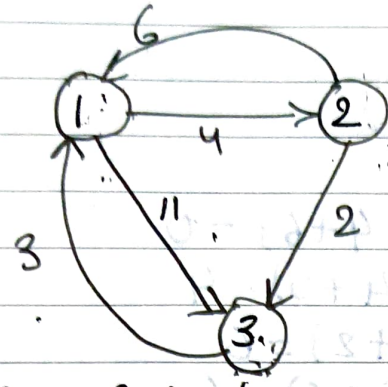
All Pair Shortest Path (Floyd Warshall Algo)

Eg.



A^0

	1	2	3
1	0	4	11
2	6	0	2
3	3	∞	0

Matrix Rep^n

$$d_{ij}^{k} = \min\left(d_{ij}^{k-1},\ d_{ik}^{k-1} + d_{kj}^{k-1}\right)$$

$k=1$, A^1

	1	2	3
1	0	4	11
2	6	0	2
3	3	7	0

$D^1_{12} = \min(D^0_{12},\ D^0_{11} + D^0_{12})$

$= \min(4,\ 0+4) = 4$

$D^1_{13} = \min(D^0_{13},\ D^0_{11} + D^0_{13}) = \min(11,\ 0+11) = 11$

$D_{21} = \min(D^0_{21},\ D^0_{21} + D^0_{11}) = \min(6,\ 6+0) = 6$

$D_{23} = \min(D^0_{23},\ D^0_{21} + D^0_{13}) = \min(2,\ 6+11) = 2$

$D_{31} = \min(D^0_{31},\ D^0_{31} + D^0_{11}) = \min(3,\ 3+0)$

$D_{32} = \min(D^0_{32},\ D^0_{31} + D^0_{12}) = \min(\infty,\ 3+4)$

$k=2$ A^2

	1	2	3
1	0	4	6
2	6	0	2
3	3	7	0

$D^2_{11} = (D^1_{11}, D^1_{12} + D^1_{21}) = (0, 4+6) = 0$

$D_{12} = (D_{12}, D_{12} + D_{22}) = (4, 4+0) = 4$

$D_{13} = (D_{13}, D_{12} + D_{23}) = (11, 4+2) = 6$

$D_{21} = (D_{21}, D_{22} + D_{21}) = (6, 0+6) = 6$

$D_{22} = 0$

$D_{23} = (D_{23}, D_{22} + D_{23}) = (2, 0+2) = 2$

$D_{31} = (D_{31}, D_{32} + D_{21}) = (3, 7+6) = 3$

$D_{32} = (D_{32}, D_{32} + D_{22}) = (7, 7+0) = 7$

$D_{33} = 0$

$k=3$ A^3

	1	2	3
1	0	4	6
2	5	0	2
3	3	7	0

$D^3_{12} = (D^2_{12}, D^2_{13} + D^2_{32}) = (4, 6+7) = 4$

Check the Soln

cost

3 - 2 => 7

3 - 1 = 3

1 - 2 = 4

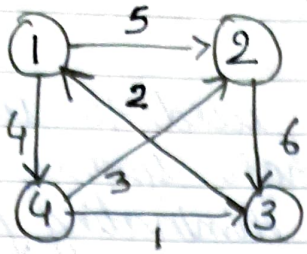
7 ✓

2 - 3 => 2

2 - 3 = 2 ✓

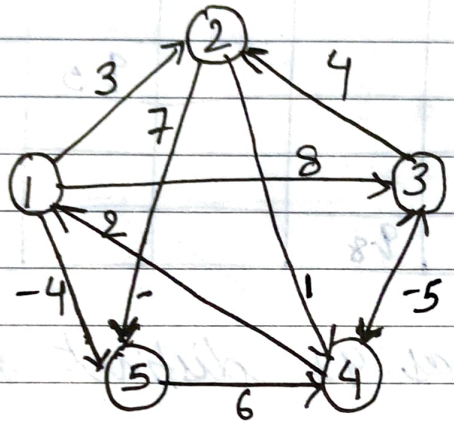
2 - 6, 6 - 3 = 17 ✗

Q.



$$D = \begin{bmatrix} 0 & 5 & 5 & 4 \\ 8 & 0 & 6 & 12 \\ 2 & 7 & 0 & 6 \\ 3 & 3 & 1 & 0 \end{bmatrix}$$

Q.



$$D = \begin{bmatrix} 0 & 1 & -3 & 2 & -4 \\ 3 & 0 & -4 & 1 & -1 \\ 7 & 4 & 0 & 5 & 3 \\ 2 & -1 & -5 & 0 & -2 \\ 8 & 5 & 1 & 6 & 0 \end{bmatrix}$$