

3.42 Digital Signal Processing

In the case of circular convolution the situation is entirely different. If $x(n)$ contains L number of samples and $h(n)$ has M number of samples and that $L > M$, then we perform circular convolution between the two using $N = \text{Max}(L, M)$, by adding $L - M$ number of zero samples to the sequence $h(n)$, so that both sequences are periodic with N .

Linear convolution can be used to find the response of a filter. Circular convolution cannot be used to find the response of a linear filter without zero padding.

3.8 Methods to evaluate circular convolution of two sequences

The methods used to find the circular convolution of two sequences are (1) Concentric circle method (2) Matrix multiplication method.

3.8.1 Concentric Circle Method

Given two sequences $x_1(n)$ and $x_2(n)$, the circular convolution of these two sequences $x_3(n) = x_1(n) \otimes x_2(n)$ can be found by using the following steps.

1. Graph N samples of $x_1(n)$ as equally spaced points around an outer circle in counterclockwise direction.
2. Start at the same point as $x_1(n)$ graph N samples of $x_2(n)$ as equally spaced points around an inner circle in clockwise direction.
3. Multiply corresponding samples on the two circles and sum the products to produce output.
4. Rotate the inner circle one sample at a time in counterclockwise direction and go to step 3 to obtain the next value of output.
5. Repeat step No.4 until the inner circle first sample lines up with the first sample of the exterior circle once again.

3.8.2 Matrix Multiplication Method

In this method, the circular convolution of two sequences $x_1(n)$ and $x_2(n)$ can be obtained by representing the sequences in matrix form as shown below

$$\begin{bmatrix} x_2(0) & x_2(N-1) & x_2(N-2) & \dots & x_2(2) & x_2(1) \\ x_2(1) & x_2(0) & x_2(N-1) & \dots & x_2(3) & x_2(2) \\ x_2(2) & x_2(1) & x_2(0) & \dots & x_2(4) & x_2(3) \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ x_2(N-2) & x_2(N-3) & x_2(N-4) & \dots & x_2(0) & x_2(N-1) \\ x_2(N-1) & x_2(N-2) & x_2(N-3) & \dots & x_2(1) & x_2(0) \end{bmatrix} \begin{bmatrix} x_1(0) \\ x_1(1) \\ x_1(2) \\ \vdots \\ x_2(N-2) \\ x_1(N-1) \end{bmatrix} = \begin{bmatrix} x_3(0) \\ x_3(1) \\ x_3(2) \\ \vdots \\ x_3(N-2) \\ x_3(N-1) \end{bmatrix} \quad (3.55)$$