

3. The number of stages in the flowgraph is given by $M = \log_2 N$.
4. Each stage consists of $\frac{N}{2}$ butterflies.
5. Inputs/outputs for each butterfly are separated by 2^{m-1} samples, where m represents the stage index, i.e., for first stage $m = 1$ and for second stage $m = 2$ so on.
- ✓ 6. The number of complex multiplications is given by $\frac{N}{2} \log_2 N$.
- ✓ 7. The number of complex additions is given by $N \log_2 N$.
8. The twiddle factor exponents are a function of the stage index m and is given by

$$k = \frac{Nt}{2^m} \quad t = 0, 1, 2, \dots, 2^{m-1} - 1. \quad (4.22)$$

9. The number of sets or sections of butterflies in each stage is given by the formula 2^{M-m} .
10. The exponent repeat factor (ERF), which is the number of times the exponent sequence associated with m is repeated is given by 2^{M-m} .

Example 4.1 Draw the Flow graph of 16-point DIT-FFT.

Solution

We can draw the flowgraph of 16-point DIT-FFT using the steps given in section 4.5.

Table 4.3 Bit-reversal process

Index	Binary Representation	Bit-reversal Order	Bit-reversal Index
0	0000	0000	0
1	0001	1000	8
2	0010	0100	4
3	0011	1100	12
4	0100	0010	2
5	0101	1010	10
6	0110	0110	6
7	0111	1110	14
8	1000	0001	1
9	1001	1001	9
10	1010	0101	5
11	1011	1101	13
12	1100	0011	3
13	1101	1011	11
14	1110	0111	7
15	1111	1111	15

4.12 Digital Signal Processing

1. The number of input samples, $N = 16$
2. The input sequence is shuffled through bit-reversal shown in table 4.3 and applied as input to the flow graph.
3. The number of stages $M = \log_2 16 = 4$.
4. The number of butterflies per stage is $\frac{N}{2} = 8$.
5. The inputs/outputs for each butterfly in stage m is separated by 2^{m-1} samples.

Stage 1 Inputs/outputs for each butterfly are separated by 1 sample.

Stage 2 Inputs/outputs for each butterfly are separated by 2 samples.

Stage 3 Inputs/outputs for each butterfly are separated by 4 samples.

Stage 4 Inputs/outputs for each butterfly are separated by 8 samples.

6. The number of complex multiplications is given by

$$\frac{N}{2} \log_2 N = 8 \log_2 16 = 32$$

7. The number of complex additions is given by $16 \log_2 16 = 64$.
8. The number of sets or sections of butterflies in each stage is given by 2^{M-m} .

For stage 1 the number of sets of butterflies are $2^{4-1} = 8$

For stage 2 the number of sets of butterflies are $2^{4-2} = 4$

For stage 3 the number of sets of butterflies are $2^{4-3} = 2$

For stage 4 there is only set of butterflies.

9. The twiddle factor exponents for each stage are given by

$$k = \frac{Nt}{2^m} \quad t = 0, 1, 2, \dots, 2^{m-1} - 1.$$

For Stage 1 the exponent is 0

For Stage 2 the exponents are 0, 4

For Stage 3 the exponents are 0, 2, 4, 6

For Stage 4 the exponents are 0, 1, 2, 3, 4, 5, 6, 7

10. The exponent repeat factor (ERF) which is the number of times the exponent sequence associated with m repeat is given by 2^{M-m} .

For stage 1 ERF = 8

For stage 2 ERF = 4

For stage 3 ERF = 2

For stage 4 ERF = 1

From the steps 8, 9, 10 we can draw the following conclusions.

For stage 1 the twiddle factor exponent is zero and is repeated 8 times.

(\therefore ERF = 8). Therefore, all the 8 sets of butterflies have twiddle factor as W_{16}^0 .