

PT-A

23/9/20.

DAA

red

Q4.

$$1) \quad T(n) = 2T\left(\frac{n}{2}\right) + \frac{n}{\log n} \rightarrow (1)$$

$$2) \quad T\left(\frac{n}{2}\right) = 2T\left(\frac{1}{2} \times \frac{n}{2}\right) + \frac{n/2}{\log n/2}$$

$$3) \quad T\left(\frac{n}{2}\right) = 2T\left(\frac{n}{2^2}\right) + \frac{n/2}{\log n/2}$$

put in (1)

$$T(n) = 2 \left[2T\left(\frac{n}{2^2}\right) + \frac{n/2}{\log n/2} \right] + \frac{n}{\log n}$$

$$T(n) = 2^2 T\left(\frac{n}{2^2}\right) + \frac{n}{\log n/2} + \frac{n}{\log n} \rightarrow (2)$$

$$T\left(\frac{n}{2^2}\right) = 2T\left(\frac{1}{2} \times \frac{n}{2^2}\right) + \frac{n/2^2}{\log n/2^2} \text{ from (1)}$$

$$T\left(\frac{n}{2^2}\right) = 2T\left(\frac{n}{2^3}\right) + \frac{n/2^2}{\log n/2^2}$$

put in (2)

$$\Rightarrow T(n) = 2^2 \left[2T\left(\frac{n}{2^3}\right) + \frac{n/2^2}{\log n/2^2} \right] + \frac{n}{\log n/2} + \frac{n}{\log n}$$

Mahima Rai
2018BIT1123
1803213087

classmate

Date _____
Page _____

IT-A

23/9/20.

DAA

Q4.

$$a) T(n) = 2T\left(\frac{n}{2}\right) + \frac{n}{\log n} \rightarrow (1)$$

$$b) T\left(\frac{n}{2}\right) = 2T\left(\frac{1}{2} \times \frac{n}{2}\right) + \frac{n/2}{\log n/2}$$

$$c) T\left(\frac{n}{2}\right) = 2T\left(\frac{n}{2^2}\right) + \frac{n/2}{\log n/2}$$

put in (1)

$$T(n) = 2 \left[2T\left(\frac{n}{2^2}\right) + \frac{n/2}{\log n/2} \right] + \frac{n}{\log n}$$

$$T(n) = 2^2 T\left(\frac{n}{2^2}\right) + \frac{n}{\log n/2} + \frac{n}{\log n} \rightarrow (2)$$

$$T\left(\frac{n}{2^2}\right) = 2T\left(\frac{1}{2} \times \frac{n}{2^2}\right) + \frac{n/2^2}{\log n/2^2} \text{ from (1)}$$

$$T\left(\frac{n}{2^2}\right) = 2T\left(\frac{n}{2^3}\right) + \frac{n/2^2}{\log n/2^2}$$

put in (2)

$$T(n) = 2^2 \left[2T\left(\frac{n}{2^3}\right) + \frac{n/2^2}{\log n/2^2} \right] + \frac{n}{\log n/2} + \frac{n}{\log n}$$

$$T(n) = 2^3 T\left(\frac{n}{2^3}\right) + \frac{n}{\log n/2^2} + \frac{n}{\log n/2} + \frac{n}{\log n}$$

$$\log_2 2^{k-(k-1)}$$

$$\log_2 \frac{2^k}{2^{k-1}} \log_2 2^{k-(k-1)}$$

classmate

Date _____
Page _____

$$T(n) = 2^3 T\left[\frac{n}{2^3}\right] + n \left[\frac{1}{\log_2 \frac{n}{2^2}} + \frac{1}{\log_2 \frac{n}{2}} + \frac{1}{\log_2 n} \right]$$

k times.

$$T(n) = 2^k T\left[\frac{n}{2^k}\right] + n \left[\frac{1}{\log_2 \frac{n}{2^{k-1}}} + \frac{1}{\log_2 \frac{n}{2^{k-2}}} + \dots + \frac{1}{\log_2 \frac{n}{2}} + \frac{1}{\log_2 n} \right] \rightarrow (3)$$

Now

$$T(1) = 1$$

$$\frac{n}{2^k} = 1 \quad \text{or } n = 2^k \quad \text{or } k = \log_2 n$$

put in (3)

$$T(n) = n T(1) + n \left[\frac{1}{\log_2 2^k} + \frac{1}{\log_2 2^{k-1}} + \dots + \frac{1}{\log_2 2} \right]$$

$$\Rightarrow n + n \left[\frac{1}{\log_2 2^1} + \frac{1}{\log_2 2^2} + \dots + \frac{1}{\log_2 2^k} \right]$$

$$\log_2 2^{k-(k-1)}$$

$$n + n \left[\frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{k} \right]$$

$$n + n \log k$$

$$\therefore T.C. = n \log k$$

$$\Rightarrow n \log \log_2 n \quad \text{Ans.}$$

Q4. b). $T(n) = T(\sqrt{n}) + O(\log n)$

$$\Rightarrow T(n) = T(\sqrt{n}) + \log n$$

$$\Rightarrow \text{let } n = 2^k \quad k = \log_2 n$$

$$T(2^k) = T(2^{k/2}) + \log_2 2^k \quad \underline{k+1}$$

$$\Rightarrow T(2^k) = T(2^{k/2}) + k$$

\hookrightarrow let this be equal to S.

$$S(k) = S\left(\frac{k}{2}\right) + k$$

Applying masters

$$a = 1 \quad b = 2 \quad f(k) = k$$

$$\Rightarrow k \log_b a = k \log_2 1 = k^0 = k^0 \quad \text{①}$$

$$k^0$$

$$f(k) = k$$

$$k \log_b a = k^0$$

$$f(k) > k \log_b a$$

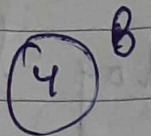
$$\therefore T.C. = \theta(f(k)) = \theta(\log_2 n)$$

QB 1

4, 3, 6, 1, 11, 5, 8, 9.

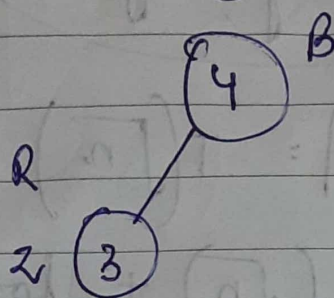
Step 1

4 →



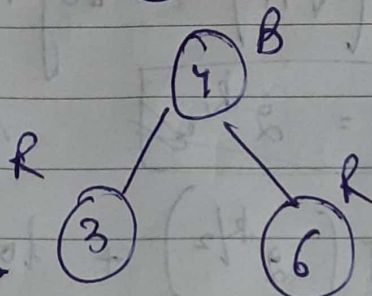
(2)

3 →



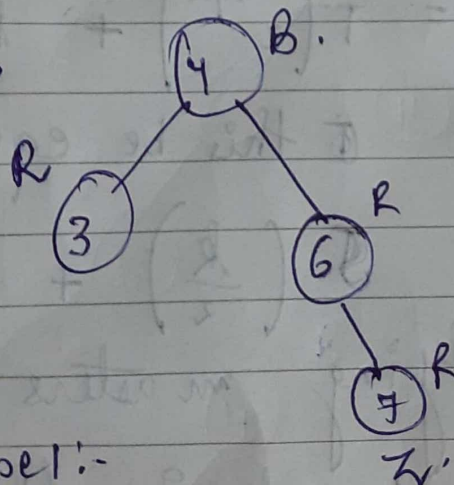
(3)

6 →



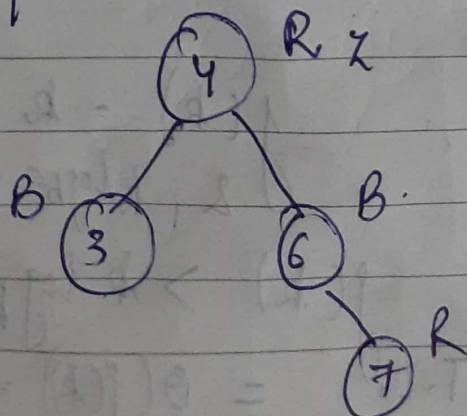
(4)

7 →

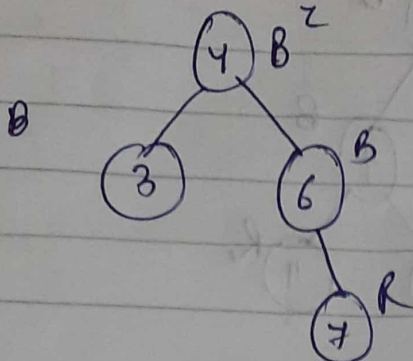


Apply case 1:-

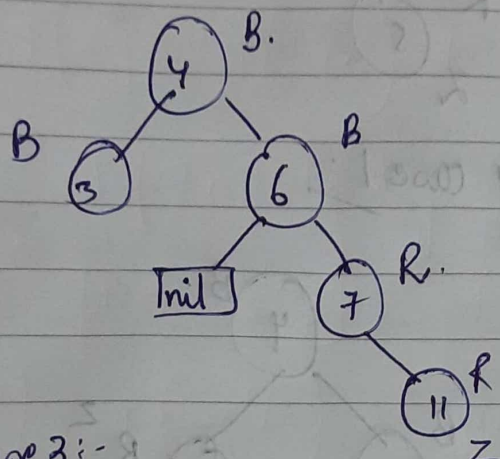
make 3 & 6 Black & grandparent red & make it new root



As root node can't be red.

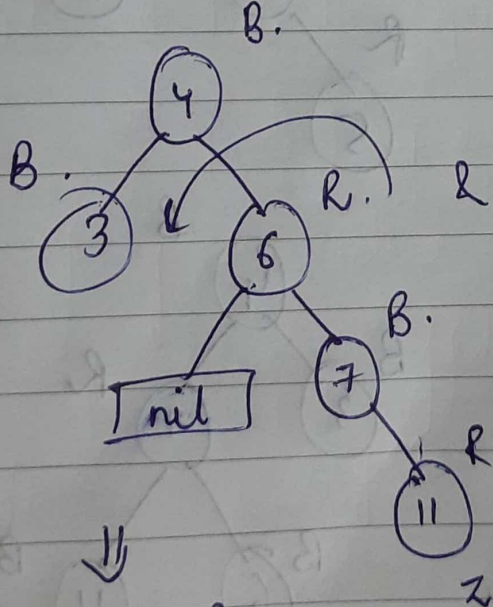


⑤ 11 →

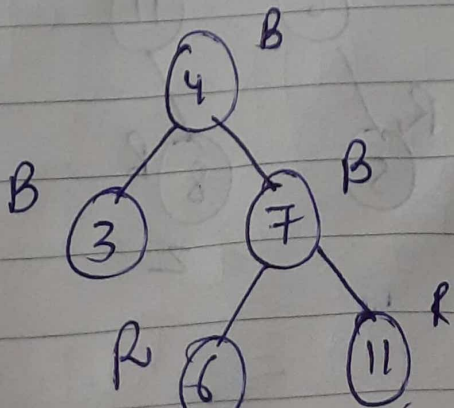


Apply case 3:-

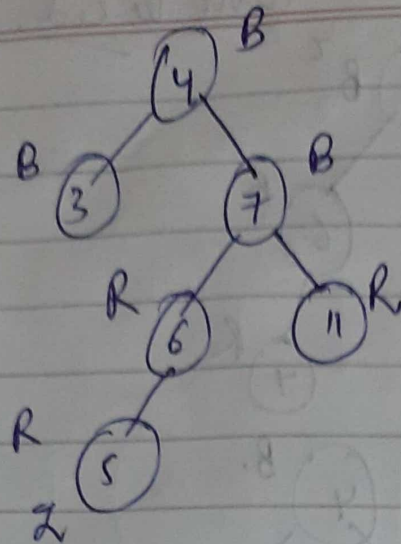
Z's uncle is black so make 7 black & 6 red.



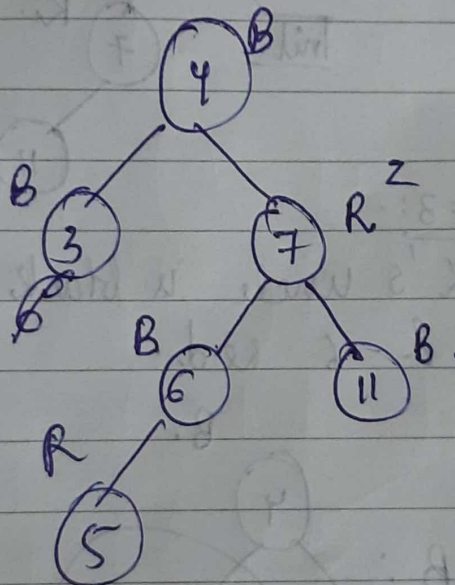
& left rotate around 6.



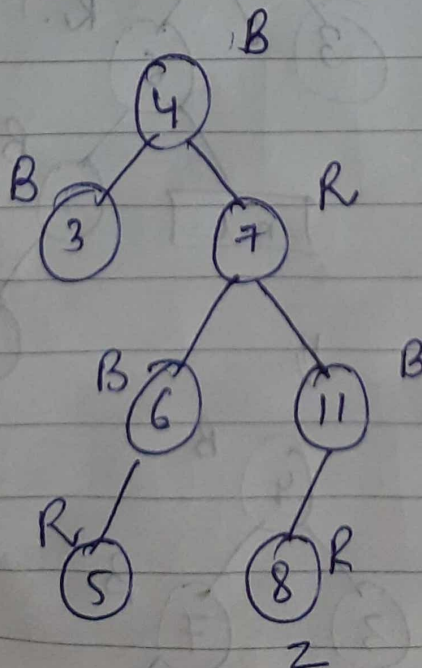
⑥ 5 →



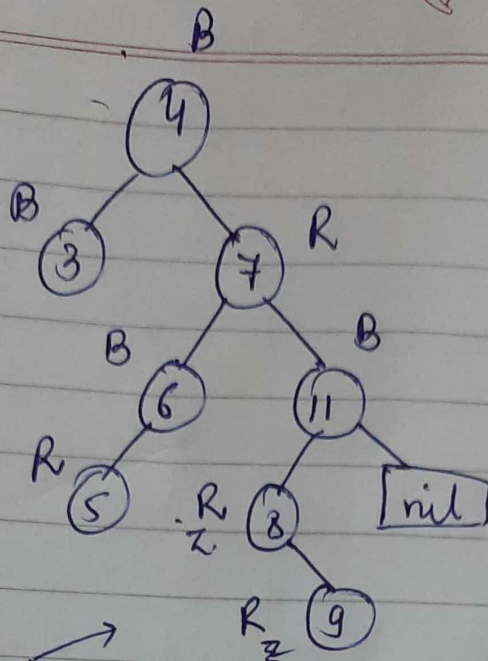
Apply case 1:



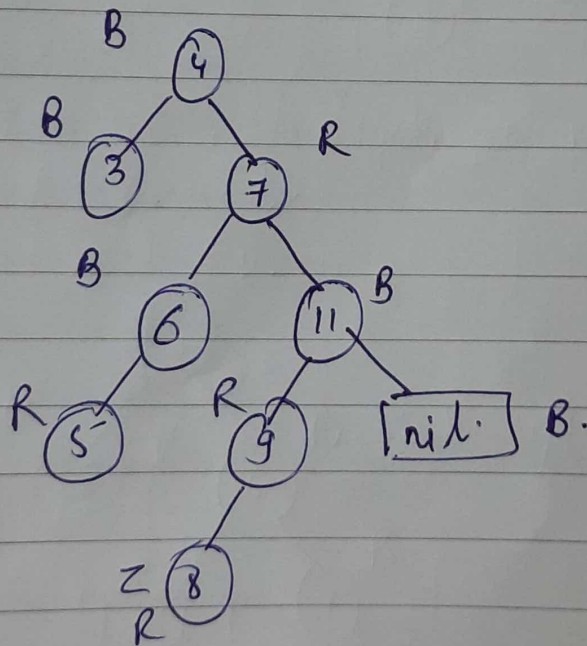
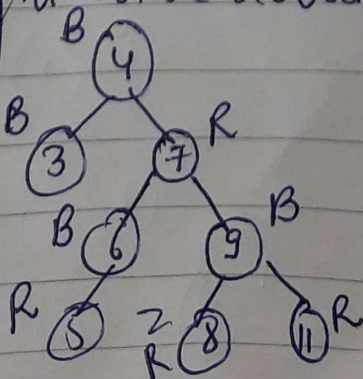
⑦ 8 →



8 → 9

Apply case 2:

set 8 as new z & left rotate.

Case 3:-color node 9 black & 11 red
& right rotate around 11.

Q3.

Mahima Rai
1803213087

classmate

Date _____

Page _____

Insert fix up (T, z)

case 1

①

while $z \neq \text{root}[T]$ & $\text{color}[P[z]] = \text{red}$

②

do if $P[z] = \text{left}[P[P[z]]]$

③

then $y \leftarrow \text{right}[P[P[z]]]$

④

if $\text{color}[y] = \text{red}$

⑤

then $\text{color}[P[z]] \leftarrow \text{black}$

⑥

 $\text{color}[y] \leftarrow \text{red}$ Black —

⑦

 $\text{color}[P[P[z]]] \leftarrow \text{red}$

⑧

 $z \leftarrow P[P[z]]$

case 2

⑨

else if $z = \text{right}[P[z]]$

⑩

then $z \leftarrow P[z]$

⑪

left rotate (T, z)

⑫

 ~~$\text{color}[z] \leftarrow \text{black}$~~

case 3

⑬

 $\text{color}[P[z]] \leftarrow \text{black}$

⑭

 $\text{color}[P[P[z]]] \leftarrow \text{red}$

⑮

right rotate (T, $P[P[z]]$)

⑯

else (same as then clause)

⑰

with left & right exchange

⑱

 $\text{color}(\text{root}[T]) \leftarrow \text{black}$

Q2. R-B tree. (Binary Search Tree)

for every node, left child should be $<$ key value & right child should be $>$ root value.

→ Every node in R-B tree is either red or black.

→ Root node is always black.

→ Leaf nodes are always black.

→ If any node is red, both of its children must be black.

Ans

for each node, every path from node to leaf contains same no. of black nodes.
(all path from node to leaf has same black height).

bh x.

Black height of node:- ^{of any node x .} It is the no. of black nodes including leaf nodes on the path x to leaf but not counting x .

The height should be same for all paths from x to $l.n.$
Black height of tree:- Black height of root

Use of B-B tree

- ① CPU scheduling (fair play)
- ② map fun()
- ③ set fun()