ~~Bp~~ BPSK signal is $2f_b$.