5.14 Realization of Digital filters

A digital filter transfer function can be realized in a variety of ways. There are two types of realizations 1. Recursive, 2. Nonrecursive

1. For recursive realization the current output $y(n)$ is a function of past outputs, past and present inputs. This form corresponds to an Infinite Impulse Response (IIR) digital filter. In this section we discuss this type of realization.

2. For non-recursive realization current output sample $y(n)$ is a function of only past and present inputs. This form corresponds to a Finite Impulse Response (FIR) digital filter.

IIR filter can be realized in many forms. They are

1. Direct form - I realization,
2. Direct form - II realization,
3. Transposed direct form realization,
4. Cascade form realization,
5. Parallel form realization,
6. Lattice - ladder structure.

5.14.1 *Direct Form I realization*

Let us consider an LTI recursive system described by the difference equation.

$$y(n) = -\sum_{k=1}^{N} a_k y(n-k) + \sum_{k=0}^{M} b_k x(n-k) \tag{5.102}$$

$$\begin{aligned} &= -a_1 y(n-1) - a_2 y(n-2) \ldots - a_{N-1} y(n-N+1) \\ &\quad - a_N y(n-N) + b_0 x(n) + b_1 x(n-1) + \ldots + b_M x(n-M) \end{aligned} \tag{5.103}$$

Let

$$b_0 x(n) + b_1 x(n-1) + \ldots + b_M x(n-M) = w(n). \tag{5.104}$$

then $y(n) = -a_1 y(n-1) - a_2 y(n-2) + \ldots - a_N y(n-N) + w(n)$ (5.105)
The Eq. (5.104) can be realized as shown in Fig. 5.28.

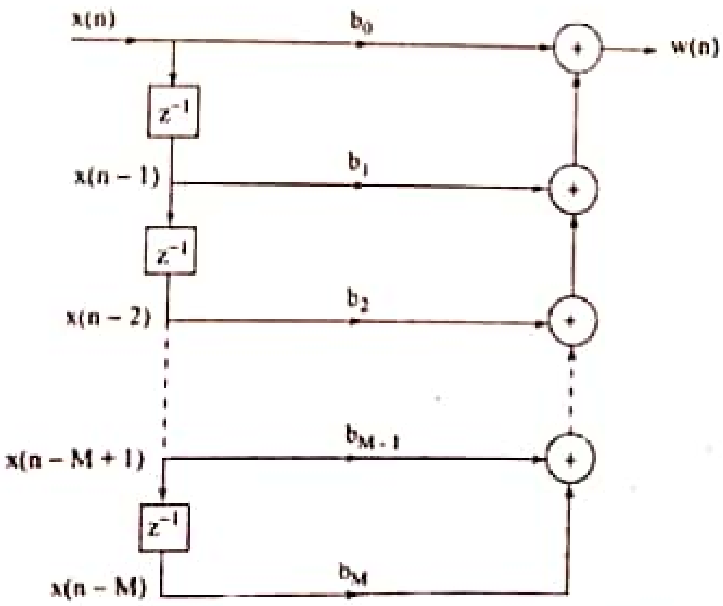

Fig. 5.28 Realization structure of Eq. (5.104)

Similarly the Eq.(5.105) can be realized as shown in Fig. 5.29.
To realize the difference Eq.(5.103) combine Fig. (5.28) and Fig. (5.29).

The structure shown in Fig. 5.30 is called direct form I, which used separate delays for both input and output. This realization requires $M + N + 1$ multiplications $M + N$ additions and $M + N + 1$ memory locations.

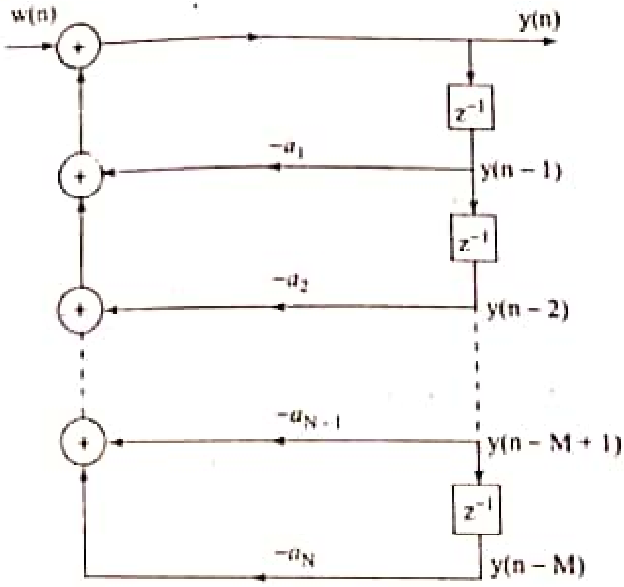

Fig. 5.29 Realization Structure of Eq. (5.105)

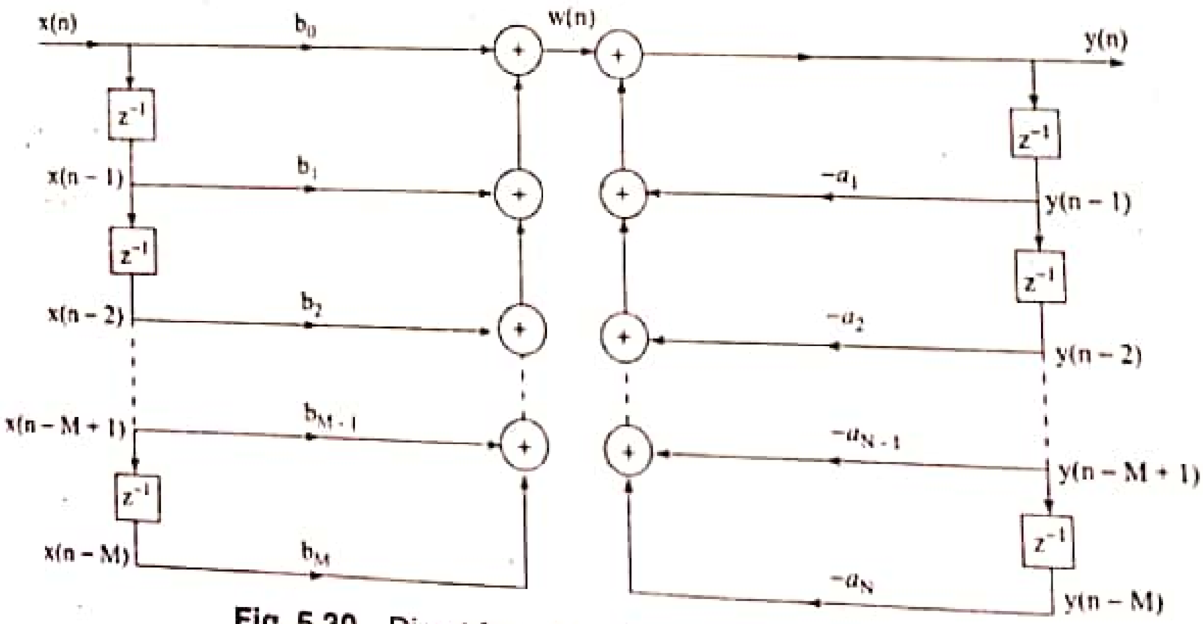

Fig. 5.30 Direct form I Realization of Eq.(5.103).

Example 5.20 Realize the second order digital filter $y(n) = 2r\cos(\omega_0)y(n-1) - r^2y(n-2) + x(n) - r\cos(\omega_0)x(n-1)$

Solution

Let

$$x(n) - r\cos(\omega_0)x(n-1) = w(n) \tag{5.106}$$

then

$$y(n) = 2r\cos(\omega_0)y(n-1) - r^2y(n-2) + w(n) \tag{5.107}$$

Realizing Eq.(5.106) we get

Fig. 5.31 Realization of Equation 5.106

Realizing Eq.(5.107) we obtain

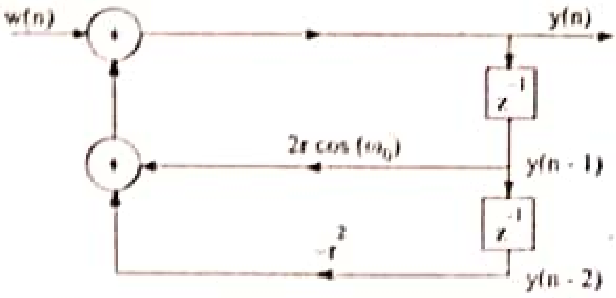

Fig. 5.32 Realization of Equation 5.107

If we combine both figures, we obtain the realization of the second order digital filter as shown in Fig. 5.33.

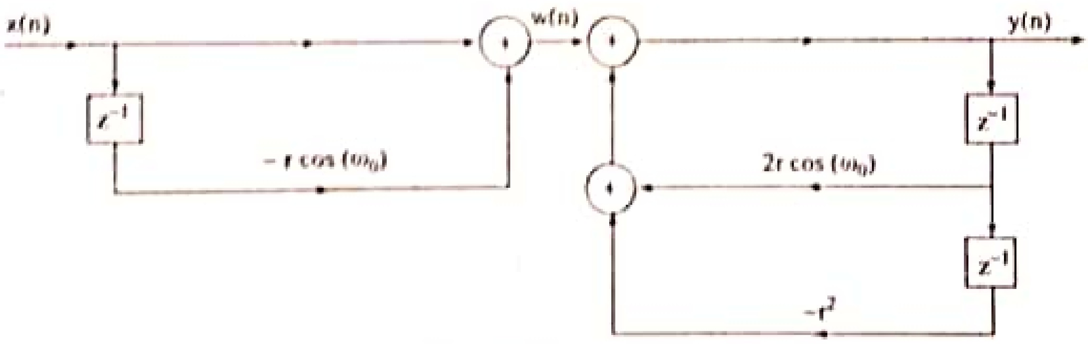

Fig. 5.33 Realization of Example 5.20

Example 5.21 Obtain the direct form-I realization for the system described by difference equation $y(n) = 0.5y(n-1) - 0.25y(n-2) + x(n) + 0.4x(n-1)$

Solution

Let

$$x(n) + 0.4x(n-1) = w(n) \tag{5.108}$$

then

$$y(n) = 0.5y(n-1) - 0.25y(n-2) + w(n) \tag{5.109}$$

Realizing Eq. (5.108) and Eq. (5.109) and combining we get

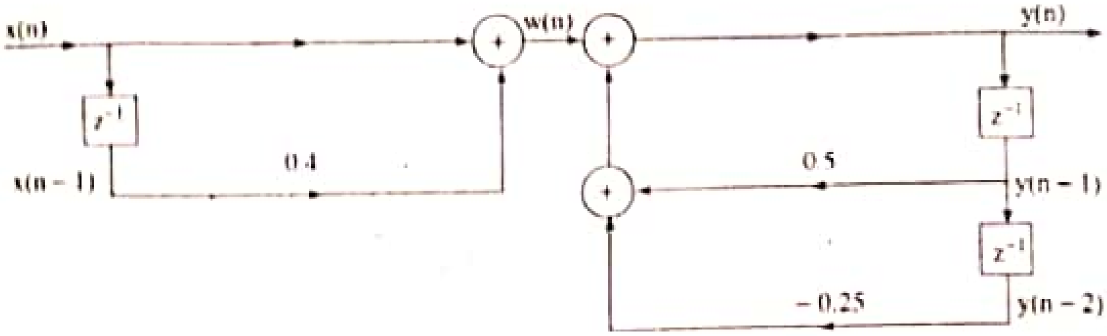

Fig. 5.34 Realization of Example 5.21

Practice Problem 5.11 Obtain the direct form-I realization for the systems described by the following difference equations

(i) $y(n) = 2y(n-1) + 3y(n-2) + x(n) + 2x(n-1) + 3x(n-2)$

(ii) $y(n) = 0.5y(n-1) + 0.06y(n-2) + 0.3x(n) + 0.5x(n-1)$

5.14.2 *Direct form II realization*

Consider the difference equation of the form

$$y(n) = -\sum_{k=1}^{N} a_k y(n-k) + \sum_{k=0}^{M} b_k x(n-k) \tag{5.110}$$

The system function of above difference equation can be expressed as

$$H(z) = \frac{Y(z)}{X(z)} = \frac{\sum_{k=0}^{M} b_k z^{-k}}{1 + \sum_{k=1}^{N} a_k z^{-k}} \tag{5.111}$$

Let $\dfrac{Y(z)}{X(z)} = \dfrac{Y(z)}{W(z)} \cdot \dfrac{W(z)}{X(z)}$ where

$$\frac{W(z)}{X(z)} = \frac{1}{1 + \sum_{k=1}^{N} a_k z^{-k}}$$

which gives us

$$W(z) = X(z) - a_1 z^{-1} W(z) - a_2 z^{-2} W(z) \ldots - a_N z^{-N} W(z) \tag{5.112}$$

and $\frac{Y(z)}{W(z)} = \sum_{k=0}^{M} b_k z^{-k}$ from which (5.113a)

$$Y(z) = b_0 W(z) + b_1 z^{-1} W(z) + b_2 z^{-2} W(z) + \ldots + b_M z^{-M} W(z) \quad (5.113b)$$

Now Eq.(5.112) and Eq:(5.113b) can be expressed in difference equation form i.e.,

$$w(n) = x(n) - a_1 w(n-1) - a_2 w(n-2) - \ldots - a_N w(n-N) \quad (5.114)$$

and

$$y(n) = b_0 w(n) + b_1 w(n-1) + b_2 w(n-2) + \ldots + b_M w(n-M) \quad (5.115)$$

From Eq. (5.114) and Eq.(5.115) we observe that the same delay terms $w(n-1)$, $w(n-2)\ldots$ etc, are used to express $w(n)$ and $y(n)$.

The realization of Eq.(5.114) and Eq.(5.115) are shown in Fig. (5.35) and Fig. (5.36) respectively.

To obtain the realization of difference Equation (5.110) combine Fig. (5.35) and Fig. (5.36).

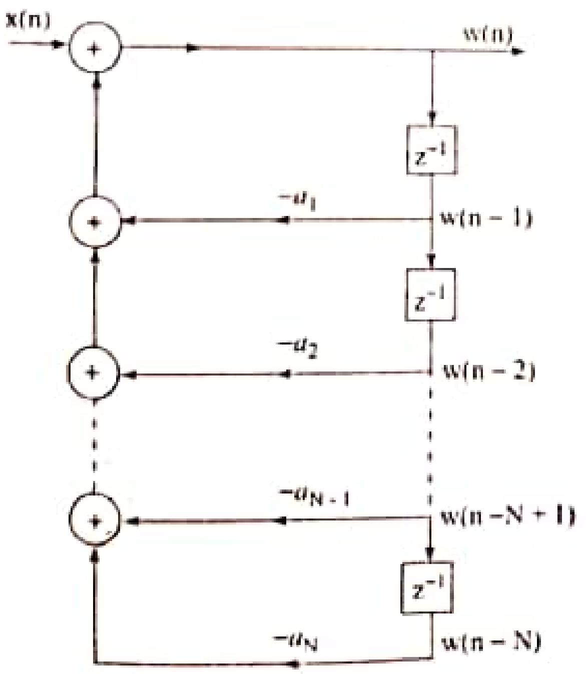

Fig. 5.35 Realization of Eq. (5.114).

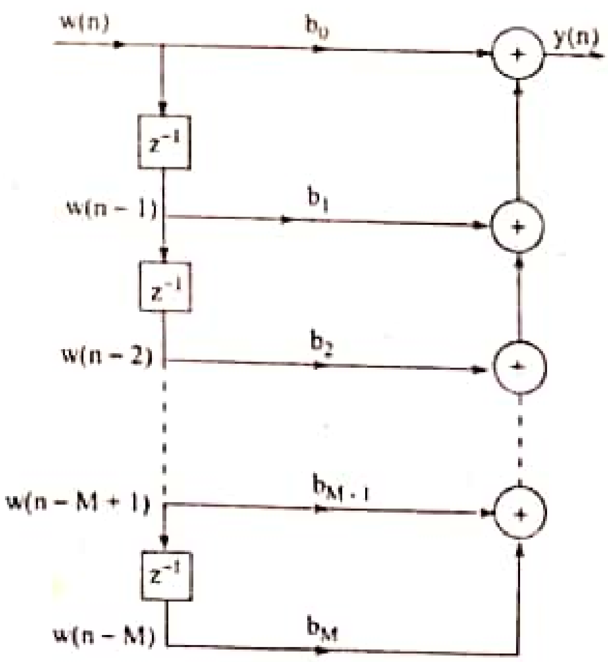

Fig. 5.36 Realization of Eq. (5.115).

From Fig. 5.37 we find that the two delay elements contain the same input $w(n)$ and hence the same output $w(n-1)$. Consequently we can merge these delays into one delay and can redraw the Fig. 5.37 as shown in Fig. 5.38.

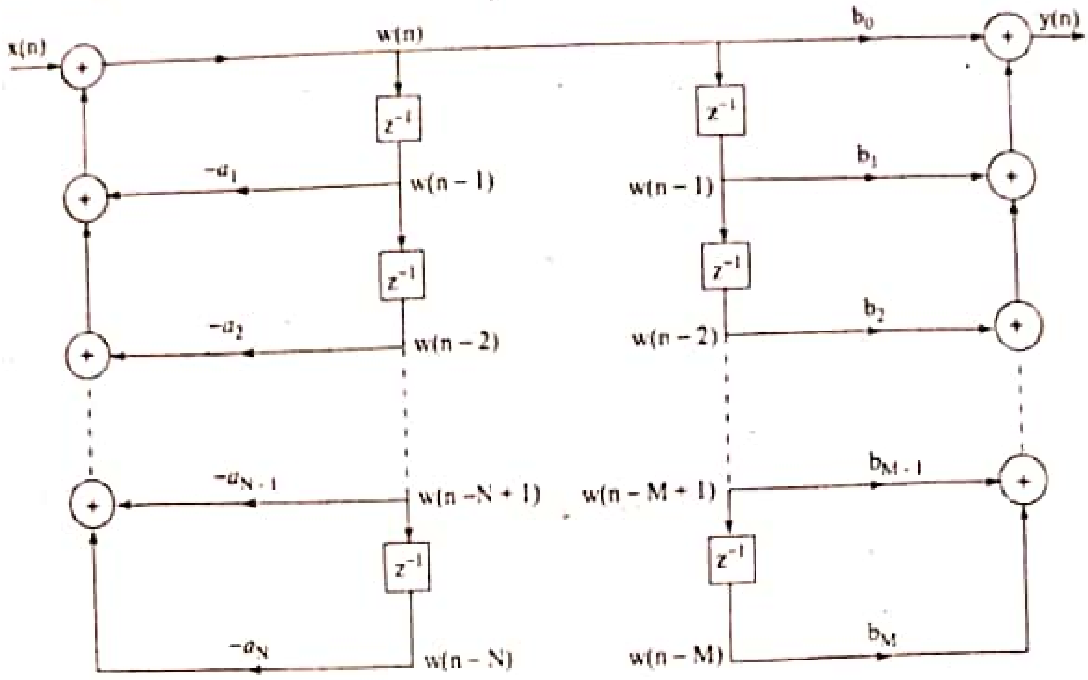

Fig. 5.37 Realization structure of Eq.(5.110)

The realization structure shown in Fig. 5.38 is called a direct form II realization. This structure requires $M+N+1$ multiplications, $M+N$ additions and the maximum of $\{M, N\}$ memory locations. Since the direct form II realization minimizes the number of memory locations, it is said to be canonic.

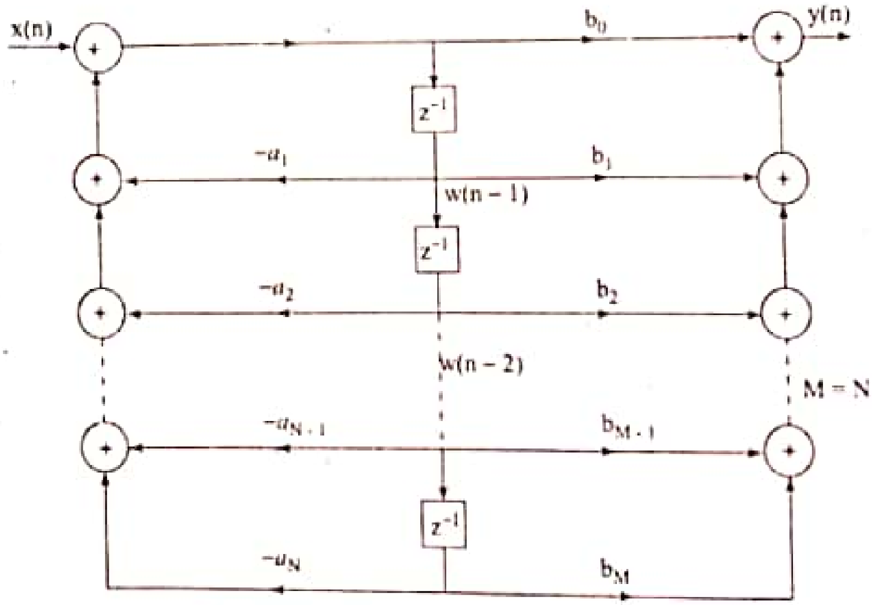

Fig. 5.38 Direct form II realization

Example 5.22 Realize the second order system $y(n) = 2r\cos(\omega_0)y(n-1) - r^2y(n-2) + x(n) - r\cos(\omega_0)x(n-1)$ in direct form II.

Solution

Given $y(n) = 2r\cos(\omega_0)y(n-1) - r^2y(n-2) + x(n) - r\cos(\omega_0)x(n-1)$

The system function

$$\frac{Y(z)}{X(z)} = \frac{1 - r\cos(\omega_0)z^{-1}}{1 - 2r\cos(\omega_0)z^{-1} + r^2z^{-2}} \tag{5.116}$$

Let $$\frac{Y(z)}{X(z)} = \frac{Y(z)}{W(z)} \cdot \frac{W(z)}{X(z)}$$

where $$\frac{Y(z)}{W(z)} = 1 - r\cos(\omega_0)z^{-1} \tag{5.117a}$$

and $$\frac{W(z)}{X(z)} = \frac{1}{1 - 2r\cos(\omega_0)z^{-1} + r^2z^{-2}} \tag{5.117b}$$

From Eq. (5.117a) we obtain $Y(z) = W(z) - r\cos(\omega_0)z^{-1}W(z)$ which gives us

$$y(n) = w(n) - r\cos(\omega_0)w(n-1) \tag{5.118a}$$

and from Eq. (5.117b) we have
$W(z) = X(z) + 2r\cos(\omega_0)z^{-1}W(z) - r^2z^{-2}W(z)$ which gives us

$$w(n) = x(n) + 2r\cos(\omega_0)w(n-1) - r^2w(n-2) \tag{5.118b}$$

We realize Eq. (5.118a) and Eq. (5.118b) and combine them to get the direct form II realization.

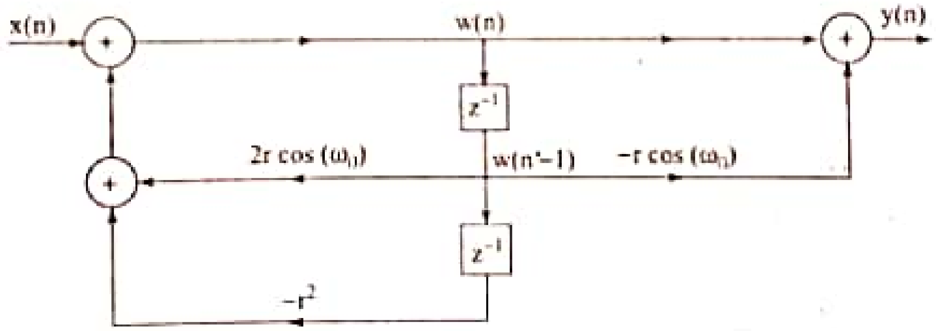

Fig. 5.39 Direct form II realization of example (5.21)

Example 5.23 Determine the direct form II realization for the following system $y(n) = -0.1y(n-1) + 0.72y(n-2) + 0.7x(n) - 0.252x(n-2)$

(AU ECE May'07)

Solution

The system function is given by

$$\frac{Y(z)}{X(z)} = \frac{0.7 - 0.252z^{-2}}{1 + 0.1z^{-1} - 0.72z^{-2}} \tag{5.119}$$

Let

$$\frac{Y(z)}{W(z)} = 0.7 - 0.252z^{-2}$$

$$Y(z) = 0.7W(z) - 0.252z^{-2}W(z)$$

Then

$$y(n) = 0.7w(n) - 0.252w(n-2) \tag{5.120}$$

Similarly let $\dfrac{W(z)}{X(z)} = \dfrac{1}{1 + 0.1z^{-1} - 0.72z^{-2}}$

$$W(z) = X(z) - 0.1z^{-1}W(z) + 0.72z^{-2}W(z)$$

then $w(n) = x(n) - 0.1w(n-1) + 0.72w(n-2)$ (5.121)

If we realize Eq. (5.120) and Eq. (5.121) and combine them we get direct form II realization of the system shown in Fig. 5.40.

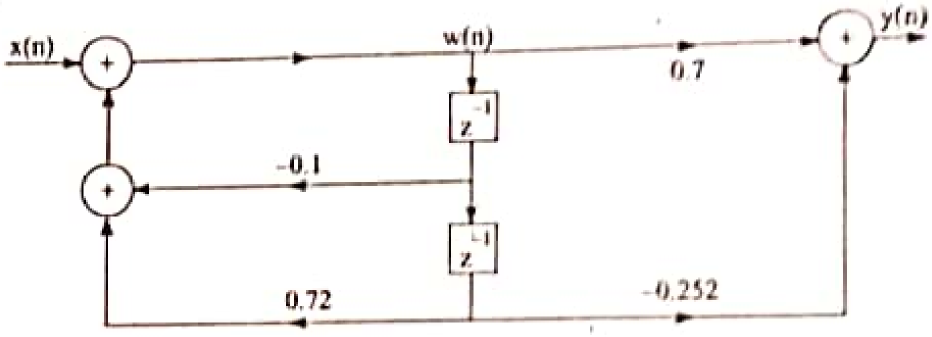

Fig. 5.40 Direct form II realization of example (5.23)

Practice Problem 5.12 Obtain the direct form-II realization for the systems described by the following difference equation

(i) $y(n) + y(n-1) - 4y(n-3) = x(n) + 3x(n-2)$

(ii) $y(n) = \dfrac{3}{4}y(n-1) - \dfrac{1}{8}y(n-2) + x(n) + \dfrac{1}{2}x(n-1)$

5.14.3 *Signal flowgraph*

A signal flowgraph is a graphical representation of the relationship between the variables of a set of linear difference equations. The basic elements of a signal flowgraph are branches and nodes. The signal flow graph is basically a set of directed branches that connect at nodes. A node represents a system variable, which is equal to the sum of incoming signals from all branches connecting to the node. There are two types of nodes. Source nodes are nodes that have no entering branches. Sink nodes are nodes that have only entering branches. A signal travels along a branch from one node to another node. The signal out of a branch is equal to the branch gain times the signal into the branch. The arrow head shows the direction of the branch and the branch gain is indicated next to the arrow head. The delay is indicated by the branch transmittance z^{-1}. When the branch transmittance is unity, it is left unlabeled.

Let us consider a block diagram representation of a first order digital filter shown in Fig. 5.41a. The system block diagram can be converted to the signal flow graph shown in Fig. 5.41b. We find that the flow graph contain four nodes, out of which two nodes are summing nodes, while the other two nodes represent branching points. Branch transmittances are indicated next to the arrowhead and the delay is indicated by the branch transmittance z^{-1}.

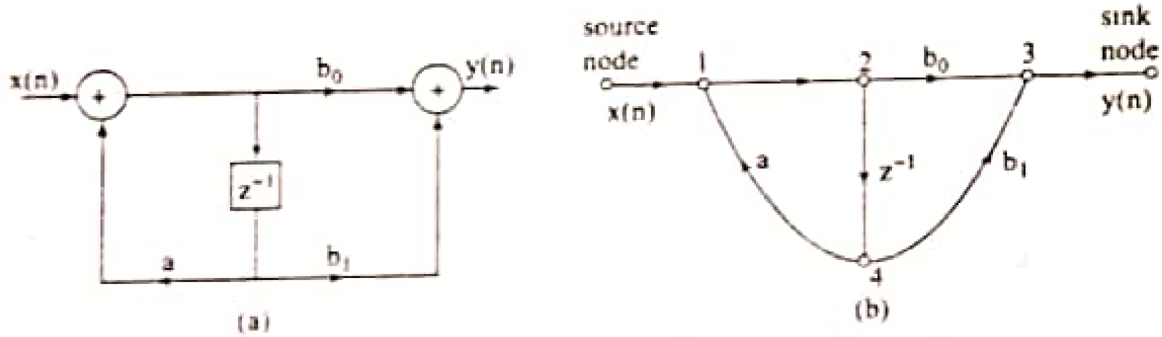

Fig. 5.41 (a) Block diagram representation of first-order digital filter (b) Signal flow graph representation of first-order digital filter

5.14.4 *Transposition theorem and transposed structure*

The transpose of a structure is defined by the following operations.

(i) Reverse the direction of all branches in the signal flow graph.
(ii) Interchange the inputs and outputs.
(iii) Reverse the roles of all nodes in the flowgraph.
(iv) Summing points become branching points.
(v) Branching points become summing points.

According to transposition theorem, the system transfer function remain unchanged by transposition.