$$y_3(n) = x_3(n) \text{ (N) } h(n) = \{0,1,3,3,2\}$$

$$y_4(n) = x_4(n) \text{ (N) } h(n) = \{1,1,1,0,0\}$$

$\boxed{3,2,2,-1,0}$

add

$\boxed{1,4,6,5,2}$

add

$\boxed{0,1,3,3,2}$

add

$\boxed{1,1,1,0,0}$

$$y(n) = \{3,2,2,0,4,6,5,3,3,4,3,1\}$$

Example 3.21 Using linear convolution find $y(n) = x(n) * h(n)$ for the sequences $x(n) = (1,2,-1,2,3,-2,-3,-1,1,1,2,-1)$ and $h(n) = (1,2)$. Compare the result by solving the problem using (a) overlap-save method (b) overlap-add method.

Solution

The linear convolution of $x(n)$ and $h(n)$ is

$$y(n) = x(n) * h(n) = \{1,4,3,0,7,4,-7,-7,-1,3,4,3,-2\}$$

Overlap-save method

The input sequence can be divided into blocks of data as follows.

$\rightarrow M-1$ zeros appended

$$x_1 = \{0, \underbrace{1,2,-1,}_{\text{3 datas}}\}$$

$$x_2(n) = \{-1, \underbrace{2,3-2}_{\text{3 new datas}}\}$$

$\rightarrow M-1 = 1$ data from previous block

$$x_3(n) = \{-2,-3,-1,1\}; \quad x_4(n) = \{1,1,2,-1\}; \quad x_5(n) = \{-1,0,0,0\}$$

Given $h(n) = \{1,2\}$. Appending two zeros to the sequence we obtain

$$h(n) = \{1,2,0,0\}$$

$$y_1(n) = x_1(n) \text{ (N) } h(n)$$

$$= \begin{bmatrix} 1 & 0 & 0 & 2 \\ 2 & 1 & 0 & 0 \\ 0 & 2 & 1 & 0 \\ 0 & 0 & 2 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 2 \\ -1 \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \\ 4 \\ 3 \end{bmatrix}$$

$$y_2(n) = x_2(n) \; \textcircled{N} \; h(n)$$

$$= \begin{bmatrix} 1 & 0 & 0 & 2 \\ 2 & 1 & 0 & 0 \\ 0 & 2 & 1 & 0 \\ 0 & 0 & 2 & 1 \end{bmatrix} \begin{bmatrix} -1 \\ 2 \\ 3 \\ -2 \end{bmatrix} = \begin{bmatrix} -5 \\ 0 \\ 7 \\ 4 \end{bmatrix}$$

Similarly $y_3(n) = \{0, -7, -7, -1\}$; $y_4(n) = \{-1, 3, 4, 3\}$;
$y_5(n) = \{-1, -2, 0, 0\}$

$\boxed{-2, 1, 4, -3}$
$\downarrow$
discard

$\boxed{-5, 0, 7, 4}$
$\downarrow$
discard

$\boxed{0, -7, -7, -1}$
$\downarrow$
discard

$\boxed{-1, 3, 4, 3}$
$\downarrow$
discard

$\boxed{-1, -2, 0, 0}$
$\downarrow$
discard

$$y(n) = \{1, 4, 3, 0, 7, 4, -7, -7, -1, 3, 4, 3, -2\}$$

Overlap-add method

In this method the sequence $x(n)$ can be divided into data blocks as shown below.

$$x_1(n) = \{1, 2, -1, \underset{\downarrow}{0}\}$$

$M - 1 = 1$ zero added

$$x_2(n) = \{2, 3, -2, 0\}; \quad x_3(n) = \{-3, -1, 1, 0\}$$
$$x_4(n) = \{1, 2, -1, 0\}; \quad h(n) = \{1, 2, 0, 0\}$$

$$y_1(n) = x_1(n) \; N \; h(n) = \{1, 4, 3, -2\}$$

$$y_2(n) = x_2(n) \; N \; h(n) = \{2, 7, 4, -4\}$$

$$y_3(n) = x_3(n) \; N \; h(n) = \{-3, -7, -1, 2\}$$

$$y_4(n) = x_4(n) \; N \; h(n) = \{1, 4, 3, -2\}$$

$$\begin{array}{llll}
\boxed{1, 4, 3, -2} & & & \\
\quad\ \updownarrow \text{ add} & & & \\
& \boxed{2, 7, 4, -4} & & \\
& \quad\ \updownarrow \text{ add} & & \\
& & \boxed{-3, -7, -1, 2} & \\
& & \quad\ \updownarrow \text{ add} & \\
& & & \boxed{1, 4, 3, -2}
\end{array}
\qquad (\because M - 1 = 1)$$

$$y(n) = \{1, 4, 3, 0, 7, 4, -7, -7, -1, 3, 4, 3, -2\}$$