Solution

Given $\alpha_p = 3\,\text{dB};\ \omega_c = \omega_p = 2 \times \pi \times 1000 = 2000\pi$ rad/sec

$$\alpha_s = 10\,\text{dB}: \quad \omega_s = 2 \times \pi \times 350 = 700\pi \text{ rad/sec}$$

$$T = \frac{1}{f} = \frac{1}{5000} = 2 \times 10^{-4}\,sec$$

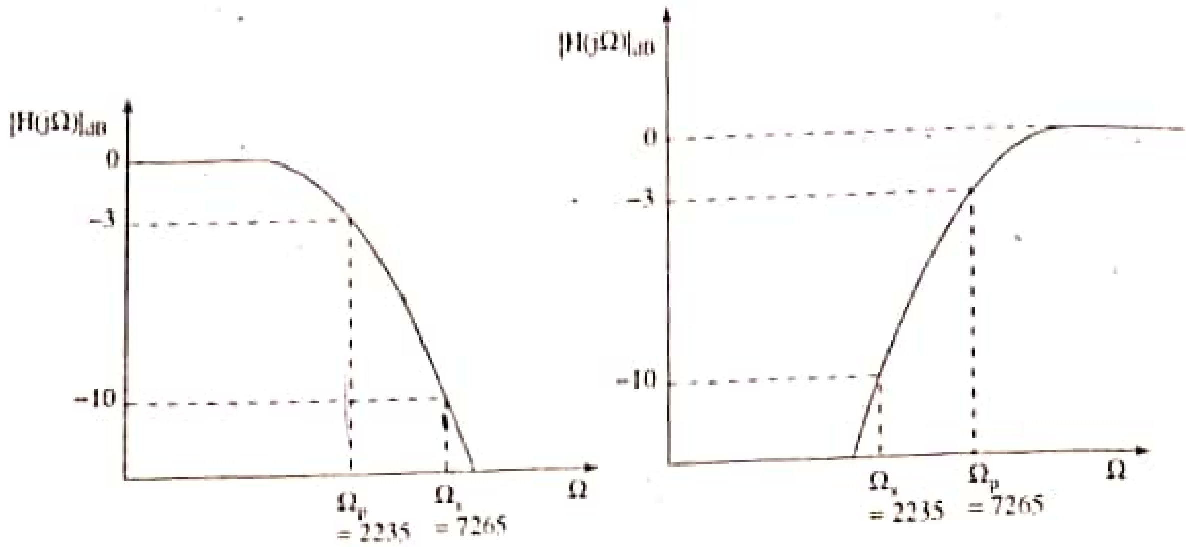

Fig. 5.27

The characteristics are monotonic in both passband and stopband. Therefore, the filter is Butterworth filter.

Prewarping the digital frequencies we have

$$\Omega_p = \frac{2}{T}\tan\frac{\omega_p T}{2} = \frac{2}{2 \times 10^{-4}}\tan\frac{(2000\pi \times 2 \times 10^{-4})}{2}$$
$$= 10^4 \tan(0.2\pi) = 7265 \text{ rad/sec}$$

$$\Omega_s = \frac{2}{T}\tan\frac{\omega_s T}{2} = \frac{2}{2 \times 10^{-4}}\tan\frac{(700\pi \times 2 \times 10^{-4})}{2}$$
$$= 10^4 \tan(0.07\pi) = 2235 \text{rad/sec}$$

First we design a lowpass filter for the given specifications and use suitable transformation to obtain transfer function of highpass filter.

The order of the filter

$$N = \frac{\log\sqrt{\dfrac{10^{0.1\alpha_s} - 1}{10^{0.1\alpha_p} - 1}}}{\log\dfrac{\Omega_s}{\Omega_p}} = \frac{\log\sqrt{\dfrac{10^{0.1(10)} - 1}{10^{0.1(3)} - 1}}}{\log\dfrac{7265}{2235}} = \frac{\log 3}{\log 3.25} = \frac{0.4771}{0.5118} = 0.932$$

Therefore, we take $N = 1$.

The first-order Butterworth filter for $\Omega_c = 1$ rad/sec is $H(s) = \dfrac{1}{1+s}$

The highpass filter for $\Omega_c = \Omega_p = 7265$ rad/sec can be obtained by using the transformation

$$s \to \frac{\Omega_c}{s}$$

$$i.e.,\ s \to \frac{(7265)}{s}$$

The transfer function of highpass filter

$$H(s) = \frac{1}{s+1}\bigg|_{s=\frac{7265}{s}}$$

$$= \frac{s}{s+7265}$$

Using bilinear transformation

$$H(z) = H(s)\Big|_{s=\frac{2}{T}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)}$$

$$= \frac{s}{s+7265}\bigg|_{s=\frac{2}{2\times10^{-4}}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)}$$

$$= \frac{10000\left(\frac{1-z^{-1}}{1+z^{-1}}\right)}{10000\left(\frac{1-z^{-1}}{1+z^{-1}}\right)+7265}$$

$$= \frac{0.5792(1-z^{-1})}{1-0.1584z^{-1}}$$

Example 5.18 Determine $H(z)$ that results when the bilinear transformation is applied to $H_a(s) = \dfrac{s^2+4.525}{s^2+0.692s+0.504}$

Solution

In bilinear transformation

$$H(z) = H(s)\Big|_{s=\frac{2}{T}\left[\frac{1-z^{-1}}{1+z^{-1}}\right]}$$

Assume $T = 1$ sec.

Then

$$H(z) = \frac{\left[2\left(\frac{1-z^{-1}}{1+z^{-1}}\right)\right]^2+4.525}{4\left[\frac{1-z^{-1}}{1+z^{-1}}\right]^2+0.692\times2\times\left[\frac{1-z^{-1}}{1+z^{-1}}\right]+0.504}$$

$$= \frac{1.4479+0.1783z^{-1}+1.4479z^{-2}}{1-1.18752z^{-1}+0.5299z^{-2}}$$

Practice Problem 5.9 An analog filter has a transfer function

$$H(s) = \frac{1}{s^2 + 6s + 9}$$

Design a digital filter using bilinear transformation method.

Practice Problem 5.10 Repeat practice problem 5.7 using bilinear transformation method.

5.13 Frequency Transformation in Digital Domain

A digital lowpass filter can be converted into a digital highpass, bandstop, bandpass or another digital filter. These transformations are given below.

5.13.1 *Lowpass to Lowpass*

$$z^{-1} \longrightarrow \frac{z^{-1} - \alpha}{1 - \alpha z^{-1}}$$

$$\text{where } \alpha = \frac{\sin[(\omega_p - \omega'_p)/2]}{\sin[(\omega_p + \omega'_p)/2]} \tag{5.98}$$

ω_p = passband frequency of lowpass filter

ω'_p = passband frequency of new lowpass filter

5.13.2 *Lowpass to highpass*

$$z^{-1} = -\left[\frac{z^{-1} + \alpha}{1 + \alpha z^{-1}}\right]$$

$$\text{where } \alpha = -\frac{\cos[(\omega'_p + \omega_p)/2]}{\cos[(\omega'_p - \omega_p)/2]} \tag{5.99}$$

ω_p = passband frequency of lowpass filter

ω'_p = passband frequency of highpass filter

5.13.3 *Lowpass to Bandpass*

$$z^{-1} \longrightarrow \frac{-\left(z^{-2} - \frac{2\alpha k}{1+k} z^{-1} + \frac{k-1}{k+1}\right)}{\frac{k-1}{k+1} z^{-2} - \frac{2\alpha k}{k+1} z^{-1} + 1}$$

where $\alpha = \dfrac{\cos[(\omega_u + \omega_l)/2]}{\cos[(\omega_u - \omega_l)/2]}$

$$k = \cot\left[\frac{\omega_u - \omega_l}{2}\right]\tan\frac{\omega_p}{2} \tag{5.100}$$

$$\omega_u = \text{upper cutoff frequency}$$
$$\omega_l = \text{lower cutoff frequency}$$

5.13.4 *Lowpass to Bandstop*

$$z^{-1} \longrightarrow \frac{z^{-2} - \dfrac{2\alpha}{1+k}z^{-1} + \dfrac{1-k}{1+k}}{\dfrac{1-k}{1+k}z^{-2} - \dfrac{2\alpha}{1+k}z^{-1} + 1}$$

where $\alpha = \dfrac{\cos[(\omega_u + \omega_l)/2]}{\cos[(\omega_u - \omega_l)/2]}$

$$k = \tan[(\omega_u - \omega_l)/2]\tan\frac{\omega_p}{2} \tag{5.101}$$

Example 5.19 Convert the single pole lowpass filter with system function $H(z) = \dfrac{0.5(1+z^{-1})}{1-0.302z^{-2}}$ into bandpass filter with upper and lower cutoff frequencies ω_u and ω_l respectively. The lowpass filter has 3 dB bandwidth $\omega_p = \dfrac{\pi}{6}$ and $\omega_u = \dfrac{3\pi}{4}, \omega_l = \dfrac{\pi}{4}$

Solution

The digital-to-digital transformation from lowpass filter to a bandpass filter is

$$z^{-1} \longrightarrow \frac{-\left(z^{-2} - \dfrac{2\alpha k}{1+k}z^{-1} + \dfrac{k-1}{k+1}\right)}{\dfrac{k-1}{k+1}z^{-2} - \dfrac{2\alpha k}{k+1}z^{-1} + 1}$$

where

$$k = \cot\left[\frac{\omega_u - \omega_l}{2}\right]\tan\frac{\omega_p}{2} = \cot\left(\frac{\frac{3\pi}{4} - \frac{\pi}{4}}{2}\right)\tan\frac{\pi}{12}$$
$$= \cot\left(\frac{\pi}{4}\right)\tan\frac{\pi}{12}$$
$$= 0.268$$

$$\alpha = \frac{\cos\dfrac{\omega_u + \omega_l}{2}}{\cos\dfrac{\omega_u - \omega_l}{2}} = \frac{\cos\left(\dfrac{\dfrac{3\pi}{4} + \dfrac{\pi}{4}}{2}\right)}{\cos\left(\dfrac{\dfrac{3\pi}{4} - \dfrac{\pi}{4}}{2}\right)} = \frac{\cos\dfrac{\pi}{2}}{\cos\dfrac{\pi}{4}} = 0$$

Substituting the values of α and k in the transformation

$$z^{-1} \rightarrow \frac{-\left(z^{-2} + \dfrac{0.268 - 1}{0.268 + 1}\right)}{\dfrac{0.268 - 1}{0.268 + 1} z^{-2} + 1}$$

i.e.,

$$z^{-1} \rightarrow \frac{-(z^{-2} - 0.577)}{-0.577 z^{-2} + 1}$$

Now the transfer function of bandpass filter can be obtained by substituting the above transformation in $H(z)$.

$$H(z) = 0.5 \frac{\left[1 + \dfrac{-z^{-2} + 0.577}{1 - 0.577 z^{-2}}\right]}{1 - 0.302\left(\dfrac{-z^{-2} + 0.577}{1 - 0.577 z^{-2}}\right)}$$

$$= 0.5 \left[\frac{1.577(1 - z^{-2})}{0.82575 - 0.275 z^{-2}}\right]$$

$$= \frac{0.955(1 - z^{-2})}{(1 - 0.333 z^{-2})}$$

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