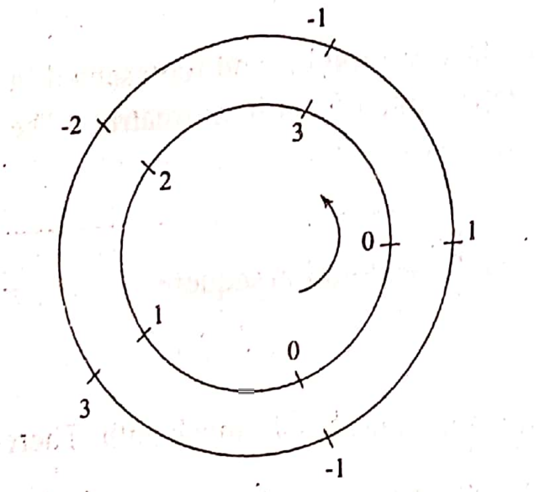
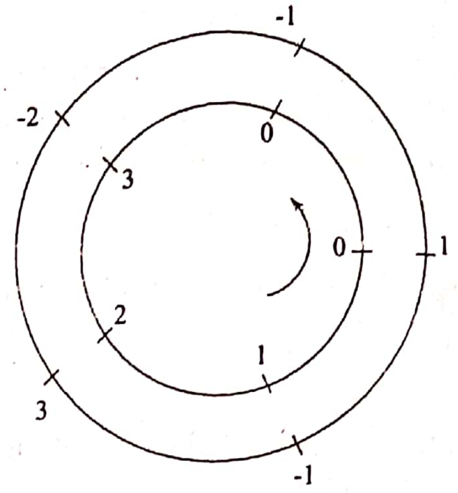




$$y(3) = (0)1 + 3(-1) + 2(-2) + 1(3) + (-1)(0); \quad y(4) = 0(1) + 0(-1) + 3(-2) + 3(2) + 1(-1)$$
$$= -4 \qquad\qquad = -1$$

$$y(n) = \{8, -2, -1, -4, -1\}$$

Matrix Method

Given

$$x_1(n) = \{1, -1, -2, 3, -1\}$$
$$x_2(n) = \{1, 2, 3, \}$$

By adding two zeros to the sequence $x_2(n)$, we bring the length of the sequence $x_2(n)$ to 5.

Now

$$x_2(n) = \{1, 2, 3, 0, 0\}$$

The matrix form can be written by substituting $N = 5$ in Eq. (3.55).

$$\begin{bmatrix} x_2(0) & x_2(4) & x_2(3) & x_2(2) & x_2(1) \\ x_2(1) & x_2(0) & x_2(4) & x_2(3) & x_2(2) \\ x_2(2) & x_2(1) & x_2(0) & x_2(4) & x_2(3) \\ x_2(3) & x_2(2) & x_2(1) & x_2(0) & x_2(4) \\ x_2(4) & x_2(3) & x_2(2) & x_2(1) & x_2(0) \end{bmatrix} \begin{bmatrix} x_1(0) \\ x_1(1) \\ x_1(2) \\ x_1(3) \\ x_1(4) \end{bmatrix} = \begin{bmatrix} y(0) \\ y(1) \\ y(2) \\ y(3) \\ y(4) \end{bmatrix}$$

Represent the sequence $x_2(n)$ in $N \times N$ matrix form and $x_1(n)$ in column matrix form and multiply to get $y(n)$.

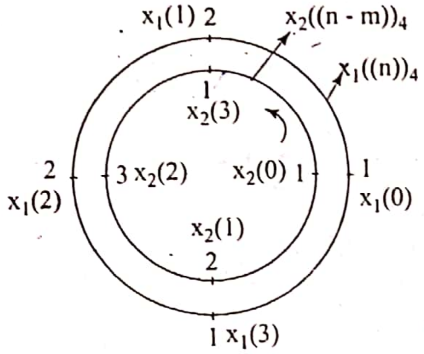
$$\begin{array}{ccccc} x_2(n) & & x_1(n) & & y(n) \\ \begin{bmatrix} 1 & 0 & 0 & 3 & 2 \\ 2 & 1 & 0 & 0 & 3 \\ 3 & 2 & 1 & 0 & 0 \\ 0 & 3 & 2 & 1 & 0 \\ 0 & 0 & 3 & 2 & 1 \end{bmatrix} & & \begin{bmatrix} 1 \\ -1 \\ -2 \\ 3 \\ -1 \end{bmatrix} & = & \begin{bmatrix} 8 \\ -2 \\ -1 \\ -4 \\ -1 \end{bmatrix} \end{array}$$

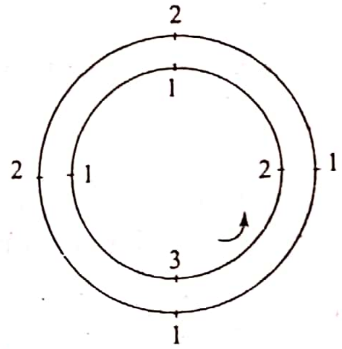
$$y(n) = \{8, -2, -1, -4, -1\}$$

Example 3.14 Find the circular convolution of the two sequences $x_1(n) = \{1, 2, 2, 1\}$ and $x_2(n) = \{1, 2, 3, 1\}$ using (a) concentric circle method (b) matrix method

Solution

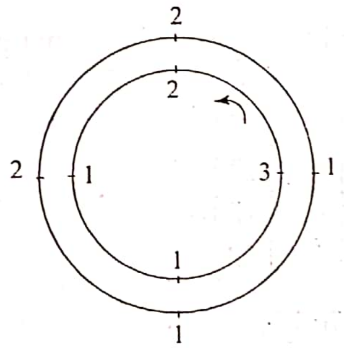
(a) Concentric Circle Method

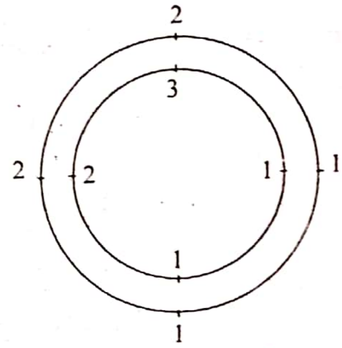




$$y(0) = 1(1) + 2(1) + 2(3) + 1(2) = 11$$

$$y(1) = 1(2) + 2(1) + 2(1) + 1(3) = 9$$





$$y(2) = 1(3) + 2(2) + 2(1) + 1(1) = 10$$

$$y(3) = 1(1) + 2(3) + 2(2) + 1(1) = 12$$

$$y(n) = \{11, 9, 10, 12\}$$

(b) Matrix Method: Represent $x_2(n)$ in $N \times N$ matrix form and $x_1(n)$ in column matrix form, i.e., substitute $N = 4$ in Eq. (3.55)

$$\begin{bmatrix} x_2(0) & x_2(3) & x_2(2) & x_2(1) \\ x_2(1) & x_2(0) & x_2(3) & x_2(2) \\ x_2(2) & x_2(1) & x_2(0) & x_2(3) \\ x_2(3) & x_2(2) & x_2(1) & x_2(0) \end{bmatrix} \begin{bmatrix} x_1(0) \\ x_1(1) \\ x_1(2) \\ x_1(3) \end{bmatrix} = \begin{bmatrix} y(0) \\ y(1) \\ y(2) \\ y(3) \end{bmatrix}$$

Substitute the sequence values and multiply as shown below

$$\begin{bmatrix} 1 & 1 & 3 & 2 \\ 2 & 1 & 1 & 3 \\ 3 & 2 & 1 & 1 \\ 1 & 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 11 \\ 9 \\ 10 \\ 12 \end{bmatrix}$$

$$y(n) = \{11, 9, 10, 12\}$$