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Problems-Mod-3
EC601, ECE 3rd Y

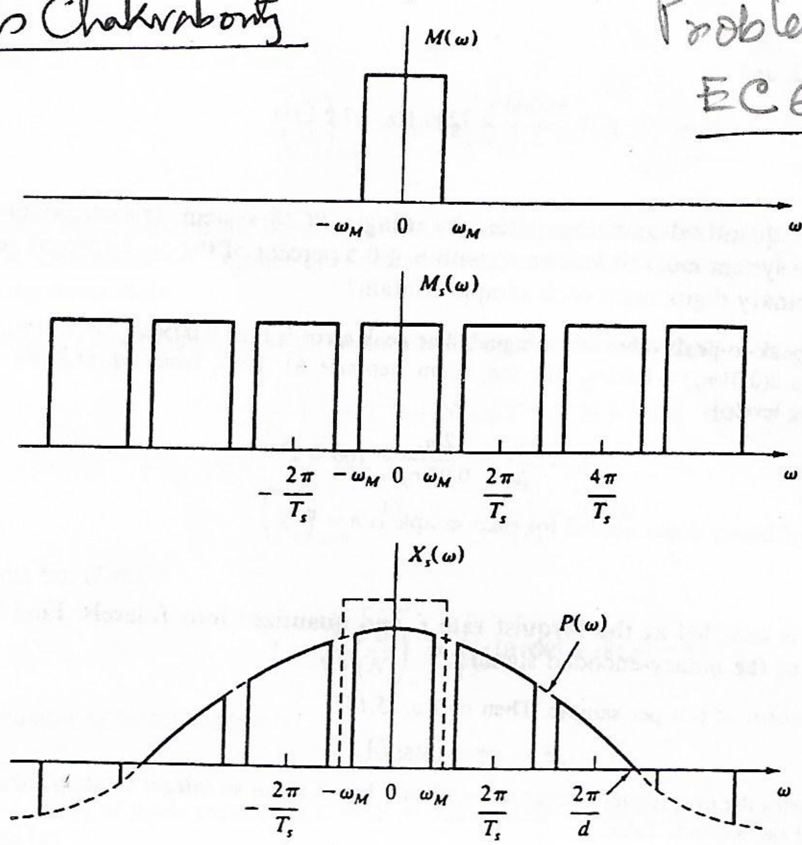

Fig. 5-23 Aperture effect in flat-top sampling

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From Eq. (5.3),

$$m_s(t) = \sum_{n=-\infty}^{\infty} m(nT_s)\delta(t - nT_s)$$

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Then using Eq. (1.36), we obtain

$$\begin{aligned} m_s(t) * p(t) &= \sum_{n=-\infty}^{\infty} m(nT_s)\delta(t - nT_s) * p(t) \\ &= \sum_{n=-\infty}^{\infty} m(nT_s)p(t) * \delta(t - nT_s) \\ &= \sum_{n=-\infty}^{\infty} m(nT_s)p(t - nT_s) = x_s(t) \end{aligned}$$

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QUANTIZING

5.12. A binary channel with bit rate $R_b = 36\,000$ bits per second (b/s) is available for PCM voice transmission. Find appropriate values of the sampling rate f_s, the quantizing level L, and the binary digits n, assuming $f_M = 3.2$ kHz.

Since we require

$$f_s \geq 2f_M = 6400 \quad \text{and} \quad nf_s \leq R_b = 36\,000$$

then

$$n \leq \frac{R_b}{f_s} \leq \frac{36\,000}{6400} = 5.6$$

So $n = 5$, $L = 2^5 = 32$, and

$$f_s = \frac{36\,000}{5} = 7200 \text{ Hz} = 7.2 \text{ kHz}$$

5.13. An analog signal is quantized and transmitted by using a PCM system. If each sample at the receiving end of the system must be known to within ± 0.5 percent of the peak-to-peak full-scale value, how many binary digits must each sample contain?

Let $2m_p$ be the peak-to-peak value of the signal. The peak error is then $0.005(2m_p) = 0.01m_p$, and the peak-to-peak error is $2(0.01m_p) = 0.02m_p$ (the maximum step size Δ). Thus, from Eq. (*5.8*) the required number of quantizing levels is

$$L = \frac{2m_p}{\Delta} = \frac{2m_p}{0.02m_p} = 100 \leqslant 2^n$$

Hence, the number of binary digits needed for each sample is $n = 7$.

5.14. An analog signal is sampled at the Nyquist rate f_s and quantized into L levels. Find the time duration τ of 1 b of the binary-encoded signal.

Let n be the number of bits per sample. Then by Eq. (*5.14*)

$$n = [\log_2 L]$$

where $[\log_2 L]$ indicates the next higher integer to be taken if $\log_2 L$ is not an integer value; nf_s binary pulses must be transmitted per second. Thus,

$$\tau = \frac{1}{nf_s} = \frac{T_s}{n} = \frac{T_s}{[\log_2 L]}$$

where T_s is the Nyquist interval.

5.15. The output *signal-to-quantizing-noise ratio*$(\text{SNR})_o$ in a PCM system is defined as the ratio of average signal power to average quantizing noise power. For a full-scale sinusoidal modulating signal with amplitude A, show that

$$(\text{SNR})_o = \left(\frac{S}{N_q}\right)_o = \frac{3}{2}L^2 \tag{5.46}$$

or

$$\left(\frac{S}{N_q}\right)_{o\text{ dB}} = 10 \log\left(\frac{S}{N_q}\right)_o = 1.76 + 20 \log L \tag{5.47}$$

where L is the number of quantizing levels.

The peak-to-peak excursion of the quantizer input is $2A$. From Eq. (*5.8*), the quantizer step size is

$$\Delta = \frac{2A}{L}$$

Then from Eq. (*5.10*) or (*5.11*), the average quantizing noise power is

$$N_q = \langle q_e^2 \rangle = \frac{\Delta^2}{12} = \frac{A^2}{3L^2}$$

The output signal-to-quantizing-noise ratio of a PCM system for a full-scale test tone is therefore

$$(\text{SNR})_o = \left(\frac{S}{N_q}\right)_o = \frac{A^2/2}{A^2/(3L^2)} = \frac{3}{2}L^2$$

5.16.

5.17.

Expressing this in decibels, we have

$$\left(\frac{S}{N_q}\right)_{0\,\mathrm{dB}} = 10\log\left(\frac{S}{N_q}\right)_o = 1.76 + 20\log L$$

5.16. In a binary PCM system, the output signal-to-quantizing-noise ratio is to be held to a minimum of 40 dB. Determine the number of required levels, and find the corresponding output signal-to-quantizing-noise ratio.

In a binary PCM system, $L = 2^n$, where n is the number of binary digits. Then Eq. (*5.47*) becomes

$$\left(\frac{S}{N_q}\right)_{0\,\mathrm{dB}} = 1.76 + 20\log 2^n = 1.76 + 6.02n \quad \mathrm{dB} \tag{5.48}$$

Now

$$\left(\frac{S}{N_q}\right)_{0\,\mathrm{dB}} = 40\ \mathrm{dB} \rightarrow \left(\frac{S}{N_q}\right)_o = 10\,000$$

Thus, from Eq. (*5.46*),

$$L = \sqrt{\frac{2}{3}\left(\frac{S}{N_q}\right)_o} = \sqrt{\frac{2}{3}(10\,000)} = [81.6] = 82$$

and the number of binary digits n is

$$n = [\log_2 82] = [6.36] = 7$$

Then the number of levels required is $L = 2^7 = 128$, and the corresponding output signal-to-quantizing-noise ratio is

$$\left(\frac{S}{N_q}\right)_{0\,\mathrm{dB}} = 1.76 + 6.02 \times 7 = 43.9\ \mathrm{dB}$$

Note: Equation (*5.48*) indicates that each bit in the code word of a binary PCM system contributes 6 dB to the output signal-to-quantizing-noise ratio. This is called the 6 *dB rule*.

5.17. A compact disc (CD) recording system samples each of two stereo signals with a 16-bit analog-to-digital converter (ADC) at 44.1 kb/s.

(*a*) Determine the output signal-to-quantizing-noise ratio for a full-scale sinusoid.

(*b*) The bit stream of digitized data is augmented by the addition of error-correcting bits, clock extraction bits, and display and control bit fields. These additional bits represent 100 percent overhead. Determine the output bit rate of the CD recording system.

(*c*) The CD can record an hour's worth of music. Determine the number of bits recorded on a CD.

(*d*) For a comparison, a high-grade collegiate dictionary may contain 1500 pages, 2 columns per page, 100 lines per column, 8 words per line, 6 letters per word, and 7 b per letter on average. Determine the number of bits required to describe the dictionary, and estimate the number of comparable books that can be stored on a CD.

(*a*) From Eq. (*5.48*),

$$\left(\frac{S}{N_q}\right)_{0\,\mathrm{dB}} = 1.76 + 6.02 \times 16 = 98.08\ \mathrm{dB}$$

The very high SNR of the disk has the effect of increasing the dynamic range of recording, resulting in the excellent clarity of sound from a CD.

(*b*) The input bit rate is

$$2(44.1)(10^3)(16) = 1.411(10^6)\,\text{b/s} = 1.411\,\text{Mb/s}$$

Including the additional 100 percent overhead, the output bit rate is

$$2(1.411)(10^6)\,\text{b/s} = 2.822\,\text{Mb/s}$$

(*c*) The number of bits recorded on a CD is

$$2.822(10^6)(3600) = 10.16(10^9)\,\text{b} = 10.16\,\text{gigabits(GB)}$$

(*d*) The number of bits required to describe the dictionary is

$$1500(2)(100)(8)(6)(7) = 100.8(10^6)\,\text{b} = 100.8\,\text{Mb}$$

Including the additional 100 percent overhead, then,

$$\frac{10.16(10^9)}{2(100.8)(10^6)} = 50.4$$

Thus, a disc contains the equivalent of about 50 comparable-books storage capacity.

5.18. (*a*) Plot the μ law compression characteristic for $\mu = 255$.
(*b*) If $m_p = 20\,\text{V}$ and 256 quantizing levels are employed, what is the voltage between levels when there is no compression? For $\mu = 255$, what is the smallest and what is the largest effective separation between levels?

(*a*) From Eq. (*5.12*), for $\mu = 255$ we have

$$y = \pm\frac{\ln(1 + 255|x|)}{\ln 256} \qquad |x| < 1$$

where $x = m/m_p$. The plot of the μ law compression characteristic for $\mu = 255$ is shown in Fig. 5-24.

(*b*) With no compression (that is, a uniform quantizing), from Eq. (*5.8*) the step size Δ is

$$\Delta = \frac{2m_p}{L} = \frac{40}{256} = 0.156\,\text{V}$$

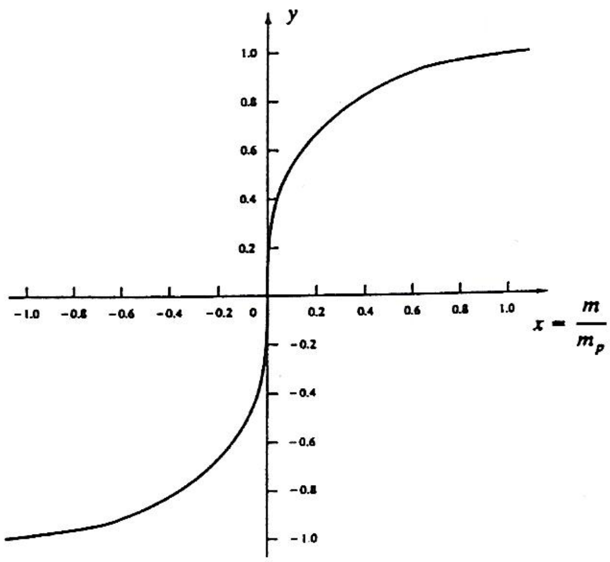

Fig. 5-24 The μ law characteristics for $\mu = 255$

With compression (that is, a nonuniform quantizing), the smallest effective separation between levels will be the one closest to the origin, and the largest effective separation between levels will be the one closest to $|x| = 1$.

Let x_1 be the value of x corresponding to $y = 1/127$, that is,

$$\frac{\ln(1 + 255|x_1|)}{\ln 256} = \frac{1}{127}$$

Solving for $|x_1|$, we obtain

$$|x_1| = 1.75(10^{-4})$$

Thus, the smallest effective separation between levels is given by

$$\Delta_{\min} = m_p|x_1| = 20(1.75)(10^{-4}) = 3.5(10^{-3})\,\text{V} = 3.5\,\text{mV}$$

Next, let x_{127} be the value of x corresponding to $y = 1 - 1/127$, that is,

$$\frac{\ln(1 + 255|x_{127}|)}{\ln 256} = \frac{126}{127}$$

Solving for $|x_{127}|$, we obtain

$$|x_{127}| = 0.957$$

Thus, the largest effective separation between levels is given by

$$\Delta_{\max} = m_p(1 - |x_{127}|) = 20(1 - 0.957) = 0.86\,\text{V}$$

5.19. When a μ law compander is used in PCM, the output signal-to-quantizing-noise ratio for $\mu \gg 1$ is approximated by

$$\left(\frac{S}{N_q}\right)_o \approx \frac{3L^2}{[\ln(1+\mu)]^2} \tag{5.49}$$

Derive the 6 dB rule for $\mu = 255$.

$$\left(\frac{S}{N_q}\right)_{o\ \text{dB}} = 10\log\left(\frac{S}{N_q}\right)_o = 10\log\frac{3L^2}{[\ln(1+\mu)]^2}$$

For $\mu = 255$,
$$\left(\frac{S}{N_q}\right)_{o\ \text{dB}} = 10\log\frac{3L^2}{(\ln 256)^2} = 20\log L - 10.1\ \text{dB} \tag{5.50}$$

In a binary PCM, $L = 2^n$, where n is the number of binary digits; then Eq. (5.50) becomes

$$\left(\frac{S}{N_q}\right)_{o\ \text{dB}} = 20\log 2^n - 10.1 = 6.02n - 10.1\ \text{dB} \tag{5.51}$$

which is the 6 dB rule for $\mu = 255$.

5.20. Consider an audio signal with spectral components limited to the frequency band of 300 to 3300 Hz. A PCM signal is generated with a sampling rate of 8000 samples/s. The required output signal-to-quantizing-noise ratio is 30 dB.

(*a*) What is the minimum number of uniform quantizing levels needed, and what is the minimum number of bits per sample needed?

(*b*) Calculate the minimum system bandwidth required.

(*c*) Repeat parts (*a*) and (*b*) when a μ law compander is used with $\mu = 255$.

(*a*) Using Eq. (5.47), we have

$$\left(\frac{S}{N_q}\right)_{0\ \mathrm{dB}} = 1.76 + 20 \log L \geqslant 30$$

$$\log L \geqslant \frac{1}{20}(30 - 1.76) = 1.412 \rightarrow L \geqslant 25.82$$

Thus, the minimum number of uniform quantizing levels needed is 26.

$$n = [\log_2 L] = [\log_2 26] = [4.7] = 5\,\mathrm{b} \text{ per sample}$$

The minimum number of bits per sample is 5.

(*b*) From Eq. (*5.15*), the minimum required system bandwidth is

$$f_{\mathrm{PCM}} = \frac{n}{2} f_s = \frac{5}{2}(8000) = 20\,000\ \mathrm{Hz} = 20\ \mathrm{kHz}$$

(*c*) Using Eq. (*5.50*),

$$\left(\frac{S}{N_q}\right)_{0\ \mathrm{dB}} = 20 \log L - 10.1 \geqslant 30$$

$$\log L \geqslant \frac{1}{20}(30 + 10.1) = 2.005 \rightarrow L \geqslant 101.2$$

Thus, the minimum number of quantizing levels needed is 102.

$$n = [\log_2 L] = [6.67] = 7$$

The minimum number of bits per sample is 7. The minimum bandwidth required for this case is

$$f_{\mathrm{PCM}} = \frac{n}{2} f_s = \frac{7}{2}(8000) = 28\,000\ \mathrm{Hz} = 28\ \mathrm{kHz}$$

DELTA MODULATION

5.21. Consider a sinusoidal signal $m(t) = A\cos \omega_m t$ applied to a delta modulator with step size Δ. Show that slope overload distortion will occur if

$$A > \frac{\Delta}{\omega_m T_s} = \frac{\Delta}{2\pi}\left(\frac{f_s}{f_m}\right) \tag{5.52}$$

where $f_s = 1/T_s$ is the sampling frequency.

$$m(t) = A\cos \omega_m t \qquad \frac{dm(t)}{dt} = -A\omega_m \sin \omega_m t$$

From Eq. (*5.20*), to avoid the slope overload, we require that

$$\frac{\Delta}{T_s} \geqslant \left|\frac{dm(t)}{dt}\right|_{\max} = A\omega_m \quad \text{or} \quad A \leqslant \frac{\Delta}{\omega_m T_s}$$

Thus, if $A > \Delta/(\omega_m T_s)$, slope overload distortion will occur.

5.22. For a sinusoidal modulating signal

$$m(t) = A\cos \omega_m t \qquad \omega_m = 2\pi f_m$$

show that the maximum output signal-to-quantizing-noise ratio in a DM system under the assumption of no slope overload is given by

$$(\mathrm{SNR})_o = \left(\frac{S}{N_q}\right)_o = \frac{3f_s^3}{8\pi^2 f_m^2 f_M} \tag{5.53}$$

where $f_s = 1/T_s$ is the sampling rate and f_M is the cutoff frequency of a low-pass filter at the output end of the receiver.

From Eq. (*5.52*), for no-slope-overload condition, we must have

$$A < \frac{\Delta}{\omega_m T_s} = \frac{\Delta}{2\pi}\left(\frac{f_s}{f_m}\right)$$

Thus, the maximum permissible value of the output signal power equals

$$P_{\max} = \frac{A^2}{2} = \frac{\Delta^2 f_s^2}{8\pi^2 f_m^2} \tag{5.54}$$

From Eq. (*5.21*,), the mean-square quantizing error, or the quantizing noise power, is $\langle q_e^2 \rangle = \Delta^2/3$. Let the bandwidth of a postreconstruction low-pass filter at the output end of the receiver be $f_M \geqslant f_m$ and $f_M \ll f_s$. Then, assuming that the quantizing noise power P_q is uniformly distributed over the frequency band up to f_s, the output quantizing noise power within the bandwidth f_M is

$$N_q = \left(\frac{\Delta^2}{3}\right)\frac{f_M}{f_s} \tag{5.55}$$

Combining Eqs. (*5.54*) and (*5.55*), we see that the maximum output signal-to-quantizing-noise ratio is

$$\left(\frac{S}{N_q}\right)_o = \frac{P_{\max}}{N_q} = \frac{3f_s^3}{8\pi^2 f_m^2 f_M}$$

5.23. Determine the output SNR in a DM system for a 1-kHz sinusoid, sampled at 32 kHz, without slope overload, and followed by a 4-kHz postreconstruction filter.

From Eq. (*5.53*), we obtain

$$(\mathrm{SNR})_o = \frac{3[(32)(10^3)]^3}{8\pi^2(10^3)^2(4)(10^3)} = 311.3 = 24.9 \text{ dB}$$

5.24. The data rate for Prob. 5.23 is 32 kb/s, which is the same bit rate obtained by sampling at 8 kHz with 4 b per sample in a PCM system. Find the average output SNR of a 4-b PCM quantizer for the sampling of a full-scale sinusoid with $f_s = 8$ kHz, and compare it with the result of Prob. 5.23.

From Eq. (*5.48*), we have

$$(\mathrm{SNR})_{0\,\mathrm{dB}} = 1.76 + 6.02(4) = 25.84\,\mathrm{dB}$$

Comparing this result with that of Prob. 5.23, we conclude that for all the simplicity of DM, it does not perform as well as even a 4-b PCM.

5.25. A DM system is designed to operate at 3 times the Nyquist rate for a signal with a 3-kHz bandwidth. The quantizing step size is 250 mV.

(*a*) Determine the maximum amplitude of a 1-kHz input sinusoid for which the delta modulator does not show slope overload.

(*b*) Determine the postfiltered output signal-to-quantizing-noise ratio for the signal of part (*a*).

(*a*)
$$m(t) = A\cos\omega_m t = A\cos 2\pi(10^3)t$$

$$\left|\frac{dm(t)}{dt}\right|_{\max} = A(2\pi)(10^3)$$

By Eq. (5.52), the maximum allowable amplitude of the input sinusoid is

$$A_{\max} = \frac{\Delta}{\omega_m T_s} = \frac{\Delta}{\omega_m} f_s = \frac{250}{2\pi(10^3)} 3(2)(3)(10^3) = 716.2\,\text{mV}$$

(b) From Eq. (5.53), and assuming that the cutoff frequency of the low-pass filter is f_m, we have

$$(\text{SNR})_o = \left(\frac{S}{N_q}\right)_o = \frac{3[(3)(6)(10^3)]^3}{8\pi^2(10^3)^3} = 221.6 = 23.5\ \text{dB}$$

SIGNALING FORMATS

5.26. Consider the binary sequence 0100101. Draw the waveforms for the following signaling formats.

(a) Unipolar NRZ signaling format

(b) Bipolar RZ signaling format

(c) AMI (alternate mark inversion) RZ signaling format

See Fig. 5-25.

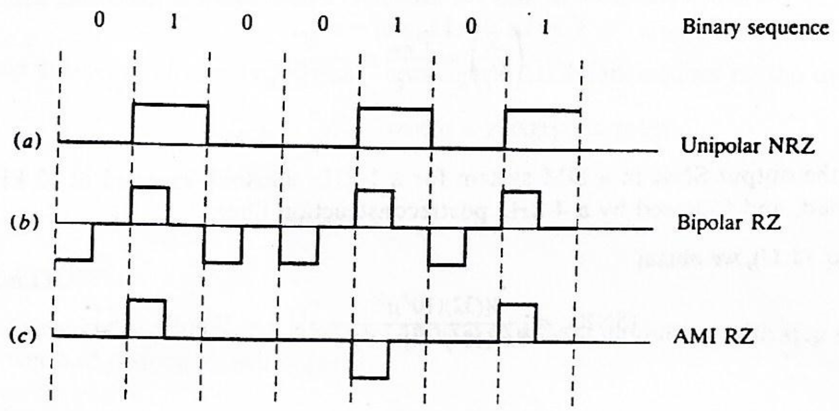

Fig. 5-25

5.27. Discuss the advantages and disadvantages of the three signaling formats illustrated in Fig. 5-25 of Prob. 5.26.

The unipolar NRZ signaling format, although conceptually simple, has disadvantages: There are no pulse transitions for long sequences of 0s or 1s, which are necessary if one wishes to extract timing or synchronizing information; and there is no way to detect when and if an error has occurred from the received pulse sequence.

The bipolar RZ signaling format guarantees the availability of timing information, but there is no error detection capability.

The AMI RZ signaling format has an error detection property; if two sequential pulses (ignoring intervening 0s) are detected with the same polarity, it is evident that an error has occurred. However, to guarantee the availability of timing information, it is necessary to restrict the allowable number of consecutive 0s.

5.28. Consider a binary sequence with a long sequence of 1s followed by a single 0 and then a long sequence of 1s. Draw the waveforms for this sequence, using the following signaling formats:

(*a*) Unipolar NRZ signaling

(*b*) Bipolar NRZ signaling

(*c*) AMI RZ signaling

(*d*) Split-phase (Manchester) signaling

See Fig. 5-26.

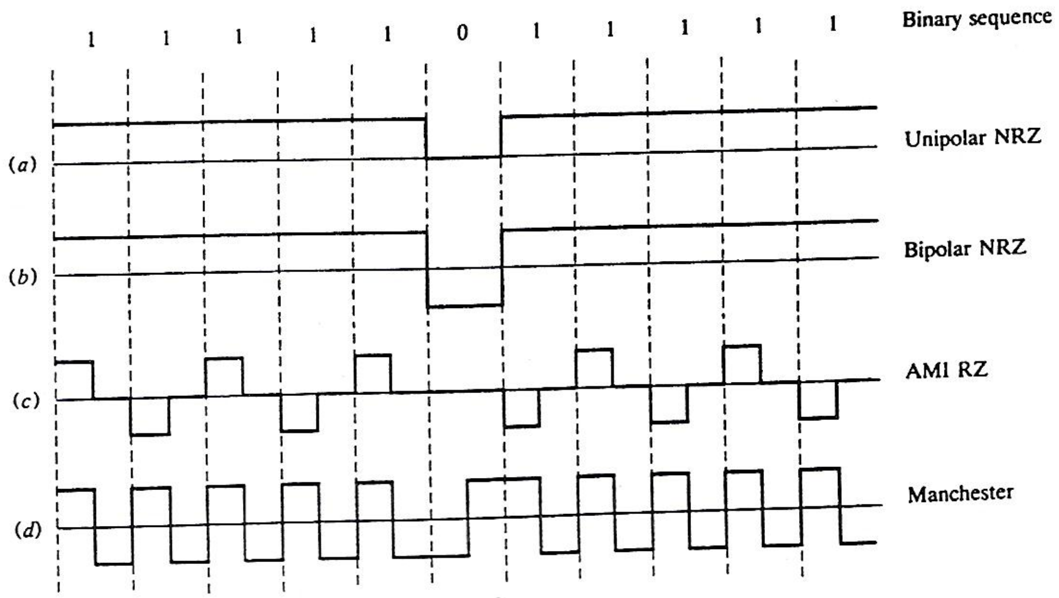

Fig. 5-26

5.29. The AMI RZ signaling waveform representing the binary sequence 0100101011 is transmitted over a noisy channel. The received waveform is shown in Fig. 5-27, which contains a single error. Locate the position of this error, and justify your answer.

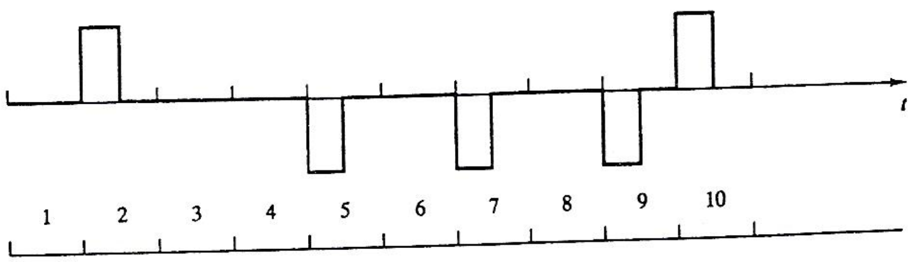

Fig. 5-27

The error is located at the bit position 7 (as indicated in Fig. 5-27), where we have a negative pulse. This bit is in error, because with the AMI signaling format, positive and negative pulses (of equal amplitude) are used alternatively for symbol 1, and no pulse is used for symbol 0. The pulse in position 7 representing the third digit 1 in the data stream should have had positive polarity.

TIME-DIVISION MULTIPLEXING

5.30. Two analog signals $m_1(t)$ and $m_2(t)$ are to be transmitted over a common channel by means of time-division multiplexing. The highest frequency of $m_1(t)$ is 3 kHz, and that of $m_2(t)$ is 3.5 kHz. What is the minimum value of the permissible sampling rate?

The highest-frequency component of the composite signal $m_1(t)+m_2(t)$ is 3.5 kHz. Hence, the minimum sampling rate is

$$2(3500) = 7000 \text{ samples/s}$$

5.31. A signal $m_1(t)$ is band-limited to 3.6 kHz, and three other signals—$m_2(t)$, $m_3(t)$, and $m_4(t)$—are band-limited to 1.2 kHz each. These signals are to be transmitted by means of time-division multiplexing.

(*a*) Set up a scheme for accomplishing this multiplexing requirement, with each signal sampled at its Nyquist rate.

(*b*) What must be the speed of the commutator (in samples per second)?

(*c*) If the commutator output is quantized with $L = 1024$ and the result is binary-coded, what is the output bit rate?

(*d*) Determine the minimum transmission bandwidth of the channel.

(*a*)

Message	Bandwidth	Nyquist rate
$m_1(t)$	3.6 kHz	7.2 kHz
$m_2(t)$	1.2 kHz	2.4 kHz
$m_3(t)$	1.2 kHz	2.4 kHz
$m_4(t)$	1.2 kHz	2.4 kHz

If the sampling commutator rotates at the rate of 2400 rotations per second, then in one rotation we obtain one sample from each of $m_2(t)$, $m_3(t)$, and $m_4(t)$ and three samples from $m_1(t)$. This means that the commutator must have at least six poles connected to the signals, as shown in Fig. 5-28.

(*b*) $m_1(t)$ has 7200 samples/s. And $m_2(t), m_3(t)$, and $m_4(t)$ each have 2400 samples/s. Hence, there are a total of 14 400 samples/s.

(*c*) $L = 1024 = 2^{10} = 2^n$

Thus, the output bit rate is $10(14\,400) = 144$ kb/s.

(*d*) The minimum channel bandwidth is

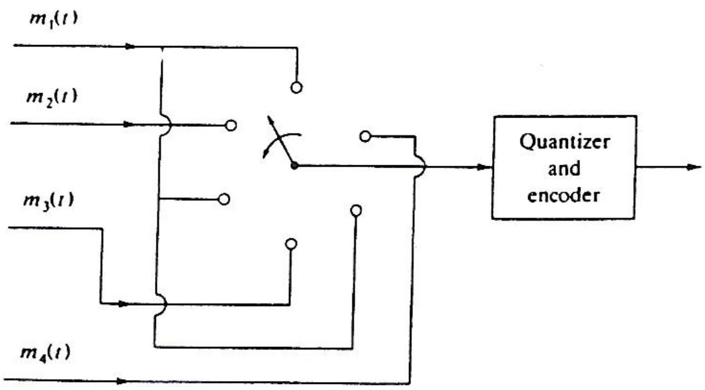

Fig. 5-28 Time-division multiplexing scheme

$$f_B = \frac{1}{2}(7.2 + 2.4 + 2.4 + 2.4) = 7.2 \text{ kHz}$$

5.32. The T1 carrier system used in digital telephony multiplexes 24 voice channels based on 8-b PCM. Each voice signal is usually put through a low-pass filter with the cutoff frequency of about 3.4 kHz. The filtered voice signal is sampled at 8 kHz. In addition, a single bit is added at the end of the frame for the purpose of synchronization. Calculate (*a*) the duration of each bit, (*b*) the resultant transmission rate, and (*c*) the minimum required transmission bandwidth (Nyquist bandwidth).

(*a*) With a sampling rate of 8 kHz, each frame of the multiplexed signal occupies a period of

$$\frac{1}{8000} = 0.000125 \text{ s} = 125 \text{ microseconds } (\mu\text{s})$$

Since each frame consists of twenty-four 8-b words, plus a single synchronizing bit, it contains a total of

$$24(8) + 1 = 193 \text{ b}$$

Thus, the duration of each bit is

$$T_b = \frac{125}{193} \mu\text{s} = 0.647 \mu\text{s}$$

(*b*) The resultant transmission rate is

$$R_b = \frac{1}{T_b} = 1.544 \text{ Mb/s}$$

(*c*) From Eq. (*5.22*), the minimum required transmission bandwidth is

$$f_{T1} = \frac{1}{2T_b} = 772 \text{ kHz}$$

PULSE SHAPING AND INTERSYMBOL INTERFERENCE

5.33. Show that (a pulse-shape function) $h(t)$, with Fourier transform given by $H(\omega)$, that satisfies the criterion

$$\sum_{k=-\infty}^{\infty} H\left(\omega + \frac{2\pi k}{T}\right) = 1 \quad \text{for } |\omega| \leq \frac{\pi}{T} \tag{5.56}$$

has $h(nT)$ given by

$$h(nT) = \begin{cases} \frac{1}{T} & n = 0 \\ 0 & n \neq 0 \end{cases} \tag{5.57}$$

The criterion (*5.56*) is known as *Nyquist's pulse-shaping criterion.*

Taking the inverse Fourier transform of $H(\omega)$, we have

$$h(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} H(\omega) e^{j\omega t} \, d\omega$$

The range of integration in the preceding equation can be divided into segments of length $2\pi/T$ as

$$h(t) = \frac{1}{2\pi} \sum_{k=-\infty}^{\infty} \int_{(2k-1)\pi/T}^{(2k+1)\pi/T} H(\omega) e^{j\omega t} \, d\omega$$

and we can write $h(nT)$ as

$$h(nT) = \frac{1}{2\pi}\sum_{k=-\infty}^{\infty}\int_{(2k-1)\pi/T}^{(2k+1)\pi/T} H(\omega)e^{j\omega nT}\,d\omega \tag{5.58}$$

By the change of variable $u = \omega - 2\pi(k/T)$, Eq. (5.58) becomes

$$h(nT) = \frac{1}{2\pi}\sum_{k=-\infty}^{\infty}\int_{-\pi/T}^{\pi/T} H\left(u + \frac{2\pi k}{T}\right)e^{j(u+2\pi k/T)nT}\,du$$

Assuming that the integration and summation can be interchanged, we have

$$h(nT) = \frac{1}{2\pi}\int_{-\pi/T}^{\pi/T}\sum_{k=-\infty}^{\infty} H\left(u + \frac{2\pi k}{T}\right)e^{junT}\,du$$

Finally, if Eq. (5.56) is satisfied, then

$$\begin{aligned} h(nT) &= \frac{1}{2\pi}\int_{-\pi/T}^{\pi/T} e^{junT}\,du \\ &= \frac{1}{T}\frac{\sin n\pi}{n\pi} = \begin{cases} \frac{1}{T} & n = 0 \\ 0 & n \neq 0 \end{cases} \end{aligned}$$

which verifies that $h(t)$ with a Fourier transform $H(\omega)$ satisfying criterion (5.56) produces zero ISI.

5.34. A certain telephone line bandwidth is 3.5 kHz. Calculate the data rate (in b/s) that can be transmitted if we use binary signaling with the raised-cosine pulses and a roll-off factor $\alpha = 0.25$.

Using Eq. (5.28), we see that the data rate is

$$R_b = \frac{1}{T} = \frac{2}{1 + 0.25}(3500) = 5600 \text{ b/s}$$

5.35. A communication channel of bandwidth 75 kHz is required to transmit binary data at a rate of 0.1 Mb/s using raised-cosine pulses. Determine the roll-off factor α.

$$T_b = \frac{1}{0.1(10^6)} = 10^{-5}\text{s}$$

$$f_B = 75 \text{ kHz} = 75(10^3)\,\text{Hz}$$

Using Eq. (5.27), we have

$$1 + \alpha = 2f_B T_b = 2(75)(10^3)(10^{-5}) = 1.5$$

Hence, we obtain

$$\alpha = 0.5$$

5.36. In a certain telemetry system, eight message signals having 2-kHz bandwidth each are time-division multiplexed using a binary PCM. The error in sampling amplitude cannot be greater than 1 percent of the peak amplitude. Determine the minimum transmission bandwidth required if raised-cosine pulses with roll-off factor $\alpha = 0.2$ are used. The sampling rate must be at least 25 percent above the Nyquist rate.

From Eq. (5.9), the maximum quantizing error must satisfy

$$(q_e)_{\max} = \frac{\Delta}{2} = \frac{m_p}{L} \leq 0.01 m_p$$

$$M(\omega) = \sum_{n=-\infty}^{\infty} M\left(\frac{n\pi}{T}\right) \frac{\sin(\omega T - n\pi)}{\omega T - n\pi}$$

This is known as the *sampling theorem in the frequency domain.*

Hint: Interchange the roles of t and ω in the sampling theorem proof (Prob. 5.2).

5.41. American Standard Code for Information Interchange (ASCII) has 128 characters that are binary-coded. If a computer generates 100 000 characters per second, determine
(*a*) The number of bits required per character
(*b*) The data rate (or bit rate) R_b required to transmit the computer output

Ans. (*a*) 7 b per character, (*b*) $R_b = 0.7\,\text{Mb/s}$

5.42. A PCM system uses a uniform quantizer followed by a 7-b binary encoder. The bit rate of the system is 50 Mb/s. What is the maximum message bandwidth for which system operation is satisfactory?

Ans. 3.57 MHz

5.43. Consider binary PCM transmission of a video signal with $f_s = 10\,\text{MHz}$. Calculate the signaling rate needed to achieve $(\text{SNR})_o \geqslant 45$ dB.

Ans. 80 Mb/s

5.44. Show that in a PCM system, the output signal-to-quantizing-noise ratio can be expressed as

$$\left(\frac{S}{N_q}\right)_o = \frac{3}{2}\left(4^{f_B/f_m}\right)$$

where f_B is the channel bandwidth and f_m is the message bandwidth.

Hint: Use Eqs. (*5.14*), (*5.15*), and (*5.46*).

5.45. The bandwidth of a TV radio plus audio signal is 4.5 MHz. If this signal is converted to PCM with 1024 quantizing levels, determine the bit rate of the resulting PCM signal. Assume that the signal is sampled at a rate 20 percent above the Nyquist rate.

Ans. 108 Mb/s

5.46. A commonly used value A for the A law compander is $A = 87.6$. If $m_p = 20\,\text{V}$ and 256 quantizing levels are employed, what is the smallest and what is the largest effective separation between levels?

Ans. $\Delta_{\min} = 9.8$ mV, $\Delta_{\max} = 0.84$ V

5.47. Given the binary sequence 1101110, draw the transmitted pulse waveform for (*a*) AMI RZ signaling format and (*b*) split-phase (Manchester) signaling format.

Hint: See Fig. 5-10.

Hence, $L \geqslant 100$, and we choose $L = 128 = 2^7$. The number of bits per sample required is 7.

Since the Nyquist sampling rate is $2f_M = 4000$ samples/s, the sampling rate for each signal is

$$f_s = 1.25(4000) = 5000 \text{ samples/s}$$

There are eight time-division multiplexed signals, requiring a total of

$$8(5000) = 40\,000 \text{ samples/s}$$

Since each sample is encoded by 7 bits, the resultant bit rate is

$$\frac{1}{T_b} = 7(40\,000) = 280 \text{ kb/s}$$

From Eq. (*5.27*), the minimum transmission bandwidth required is

$$f_B = \frac{1+0.2}{2}(280) = 168 \text{ kHz}$$

Supplementary Problems

5.37. If $m(t)$ is a band-limited signal, show that

$$\int_{-\infty}^{\infty} m(t)\phi_n(t)dt = T_s m(nT_s)$$

where $\phi_n(t)$ is the function defined in Eq. (*5.39*) of Prob. 5.4.

Hint: Use the orthogonality property (*5.40*) of $\phi_n(t)$.

5.38. The signals

$$m_1(t) = 10\cos 100\pi t$$

and

$$m_2(t) = 10\cos 50\pi t$$

are both sampled with $f_s = 75$ Hz. Show that the two sequences of samples so obtained are identical.

Hint: Take the Fourier transforms of ideally sampled signals $m_{1s}(t)$ and $m_{2s}(t)$. *Note:* This problem indicates that by undersampling $m_1(t)$ and oversampling $m_2(t)$ appropriately, their sampled versions can be identical.

5.39. A signal

$$m(t) = \cos 200\pi t + 2\cos 320\pi t$$

is ideally sampled at $f_s = 300$ Hz. If the sampled signal is passed through an ideal low-pass filter with a cutoff frequency of 250 Hz, what frequency components will appear in the output?

Ans. 100-, 140-, 160-, and 200-Hz components

5.40. A duration-limited signal is a time function $m(t)$ for which

$$m(t) = 0 \quad \text{for } |t| > T$$

Let $M(\omega) = \mathscr{F}[m(t)]$. Show that $M(\omega)$ can be uniquely determined from its values $M(n\pi/T)$ at a series of equidistant points spaced π/T apart. In fact, $M(\omega)$ is given by

5.48. A given DM system operates with a sampling rate f_s and a fixed size Δ. If the input to the system is

$$m(t) = \alpha t \quad \text{for } t > 0$$

determine the value of α for which slope overload occurs.

Ans. Δf_s

5.49. Consider a DM system whose receiver does not include a low-pass filter, as in Prob. 5.22. Show that under the assumption of no slope overload distortion, the maximum output signal-to-quantizing-noise ratio increases by 6 dB when the sampling rate is doubled. What is the improvement that results from the use of a low-pass filter at the receiver output?

Ans. 9-dB improvement

5.50. Twenty-four voice signals are sampled uniformly and then time-division-multiplexed. The sampling operation uses flat-top samples with 1-μs duration. The multiplexing operation includes provision for synchronization by adding an extra pulse of appropriate amplitude and 1-μs duration. The highest frequency component of each voice signal is 3.4 kHz.
(*a*) Assuming a sampling rate of 8 kHz, calculate the spacing between successive pulses of the multiplexed signal.
(*b*) Repeat (*a*), assuming the use of Nyquist rate sampling.

Ans. (*a*) 4 μs, (*b*) 5.68 μs.

5.51. Five telemetry signals, each of bandwidth 1 kHz, are to be transmitted by binary PCM with TDM. The maximum tolerable error in sampling amplitude is 0.5 percent of the peak signal amplitude. The signals are sampled at least 20 percent above the Nyquist rate. Framing and synchronization require an additional 0.5 percent extra bits. Determine the minimum transmission data rate and the minimum required bandwidth for the TDM transmission.

Ans. $R_b = 964.8$ kb/s, $f_{TDM} = 482.9$ kHz

5.52. In a certain telemetry system, there are four analog signals: $m_1(t)$, $m_2(t)$, $m_3(t)$, and $m_4(t)$. The bandwidth of $m_1(t)$ is 3.6 kHz, but the bandwidths of the remaining signals are 1.5 kHz each. Set up a suitable scheme for accomplishing the time-division multiplexing of these signals.

Ans. Use the same scheme as the one depicted in Fig. 5-28 of Prob. 5.31 with the commutator speed raised to 3000 rotations per second.