Q. Consider the length-8 sequence defined for $0 \le n \le 8$, $x(n) = \{1, 2, -3, 0, 1, -1, 4, 2\}$ with a 8-point DFT. Evaluate the following functions of $X(K)$ without computing DFT

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ⓐ $X(0)$ ⓑ $X(4)$ ⓒ $\sum_{K=0}^{7} X(K)$ ⓓ $e^{-j\frac{3\pi K}{4}} X(K)$
ⓔ $\sum_{K=0}^{7} |X(K)|^2$

ⓐ $X(K) = \sum_{n=0}^{N-1} x(n) e^{-j2\pi nK/N}$

Put $K=0$

$$X(0) = \sum_{n=0}^{8-1} x(n) e^{0} = \sum_{n=0}^{7} x(n)$$

$$= x(0) + x(1) + x(2) + x(3) + x(4) + x(5) + x(6) + x(7)$$

$$= 1 + 2 - 3 + 0 + \cancel{1} - \cancel{1} + 4 + 2 = 6$$

ⓑ When $K=4$, $X(4) = \sum_{n=0}^{7} x(n) e^{\frac{-j2\pi n \cdot 4}{8}}$

$$= \sum_{n=0}^{7} x(n) \left(e^{\frac{-j8\pi}{8}}\right)^n$$

$$= \sum_{n=0}^{7} x(n) \left[\cos\frac{8\pi}{8} - j\sin\frac{8\pi}{8}\right]^n$$

$\because \cos n\pi = (-1)^n$

$\because \sin n\pi = 0$

$$= \sum_{n=0}^{7} x(n)(-1)^n$$

$$= x(0)(-1)^0 + x(1)(-1)^1 + x(2)(-1)^2 + x(3)(-1)^3 + x(4)(-1)^4 + x(5)(-1)^5 + x(6)(-1)^6 + x(7)(-1)^7$$

$$= 1 - \cancel{2} - 3 + 0 + 1 + 1 + 4 - \cancel{2} = 0$$

(c) IDFT

$$x(n) = \frac{1}{N}\sum_{K=0}^{N-1} X(K)\, e^{\frac{j2\pi nK}{N}}$$

$$x(n) = \frac{1}{8}\sum_{K=0}^{7} X(K)\, e^{\frac{j2\pi nK}{8}}$$

When $n=0$

$$x(0) = \frac{1}{8}\sum_{K=0}^{7} X(K)\, e^{0}$$

$$\therefore \sum_{K=0}^{7} X(K) = 8\,x(0) = 8\cdot 1 = 8 \text{ Ans}$$

(d) $$\text{DFT}\left[x((n-3))_8\right] = e^{-\frac{j3\pi K\cdot 2}{8}} X(K) = e^{-\frac{j3\pi K}{4}} X(K)$$

Takeing IDFT of $e^{-\frac{j3\pi K}{4}} X(K)$ and substitute $n=0$

$$\frac{1}{N}\sum_{K=0}^{7} e^{-\frac{j3\pi K}{4}} X(K) = x((0-3))_8$$

$$\frac{1}{8}\sum_{K=0}^{7} e^{-\frac{j3\pi K}{4}} X(K) = x(5) \quad \because [8-3] = 5$$

or $$\sum_{K=0}^{7} e^{-\frac{j3\pi K}{4}} X(K) = 8x(5) = 8\times(-1) = -8 \text{ Ans}$$

(e)

We know that

$$\sum_{n=0}^{N-1} |x(n)|^2 = \frac{1}{N}\sum_{K=0}^{N-1} |X(K)|^2$$

$$\sum_{n=0}^{7} |x(n)|^2 = \frac{1}{8}\sum_{K=0}^{7} |X(K)|^2$$

$$\therefore \sum_{K=0}^{7} |X(K)|^2 = 8\sum_{n=0}^{7} |x(n)|^2 = 8\left[x(0)^2 + x(1)^2 + x(2)^2 + x(3)^2 + x(4)^2 + x(5)^2 + x(6)^2 + x(7)^2\right] = 8[36] = 288$$

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(*) Show that with $x(n)$ as an N-point sequence and $X(K)$ as its N-point DFT

$$\sum_{n=0}^{N-1} |x(n)|^2 = \frac{1}{N}\sum_{K=0}^{N-1} |X(K)|^2$$

$$a^2+b^2 = (a+ib)(a-ib) = (a+ib)(a+ib)^*$$

Solution

$$\sum_{n=0}^{N-1} |x(n)|^2 = \sum_{n=0}^{N-1} x(n)\, x^*(n)$$

$$= \sum_{n=0}^{N-1} x(n)\left[\frac{1}{N}\sum_{K=0}^{N-1} X(K)\, e^{j2\pi Kn/N}\right]^*$$

$$= \frac{1}{N}\sum_{n=0}^{N-1} x(n)\left[\sum_{K=0}^{N-1} X^*(K)\, e^{\frac{-j2\pi nK}{N}}\right]$$

$$= \frac{1}{N}\underbrace{\sum_{n=0}^{N-1} x(n)\, e^{\frac{-j2\pi nk}{N}}}_{X(k)} \sum_{K=0}^{N-1} X^*(K)$$

$$= \frac{1}{N}\sum_{K=0}^{N-1} X(K)\, X^*(K)$$

$$= \frac{1}{N}\sum_{K=0}^{N-1} |X(K)|^2 \quad \text{proved}$$

Circular Convolution of two Sequences. $y(n) = x(n) \; Ⓝ \; h(n)$

① The linear Convolution of two Sequences $x(n)$ of L number of Samples and $h(n)$ of M number of Samples the result $y(n)$ which consains $N = L + M - 1$

where $y(n) = x(n) * h(n)$

② Circular Convolution the situation is entirely different. If $x(n)$ contains L number of Samples and $h(n)$ has M number of Samples and that $L > M$ then we perform Circular Convolution between the two using $N = Max(L, M)$ by L-M number

(*) When Circular Convolution and Linear Convolution are Same.

(*) Find the response of an FIR filter with impulse response $h(n) = \{1, 2, 4\}$ to the input Sequences $x(n) = \{1, 2\}$.

Length of $h(n)$ is 3
" " $x(n)$ is 2

Convolution Length $= 3 + 2 - 1 = 4$.

$y(n) = x(n) * h(n)$

$x(n)$ \ $h(n)$	1	2	4
1	1	2	4
2	2	4	8

$y(n) = \{1, 4, 8, 8\}$

For Circular Convolution by Zero padding

$h(n) = \{1, 2, 4, 0\}$, $x(n) = \{1, 2, 0, 0\}$

$$\begin{bmatrix} 1 & 0 & 0 & 2 \\ 2 & 1 & 0 & 0 \\ 0 & 2 & 1 & 0 \\ 0 & 0 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 4 \\ 0 \end{bmatrix} = \begin{bmatrix} 1\times1 + 0\times2 + 0\times4 + 0\times0 \\ 2\times1 + 1\times2 + 0\times4 + 0\times0 \\ 0\times1 + 2\times2 + 1\times4 + 0\times0 \\ 0\times1 + 0\times2 + 2\times4 + \cancel{1\times0} \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 4 \\ \end{bmatrix}$$

① Find the output $y(n)$ of a filter whose impulse response is $h(n)=\{1,1,1\}$ and input signal $x(n)=\{3,-1,0,1,3,2,0,1,2,1\}$ using (i) overlap-save method (ii) overlap-add method

① overlap-save method

Ans: The input sequence can be divided blocks of data as follows.

$x_1(n)=\{\underbrace{0,0}_{M-1=3-1=2\text{ Zeros}}\ \underbrace{3,-1,0}_{L=3\text{ data points}}\}$ Length of $h(n)$ is 3

$x_2(n)=\{\underbrace{-1,0}_{}\ \underbrace{1,3,2}_{}\}$ $L+M-1 = 3+3-1=5$

Two datas from previous block, 3 new data points

$x_3(n)=\{3,2,0,1,2\}$

$x_4(n)=\{1,2,1,0,0\}$

then $h(n)=\{1,1,1\}$ by Zeroes padding

$h(n)=\{1,1,1,0,0\}$

$y_1(n)=h(n)\,Ⓝ\,x_1(n)=\{-1,0,3,2,2\}$

$$=\begin{bmatrix}1&0&0&1&1\\1&1&0&0&1\\1&1&1&0&0\\0&1&1&1&0\\0&0&1&1&1\end{bmatrix}\begin{bmatrix}0\\0\\3\\-1\\0\end{bmatrix}=\begin{bmatrix}-1\\0\\3\\2\\2\end{bmatrix}$$

$y_2(n)=h(n)\,Ⓝ\,x_2(n)=\{4,1,0,4,6\}$

$y_3(n)=h(n)\,Ⓝ\,x_3(n)=\{6,7,5,3,3\}$

$y_4(n)=x_4(n)\,Ⓝ\,h(n)=\{1,3,4,3,1\}$

$\boxed{-1,0,3,2,2}$
discard $\boxed{4,1,0,4,6}$
discard $\boxed{6,7,5,3,3}$
discard $\boxed{1,3,4,3,1}$
discard

Now

$y(n)=\{3,2,2,0,4,6,5,3,3,4,3,1\}$

(ii) Overlap-Add-method

Let the length of data block be 3 Two Zeros are added to bring the length to five $(L+M-1)=5$

$x_1(n)=\{3,-1,0,\underbrace{0,0}_{M-1=3-1=2\text{ Zeros are padding}}\}$

$x_2(n)=\{1,3,2,0,0\}$

$x_3(n)=\{0,1,2,0,0\}$

$x_4(n)=\{1,0,0,0,0\}$

$y_1(n)=h(n)\,Ⓝ\,x_1(n)=\{3,2,2,-1,0\}$

$$\begin{bmatrix}1&0&0&1&1\\1&1&0&0&1\\1&1&1&0&0\\0&1&1&1&0\\0&0&1&1&1\end{bmatrix}\begin{bmatrix}3\\-1\\0\\0\\0\end{bmatrix}=\begin{bmatrix}3\\2\\2\\-1\\0\end{bmatrix}$$

$y_2(n)=h(n)\,Ⓝ\,x_2(n)=\{1,4,6,5,2\}$

$y_3(n)=h(n)\,Ⓝ\,x_3(n)=\{0,1,3,3,2\}$

$y_4(n)=h(n)\,Ⓝ\,x_4(n)=\{1,1,1,0,0\}$

3 2 2 -1 0 add

1, 4, 6, 5, 2 add

0, 1, 3, 3, 2 add

1, 1, 1, 0, 0

$y(n) = \{3, 2, 2, 0, 4, 5, 3, 3, 4, 3, 1\}$

Assignment - 1

1. Find DFT of the sequence $x(n) = 1 \quad 0 \le n \le 3$
   $= 0 \quad 4 \le n \le 7$.

2. Find IDFT of $x(k) = \{4, 1-j2.414, 0, 1-j0.414, 0, 1+j0.414, 0, 1+j2.414\}$

3. Find the circular convolution of the two sequence $x_1(n) = \{1, 1, 2, 1\}$, $x_2(n) = \{1, 2, 3, 4\}$

4. Find the circular convolution of the following sequences $x(n) = \{1, 1, 1, 2\}$, $y(n) = \{1, 2, 3, 2\}$ using DFT and IDFT method.

5. perform linear convolution of finite duration sequence $h(n) = \{1, 1, 2, 1\}$ and $x(n) = \{1, -1, 1, 2, 1, 0, 1, -4, 3, 2, 1, 0, 1, 1\}$ by (a) overlap-add method (b) overlap-save method

6. Evaluate and compare the 8-point for the following sequence using DIT-FFT algorithm.

   (a) $x_1(n) = \begin{cases} 1 & \text{for } -3 \le n \le 3 \\ 0 & \text{otherwise} \end{cases}$

   (b) $x_2(n) = \begin{cases} 1 & \text{for } \le n \le 6 \\ 0 & \text{otherwise} \end{cases}$

7. Find the IDFT of the sequence
   $X(k) = \{1, 1+j, 2, 1-2j, 0, 1+2j, 0, 1-j\}$