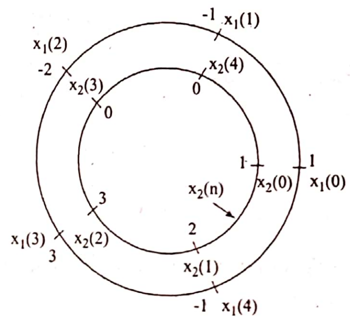
The sequence $x_2(n)$ is repeated via circular shift of samples and represented in $N \times N$ matrix form. The sequence $x_1(n)$ is represented as column matrix. The multiplication of these two matrices gives the sequences $x_3(n)$.

Example 3.13 Find the circular convolution of two finite duration sequences $x_1(n) = \{1, -1, -2, 3, -1\}$; $x_2(n) = \{1, 2, 3\}$

Solution To find circular convolution, both sequences must be of same length. Therefore we append two zeros to the sequence $x_2(n)$ and use concentric circle method to find circular convolution.

We have

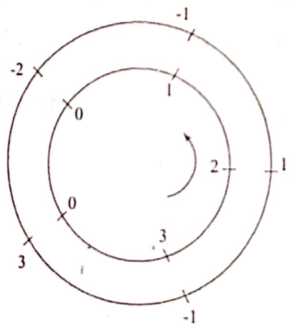
$$x_1(n) = \{1, -1, -2, 3, -1\}$$
$$x_2(n) = \{1, 2, 3, 0, 0\}$$

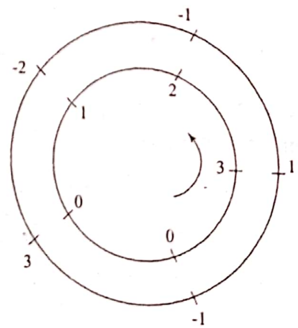


Graph all the points of $x_1(n)$ on the outer circle in the counterclockwise direction. Starting at same point as $x_1(n)$ graph all points of $x_2(n)$ on the inner circle in clockwise direction.
Multiply corresponding samples on the circle and add to obtain

$$y(0) = 1(1) + 0(-1) + 0(-2) + 3(3) + 2(-1)$$
$$= 8$$

Rotate the inner circle in counterclockwise direction by one sample, multiply the corresponding samples to obtain $y(1)$.





$y(1) = 1(2) + (-1)1 + (-2)0 + 3(0) + 3(-1);$
$= -2$

$y(2) = 3(1) + 2(-1) + 1(-2) + 0(3) + -1(0)$
$= -1$

Obtain remaining samples by repeating above procedure until the inner circle first sample lines up with the first sample of the exterior circle.