Example 5.24 Determine the direct form II and Transposed direct form II for the given system $y(n) = \frac{1}{2}y(n-1) - \frac{1}{4}y(n-2) + x(n) + x(n-1)$

Solution

The system transfer function of the given difference equation is given by

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1 + z^{-1}}{1 - 0.5z^{-1} + 0.25z^{-2}}$$

By inspection, the direct form II realization can be obtained as shown in Fig. 5.42. The signal flow graph of Fig. 5.42 is shown in Fig. 5.43.

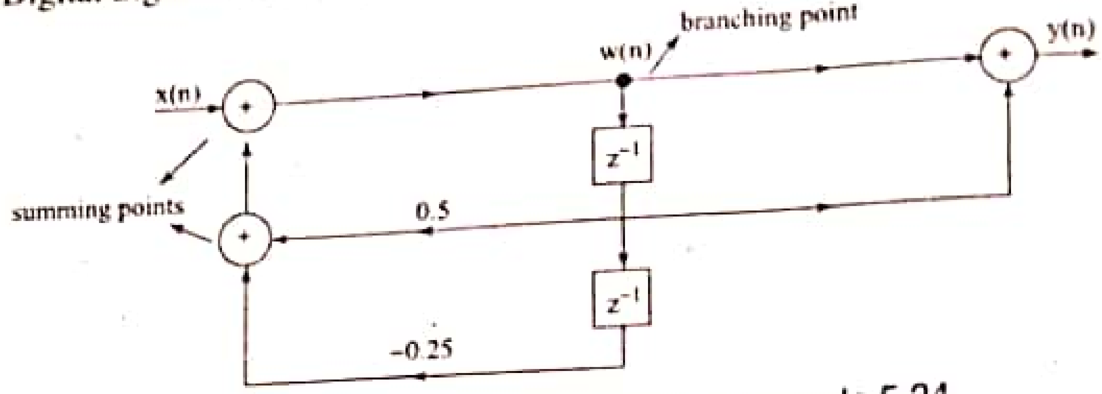


Fig. 5.42 Direct form-II realization of example 5.24

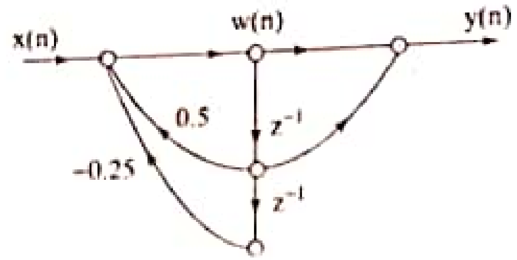


Fig. 5.43 Signal flowgraph representation of Fig. 5.42

To get transposed direct form II do the following operations.

(i) Change the direction of all branches.

(ii) Interchange the i ɔut and output.

(iii) Change the summing point to branching point and vice versa.

Then we obtain

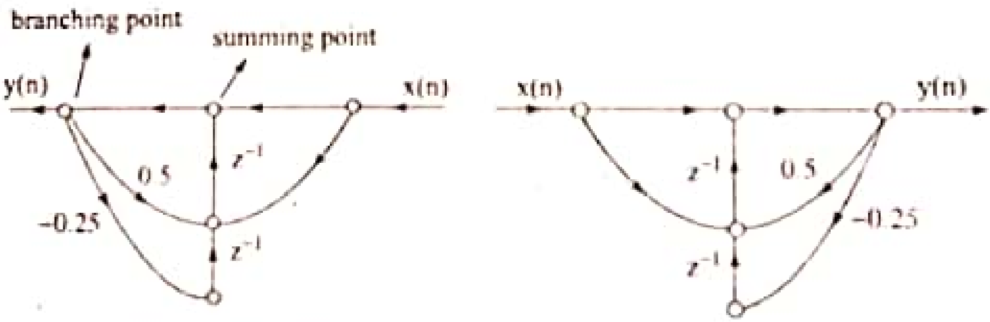


Fig. 5.44 Steps of operations in transposition.

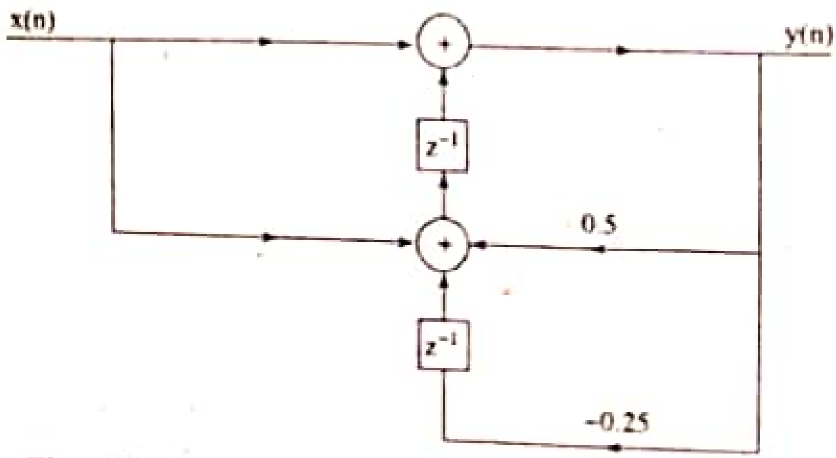


Fig. 5.45 Transposition structure of example 5.24

Practice Problem 5.13 Obtain the transposed direction form II structure of the following system

$$y(n) = \frac{1}{2}y(n-1) + \frac{1}{4}y(n-2) + x(n) + x(n-1)$$

5.14.5 *Cascade Form*

Let us consider an IIR system with system function

$$H(z) = H_1(z)H_2(z)\ldots H_k(z) \tag{5.122a}$$

This can be represented using block diagram as shown in Fig. 5.46.

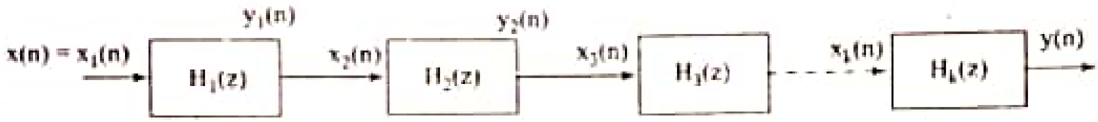


Fig. 5.46 Block diagram representation of Eq. (5.122a)

Now realize each $H_k(z)$ in direct form II and cascade all structures.
For example let us take a system whose transfer function

$$H(z) = \frac{(b_{k0} + b_{k1}z^{-1} + b_{k2}z^{-2})(b_{m0} + b_{m1}z^{-1} + b_{m2}z^{-2})}{(1 + a_{k1}z^{-1} + a_{k2}z^{-2})(1 + a_{m1}z^{-1} + a_{m2}z^{-2})} \tag{5.122b}$$

$$= H_1(z)H_2(z)$$

where $H_1(z) = \dfrac{b_{k0} + b_{k1}z^{-1} + b_{k2}z^{-2}}{1 + a_{k1}z^{-1} + a_{k2}z^{-2}}$ and

$$H_2(z) = \frac{b_{m0} + b_{m1}z^{-1} + b_{m2}z^{-2}}{1 + a_{m1}z^{-1} + a_{m2}z^{-2}}$$

Realizing $H_1(z)$ and $H_2(z)$ in direct form II, and cascading we obtain cascade form of the system function.

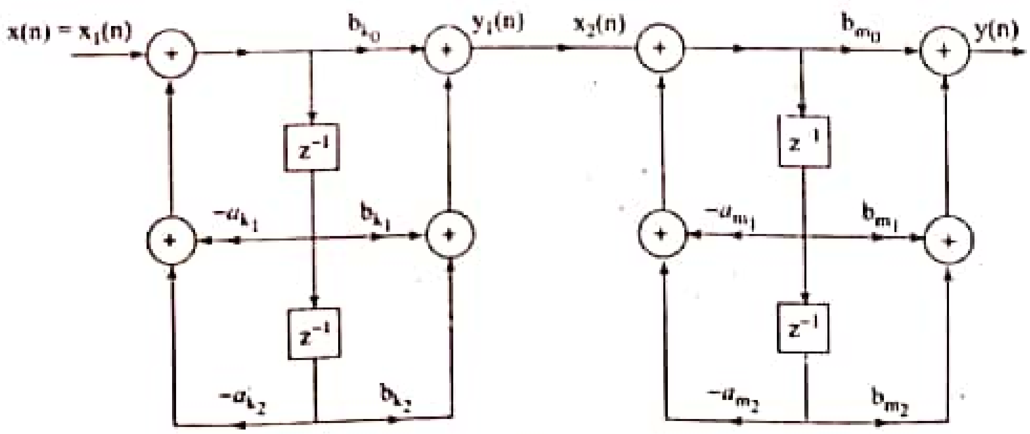


Fig. 5.47 Cascade realization of Eq. (5.122b)

Example 5.25 Realize the system with difference equation $y(n) = \frac{3}{4}y(n-1) - \frac{1}{8}y(n-2) + x(n) + \frac{1}{3}x(n-1)$ in cascade form.

Solution

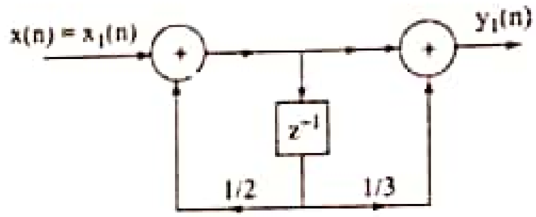
From the difference equation we obtain

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1+\frac{1}{3}z^{-1}}{1-\frac{3}{4}z^{-1}+\frac{1}{8}z^{-2}}$$

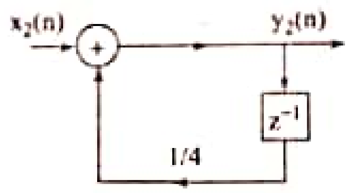
$$= \frac{1+\frac{1}{3}z^{-1}}{(1-\frac{1}{2}z^{-1})(1-\frac{1}{4}z^{-1})} = H_1(z)H_2(z)$$

where $H_1(z) = \frac{1+\frac{1}{3}z^{-1}}{1-\frac{1}{2}z^{-1}}$ and $H_2(z) = \frac{1}{1-\frac{1}{4}z^{-1}}$.

$H_1(z)$ can be realized in direct form II as



Similarly, $H_2(z)$ can be realized in direct form II as



Cascading the realization of $H_1(z)$ and $H_2(z)$ we have

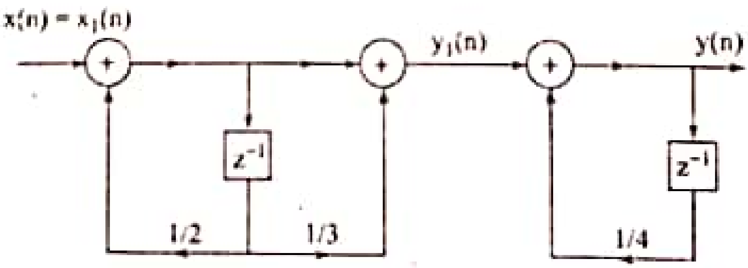


Fig. 5.48 Cascade realization of Example 5.25

Practice Problem 5.14 For the system function

$$H(z) = \frac{1+2z^{-1}+z^{-2}}{1-\frac{3}{4}z^{-1}+\frac{1}{8}z^{-2}}$$

obtain cascade structure.

5.14.6 *Parallel form structure*

A parallel form realization of an IIR system can be obtained by performing a partial expansion of

$$H(z) = c + \sum_{k=1}^{N} \frac{c_k}{1 - p_k z^{-1}} \tag{5.123}$$

where $\{p_k\}$ are the poles

The Eq. (5.123) can be written as

$$H(z) = c + \frac{c_1}{1 - p_1 z^{-1}} + \frac{c_2}{1 - p_2 z^{-1}} + \ldots + \frac{c_N}{1 - p_N z^{-1}} \tag{5.124}$$

$$H(z) = \frac{Y(z)}{X(z)} = c + H_1(z) + H_2(z) + \ldots + H_N(z) \tag{5.125}$$

Now

$$Y(z) = cX(z) + H_1(z)X(z) + H_2(z)X(z) + \ldots + H_N(z)X(z) \tag{5.126}$$

The Eq. (5.126) can be realized in parallel form as shown in Fig. 5.49.

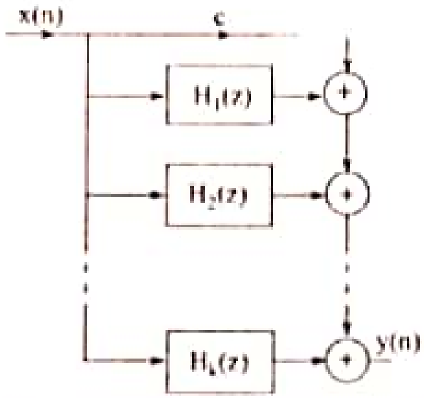


Fig. 5.49 Parallel form realization of Eq. 5.126

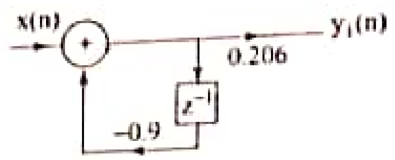
Example 5.26 Realize the system given by difference equation $y(n) = -0.1\, y(n-1) + 0.72y(n-2) + 0.7x(n) - 0.252x(n-2)$ in parallel form.

Solution

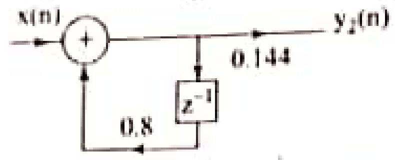
The system function of the difference equation is

$$\begin{aligned} H(z) &= \frac{0.7 - 0.252z^{-2}}{1 + 0.1z^{-1} - 0.72z^{-2}} \\ &= 0.35 + \frac{0.35 - 0.035z^{-1}}{1 + 0.1z^{-1} - 0.72z^{-2}} \\ &= 0.35 + \frac{0.206}{1 + 0.9z^{-1}} + \frac{0.144}{1 - 0.8z^{-1}} \\ &= c + H_1(z) + H_2(z) \end{aligned}$$

$H_1(z)$ can be realized in direct form II as



$H_2(z)$ can be realized in direct form II as



Now the realization of $H(z)$ is shown in Fig. 5.50.

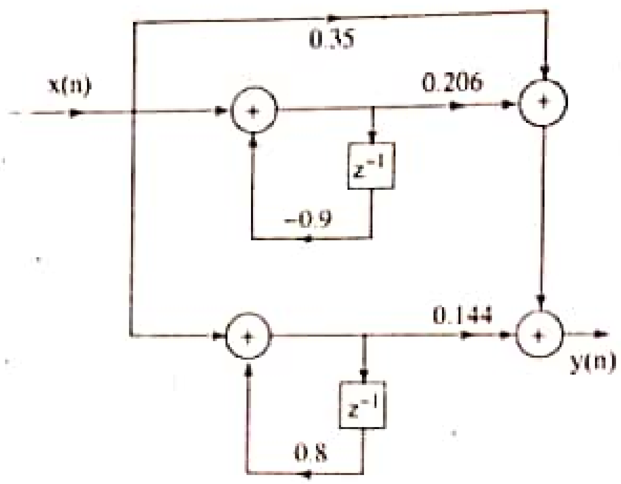


Fig. 5.50 Parallel form realization of Example 5.26

Practice Problem 5.15 For the system function

$$H(z) = \frac{1 + z^{-1} + z^{-2}}{\left(1 + \frac{1}{2}z^{-1}\right)\left(1 + \frac{1}{6}z^{-1}\right)}$$

obtain parallel structure.

Example 5.27 Obtain the direct form I, direct form II, cascade and parallel form realization for the system $y(n) = -0.1y(n-1) + 0.2y(n-2) + 3x(n) + 3.6x(n-1) + 0.6x(n-2)$

Solution

Direct form I

Let $3x(n) + 3.6x(n-1) + 0.6x(n-2) = w(n)$

$y(n) = -0.1y(n-1) + 0.2y(n-2) + w(n)$

By inspection, the direct form I realization is shown in Fig. 5.51.

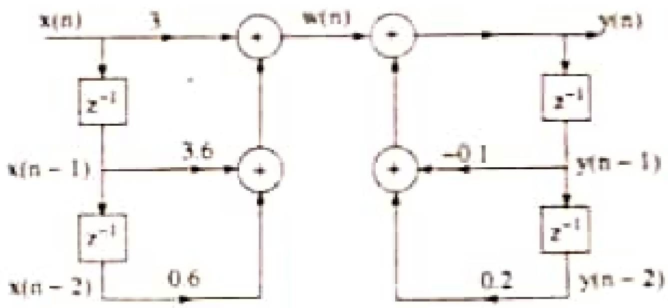


Fig. 5.51 Direct form I realization of example 5.27

Direct form II

From the given difference equation we have

$$H(z) = \frac{Y(z)}{X(z)} = \frac{3 + 3.6z^{-1} + 0.6z^{-2}}{1 + 0.1z^{-1} - 0.2z^{-2}}$$

The above system function can be realized in direct form II as shown in Fig. 5.52.

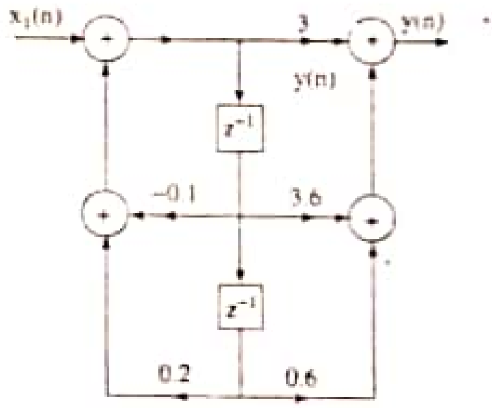


Fig. 5.52 Direct form II realization of example 5.27

Cascade form

$$\text{we have} \quad \frac{Y(z)}{X(z)} = \frac{3 + 3.6z^{-1} + 0.6z^{-2}}{1 + 0.1z^{-1} - 0.2z^{-2}}$$

$$= \frac{(3 + 0.6z^{-1})(1 + z^{-1})}{(1 + 0.5z^{-1})(1 - 0.4z^{-1})}$$

$$\text{Let} \quad H_1(z) = \frac{3 + 0.6z^{-1}}{1 + 0.5z^{-1}} \quad \text{and}$$

$$H_2(z) = \frac{1 + z^{-1}}{1 - 0.4z^{-1}}$$

Now we realize $H_1(z)$ and $H_2(z)$ and cascade both to get realization of $H(z)$

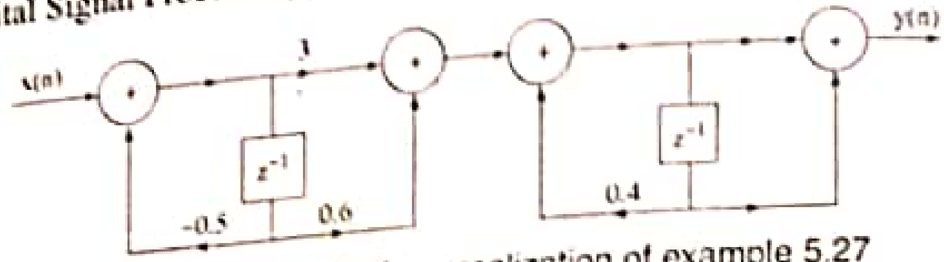


Fig. 5.53 Cascade form realization of example 5.27

Parallel form

$$H(z) = \frac{3 + 3.6z^{-1} + 0.6z^{-2}}{1 + 0.1z^{-1} - 0.2z^{-2}}$$

$$= -3 + \frac{7}{1 - 0.4z^{-1}} - \frac{1}{1 + 0.5z^{-1}}$$

$$= c + H_1(z) + H_2(z)$$

$$\begin{array}{r|l} & \quad -3 \\ -0.2z^{-2} + 0.1z^{-1} + 1 & \overline{0.6z^{-2} + 3.6z^{-1} + 3} \\ & \underline{0.6z^{-2} - 0.3z^{-1} - 3} \\ & \qquad 3.9z^{-1} + 6 \end{array}$$

$$\to \quad H(z) = -3 + \frac{3.9z^{-1} + 6}{1 + 0.1z^{-1} - 0.2z^{-2}}$$

$$= -3 + \frac{A}{1 - 0.4z^{-1}} + \frac{B}{1 + 0.5z^{-1}}$$

where $A = 7, B = -1$

Now we realize $H(z)$ in parallel form as shown in Fig. 5.54.

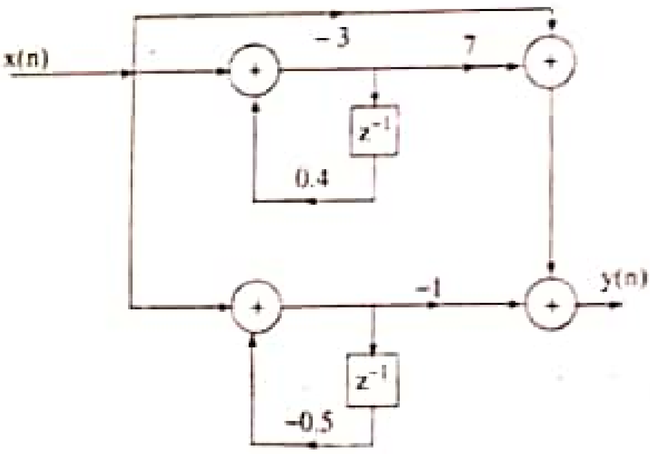


Fig. 5.54 Parallel form realization of example 5.27

Example 5.28 Obtain the cascade realization for the following systems

(a) $$H(z) = \frac{\left(1 + \frac{3}{2}z^{-1} + \frac{1}{2}z^{-2}\right)\left(1 - \frac{3}{2}z^{-1} + z^{-2}\right)}{\left(1 + z^{-1} + \frac{1}{4}z^{-2}\right)\left(1 + \frac{1}{4}z^{-1} + \frac{1}{2}z^{-2}\right)}$$

(b) $$H(z) = \frac{\left(1 - \frac{1}{2}z^{-1}\right)\left(1 - \frac{1}{2}z^{-1} + \frac{1}{4}z^{-2}\right)}{\left(1 + \frac{1}{4}z^{-1}\right)\left(1 + z^{-1} + \frac{1}{2}z^{-2}\right)\left(1 - \frac{1}{4}z^{-1} + \frac{1}{2}z^{-2}\right)}$$

Solution

(a) $$H(z) = \frac{(1+\frac{3}{2}z^{-1}+\frac{1}{2}z^{-2})(1-\frac{3}{2}z^{-1}+z^{-2})}{(1+z^{-1}+\frac{1}{4}z^{-2})(1+\frac{1}{4}z^{-1}+\frac{1}{2}z^{-2})}$$
$$= H_1(z)H_2(z)$$

where $H_1(z) = \dfrac{(1+\frac{3}{2}z^{-1}+\frac{1}{2}z^{-2})}{(1+z^{-1}+\frac{1}{4}z^{-2})}$ and

$$H_2(z) = \frac{(1-\frac{3}{2}z^{-1}+z^{-2})}{(1+\frac{1}{4}z^{-1}+\frac{1}{2}z^{-2})}$$

Realizing $H_1(z)$ and $H_2(z)$ and cascading we have

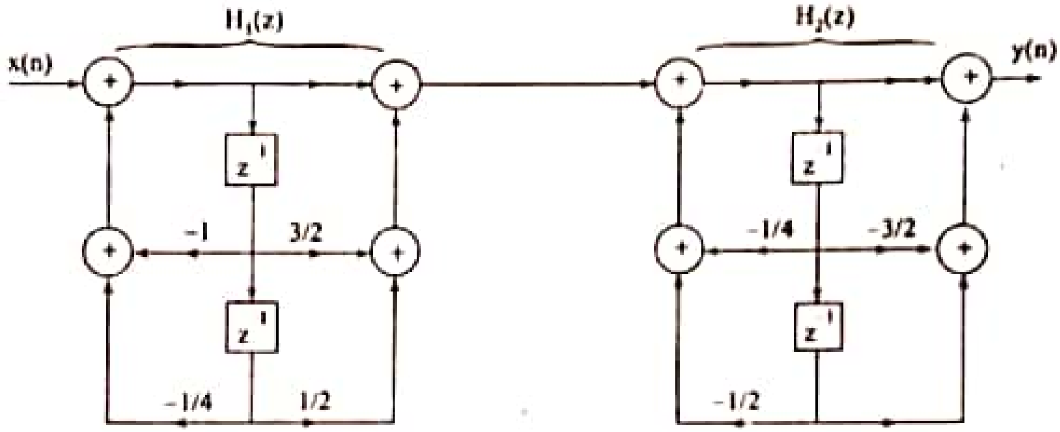


Fig. 5.55 Cascade realization of example 5.28(a)

(b) Let $H(z) = H_1(z)H_2(z)H_3(z)$ where $H_1(z) = \dfrac{1-\frac{1}{2}z^{-1}}{1+\frac{1}{4}z^{-1}}$,

$$H_2(z) = \frac{1-\frac{1}{2}z^{-1}+\frac{1}{4}z^{-2}}{1+z^{-1}+\frac{1}{2}z^{-2}}$$

$$H_3(z) = \frac{1}{1-\frac{1}{4}z^{-1}+\frac{1}{2}z^{-2}}.$$

Realizing $H_1(z)$, $H_2(z)$ and $H_3(z)$ and cascading we have

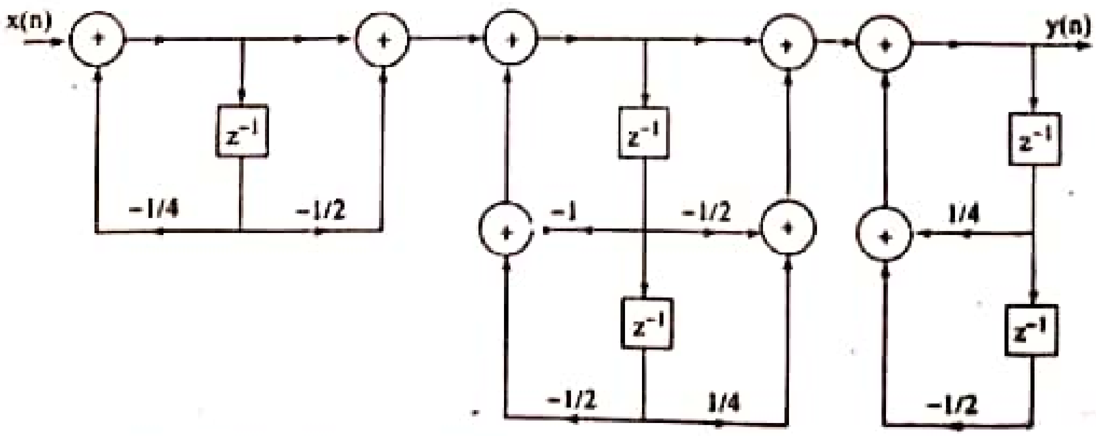


Fig. 5.56 Cascade form realization of example 5.28(b)