FAST FOURIER TRANSFORM (FFT)

① N point DFT, N^2 complex multiplication and $N(N-1)$ Complex additions are required $\overset{DFT}{\text{and}}$ IDFT Also

② N point FFT and IFFT, $\frac{N}{2}\log_2 N$ complex multiplication and $N\log_2 N$ complex additions.

For $N=8$ point DFT

ⓐ complex multiplication $= 8^2 = 64$.

ⓑ " Addition $8(8-1) = 56$

For $N=8$ point FFT

ⓐ Complex multiplication $= \frac{8}{2}\log_2 8 = 4\times3 = 12$

ⓑ " Addition $= 8\log_2 8 = 8\times 3 = 24$

⊛ Given $x(n) = \{1,2,3,4,4,3,2,1\}$, find $X(k)$ using DIF FFT algorithm.

For $N = 8 = 2^3$ the stage $= 3$

The twiddle factor exponents for each stage are given by

$$K = \frac{Nt}{2^m}, \quad t = 0,1,2,\ldots 2^{m-1}-1$$

(i) For stage-1 $m=1$ then $t=0$,

$$K = \frac{8.0}{2^1} = 0, \quad W_8^0 = 1.$$

(ii) For Stage-2, $m = 2$ then $t = 0, 1$

$$K = \frac{8.0}{2^2} = 0, \quad K = \frac{8.1}{2^2} = \frac{8}{4} = 2$$

$$W_8^0 = 1, \quad W_8^2 = e^{\frac{-j2\times2\pi}{8}} = e^{-j\frac{\pi}{2}} = \cos\frac{\pi}{2} - j\sin\frac{\pi}{2}$$

$$= -j$$

(iii) For Stage-3 $m = 3$, then $t = 0, 1, 2, 3$

$2^{3-1} - 1 = 2^2 - 1 = 4 - 1 = 3$,

$$K = \frac{8.0}{2^3} = 0, \quad W_8^0 = 1$$

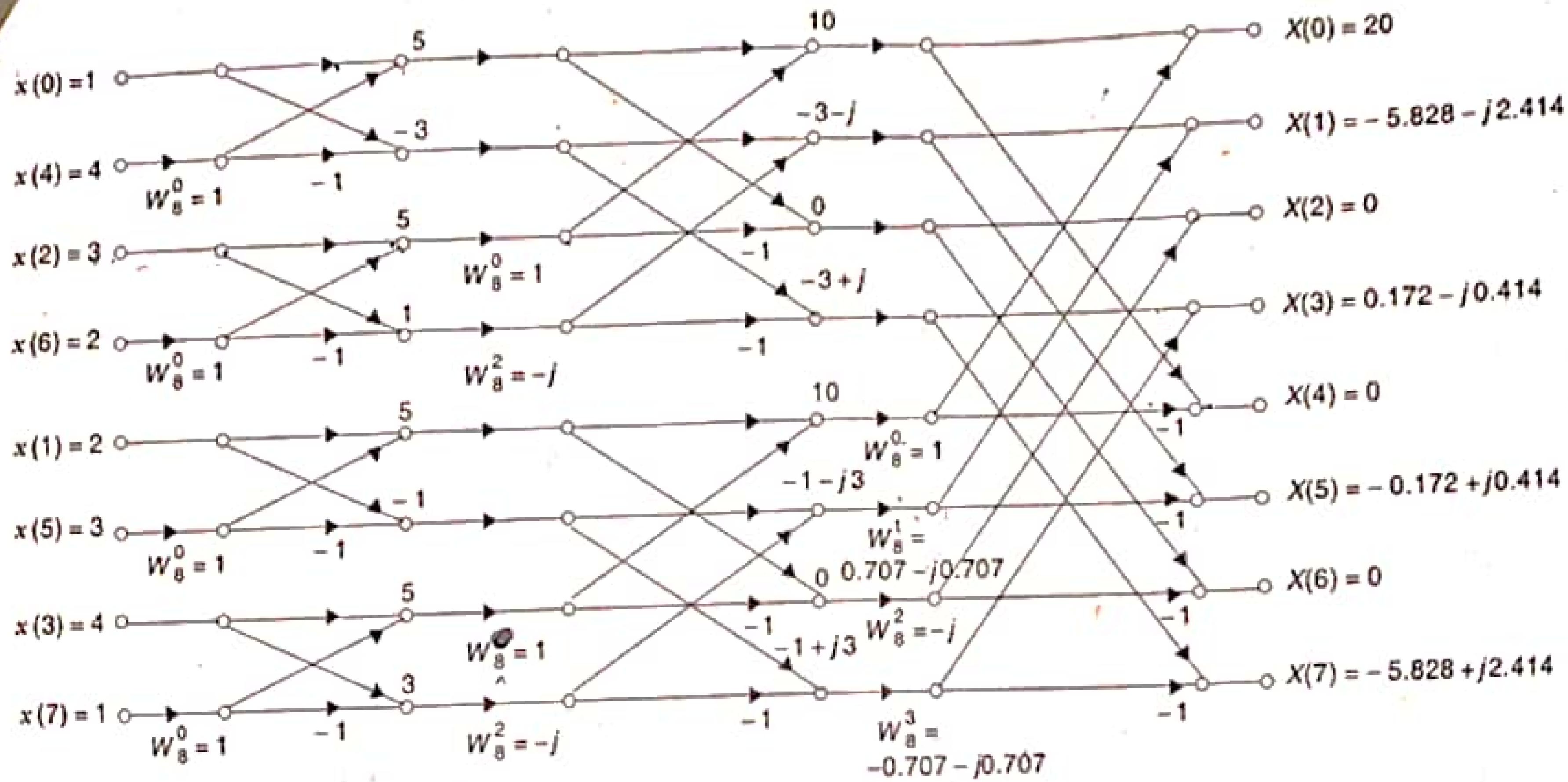
$$K = \frac{8.1}{8} = 1 \quad W_8^1 = 0.707 - j0.707$$

$$K = \frac{8\times2}{8} = 2 \quad W_8^2 = e^{-j\frac{\pi}{2}} = \cos\frac{\pi}{2} - j\sin\frac{\pi}{2}$$

$$= -j$$

$$K = \frac{8\times3}{8} = 3 \quad W_8^3 = e^{\frac{-j(2\pi)3}{8}} = \cos\frac{3\pi}{4} - j\sin\frac{3\pi}{4}$$

$$= -0.707 - j0.707.$$



x(0) = 1
x(4) = 4
x(2) = 3
x(6) = 2
x(1) = 2
x(5) = 3
x(3) = 4
x(7) = 1
$W_8^0 = 1$
$W_8^0 = 1$
$W_8^0 = 1$
$W_8^0 = 1$
−1
5
−3
5
1
5
−1
5
3
$W_8^0 = 1$
$W_8^2 = -j$
$W_8^0 = 1$
$W_8^2 = -j$
10
−3 − j
0
−3 + j
10
−1 − j3
0
−1 + j3
$W_8^0 = 1$
$W_8^1 =$ 0.707 − j0.707
$W_8^2 = -j$
$W_8^3 =$ −0.707 − j0.707
X(0) = 20
X(1) = − 5.828 − j2.414
X(2) = 0
X(3) = 0.172 − j0.414
X(4) = 0
X(5) = − 0.172 + j0.414
X(6) = 0
X(7) = − 5.828 + j2.414

Fig. E6.15

then page → 3.30 Assigement-2

* (1) Find $X(K)$ by DIT FFT Algorithm of Given $x(n) = \{0, 1, 2, 3, 4, 5, 8, 7\}$

(2) Given $x(n) = 2^n$ and $N = 8$ find $X(K)$ using DIT FFT algorithm.

(3) Evaluate and Compare the 8-point for the following sequence using DIT-FFT algorithm.

(a) $x_1(n) = \begin{cases} 1 & \text{for } -3 \leq n \leq 3 \\ 0 & \text{otherwise} \end{cases}$

(b) $x_2(n) = \begin{cases} 1 & \text{for } 0 \leq n < 6 \\ 0 & \text{otherwise} \end{cases}$

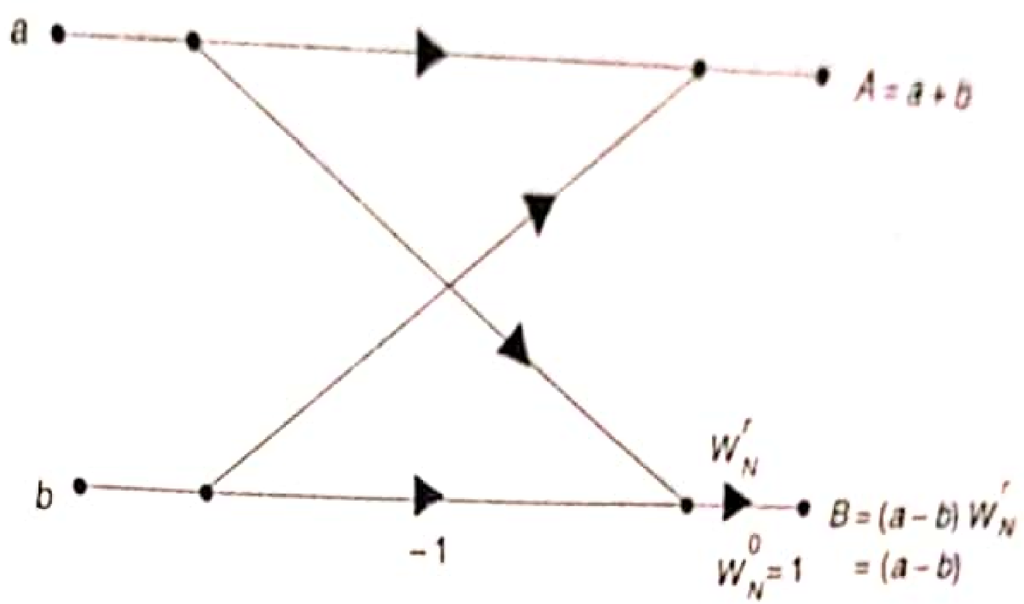


Fig. 6.14 *Basic Butterfly for DIF FFT*

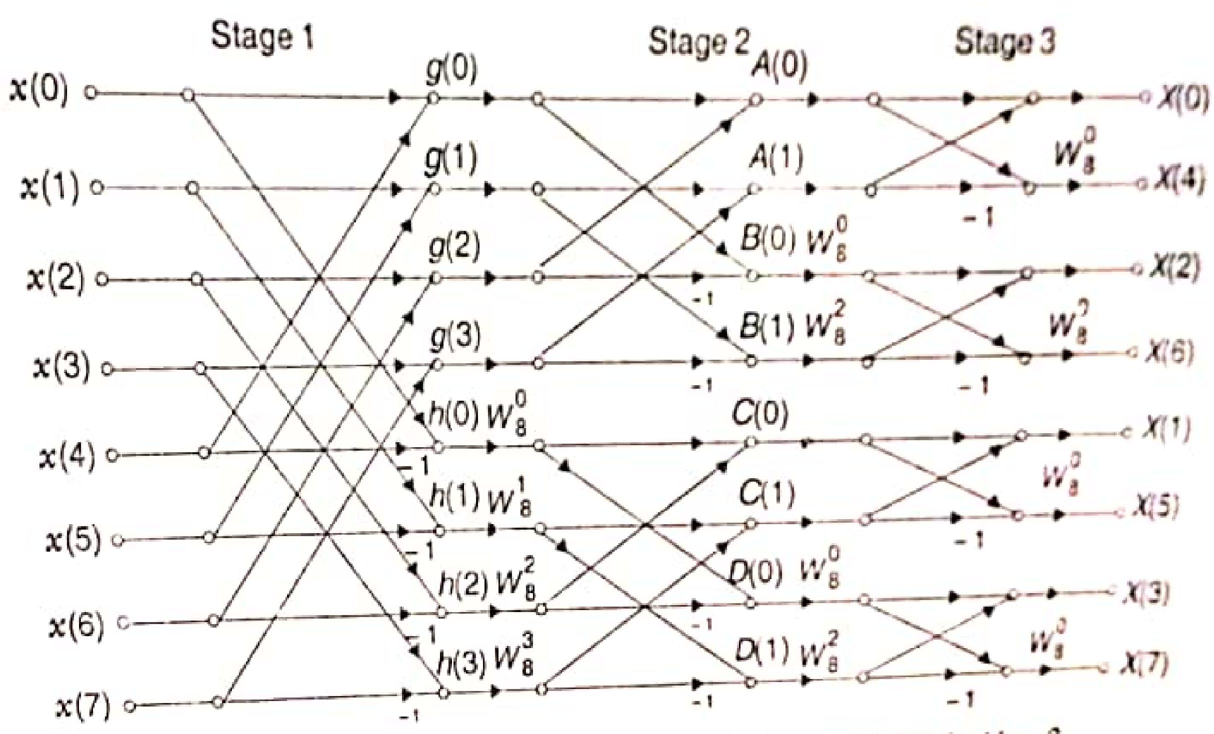


Fig. 6.15 *Reduced Flow Graph of Final Stage DIF FFT for N = 8*

(4) Given $x(n) = \{1, 2, 3, 4, 4, 3, 2, 1\}$, find $X(k)$ using DIF FFT algorithm.

Solution $N = 8$, the stage $= 3$.

The twiddle factor exponents are a function of the stage index m and is given by $K = \frac{Nt}{2^{M-m+1}}$, $t = 0, 1, 2, \dots 2^{M-m} - 1$

1 For stage-1

$M = 3$, $m = 1$, then $t = 0, 1, 2, 3$

$K = \frac{8 \times 0}{2^{3-1+1}} = 0$, $W_8^0 = 1$

$K = \frac{8 \times 1}{8} = 1$ $W_8^1 = 0.707 - j0.707$

$K = \frac{8 \times 2}{8} = 2$ $W_8^2 = -j$

$K = \frac{8 \times 3}{8} = 3$ $W_8^3 = -0.707 - j0.707$.

(2) For stage-2

$M = 3$, $m = 2$, then $t = 0, 1$

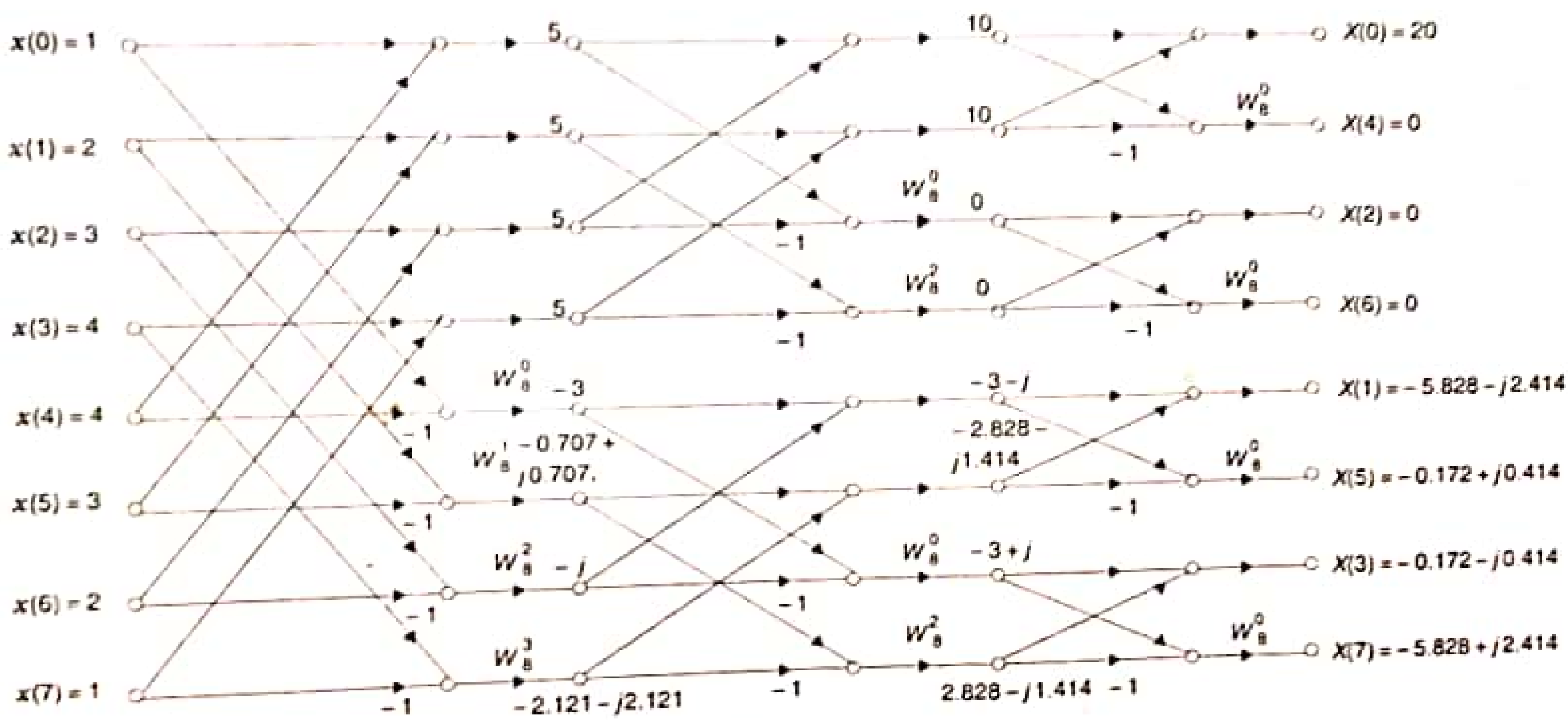
$K = \frac{8 \times 0}{2^{3-2+1}} = 0$, $W_8^0 = 1$

$K = \frac{8 \times 1}{2^2} = 2$ $W_8^2 = -j$

(3) For stage-3

$M = 3$, $m = 3$. then $t = 0$, $2^{3-3} - 1 = 1 - 1 = 0$

$K = \frac{8 \times 0}{2^{3-3+1}} = 0$ $W_8^0 = 1$



x(0) = 1
x(1) = 2
x(2) = 3
x(3) = 4
x(4) = 4
x(5) = 3
x(6) = 2
x(7) = 1
5
5
5
5
-3
-0.707 + j0.707
-j
-2.121 - j2.121
10
10
0
0
-3 - j
-2.828 - j1.414
-3 + j
2.828 - j1.414
-1
X(0) = 20
X(4) = 0
X(2) = 0
X(6) = 0
X(1) = -5.828 - j2.414
X(5) = -0.172 + j0.414
X(3) = -0.172 - j0.414
X(7) = -5.828 + j2.414

Fig. E6.19

(*) Given $x(k) = \{20, -5.828 - j2.414, 0, -0.172 - j0.414, 0, -0.172 + j0.414, 0, -5.828 + j2.414\}$ find $x(n)$.

Solution: We know that $W_N^K = e^{-j\left(\frac{2\pi}{N}\right)K}$ Given $N = 8$

Hence

$W_8^{-0} = 1$, $W_8^{-1} = 0.707 + j0.707$

$W_8^{-2} = j$, $W_8^{-3} = -0.707 + j0.707$

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$\cancel{X(K)=}$

$x(n) = \{1, 2, 3, 4, 4, 3, 2, 1\}$

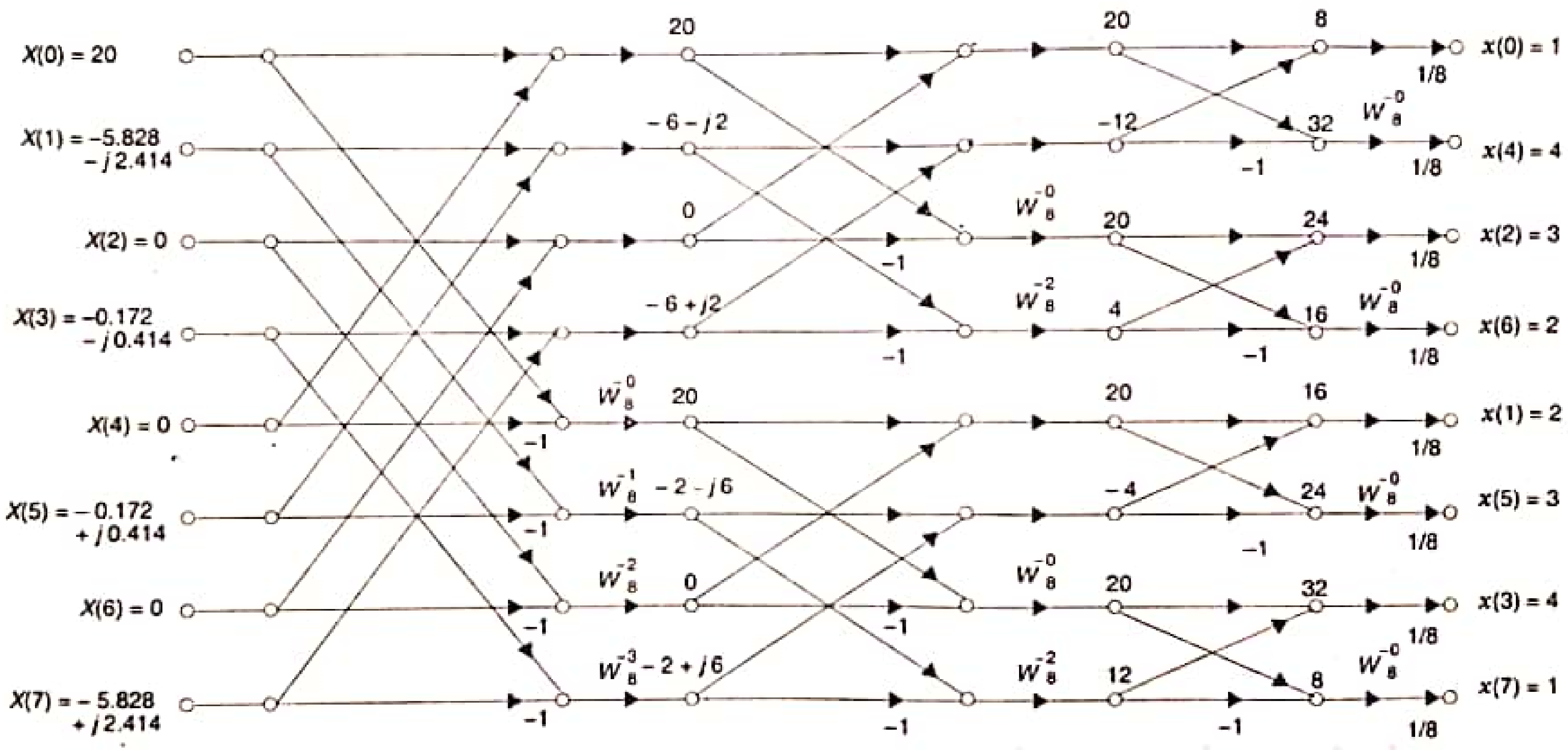
X(0) = 20
X(1) = −5.828 − j2.414
X(2) = 0
X(3) = −0.172 − j0.414
X(4) = 0
X(5) = −0.172 + j0.414
X(6) = 0
X(7) = −5.828 + j2.414
20
−6 − j2
0
−6 + j2
20
−2 − j6
0
−2 + j6
20
−12
20
4
20
−4
20
12
8
32
24
16
16
24
32
8
1/8
x(0) = 1
x(4) = 4
x(2) = 3
x(6) = 2
x(1) = 2
x(5) = 3
x(3) = 4
x(7) = 1

Fig. E6.24