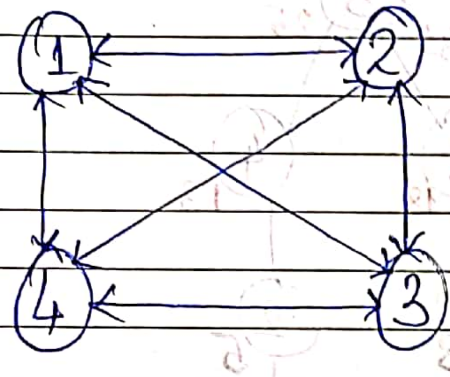
Travelling Salesman Problem.

Given a list of cities and the distances between each pair of cities, what is the shortest possible route that visits each city and returns to the origin city?

Example :-

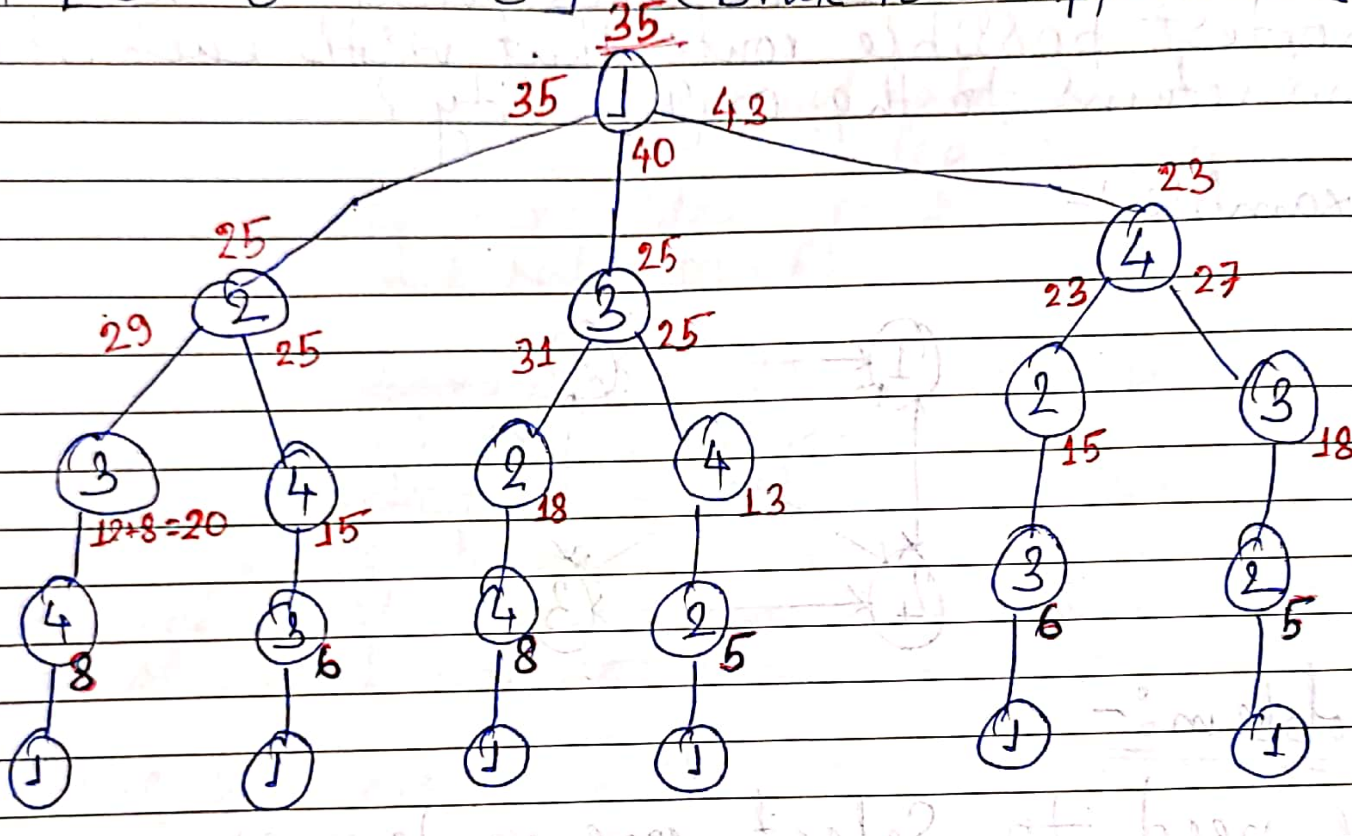


Problem :-

We need to select one vertex as a starting vertex and find out a path such that it is going through all the vertices and returning back to starting vertex.

		2	3	4
1	0	10	15	20
2	5	0	9	10
3	6	13	0	12
4	8	8	9	0

Dynamic Algorithm ~~Says~~ Says trying out all possible way. (brute-force Approach).



~~g(1)~~ If 1 is the starting vertex →

$$g(1,\{2,3,4\}) = \min_{k \in \{2,3,4\}} \left(c_{1k} + g(k, \{2,3,4\} - \{k\})\right).$$

General formula:-

$$g(i,S) = \min_{k \in S} \{c_{ik} + g(k, S - \{k\})\}.$$

S ↳ Set of vertices.

C_i,

$g(2, \phi) = 5$

$g(3, \phi) = 6$

$g(4, \phi) = 8$

$g(2, \{3\}) = 15 = C_{23} + g(3, \phi) = 9 + 6 = 15$

$g(2, \{4\}) = 18 = C_{24} + g(4, \phi) = 10 + 8 = 18$

$g(3, \{2\}) = 18 = C_{32} + g(2, \phi) = 13 + 5 = 18$

$g(3, \{4\}) = 20 = C_{34} + g(4, \phi) = 12 + 8 = 20$

$g(4, \{2\}) = 13 = C_{42} + g(2, \phi) = 8 + 5 = 13$

$g(4, \{3\}) = 15 = C_{43} + g(3, \phi) = 9 + 6 = 15$

Now we compute $g(i, S)$ with $|S| = 2$.

$$g(2, \{3, 4\}) = \min(C_{23} + g(3, \{4\}), C_{24} + g(4, \{3\})).$$

$$= \min(9 + 20, 10 + 15) = 25$$

$$g(3, \{2, 4\}) = \min(C_{32} + g(2, \{4\}), C_{34} + g(4, \{2\}))$$

$$= \min(13 + 18, 12 + 13) = 25$$

$$g(4, \{2, 3\}) = \min(C_{42} + g(2, \{3\}), C_{43} + g(3, \{2\})).$$

$$= \min(8 + 15, 9 + 18)$$

$$= 23.$$

Finally

$$g(1, \{2,3,4\}) = \min\Big(C_{12} + g(2, \{3,4\}),\ C_{13} + g(3, \{2,4\}),\ C_{14} + g(4, \{2,3\})\Big).$$

$$= \min(10+25,\ 15+25,\ 20+23)$$

$$= 35$$

Path $1 \rightarrow 2 \rightarrow 4 \rightarrow 3 \rightarrow 1$.