

AI1110

Assignment 2

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1 ICSE 2018 GRADE 12 QUESTION 8(B)

Water is dripping out of a conical funnel of semi vertex angle $\frac{\pi}{4}$ at the uniform rate of $2 \text{ cm}^2/\text{sec}$ in the surface, through a tiny hole at the vertex of the bottom. When slant height of the water level is 4 cm, find the rate of decrease of slant height of water.

2 SOLUTION

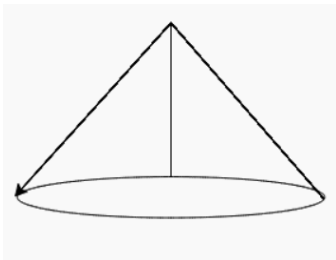


Fig. 0.

Symbol	Value	Description
V	<i>decreasing</i>	Volume of water in cone
θ	45°	semi vertex angle
l	4	slant height of cone
h	$l \sin \theta$	vertex height of the cone
r	$l \cos \theta$	radius of cone

From the table we know that r is the radius, h is the height and V is the Volume of the funnel at any time t .

$$V = \frac{1}{3} \pi r^2 h \quad (2.0.1)$$

From the table we know that l is the slant height of the funnel and θ° is semi-vertical angle. So

$$h = l \sin \theta \quad (2.0.2)$$

$$r = l \cos \theta \quad (2.0.3)$$

So, equation (2.0.1) becomes

$$V = \frac{1}{3} \pi l^3 \cos^2 \theta \sin \theta \quad (2.0.4)$$

Differentiating it w.r.t t will give us rate of change of volume

$$\frac{dV}{dt} = \left(\frac{1}{3} \pi \cos^2 \theta \sin \theta \right) 3l^2 \frac{dl}{dt} \quad (2.0.5)$$

$$\frac{dl}{dt} = \frac{dV}{\pi l^2 \cos^2 \theta \sin \theta dt} \quad (2.0.6)$$

As it is given that rate of change of water w.r.t to t is

$$\frac{dV}{dt} = -2 \left(\frac{\text{cm}^3}{\text{sec}} \right) \quad (2.0.7)$$

Putting the values of l and θ from the table in equation (2.0.6), we get

$$\frac{dl}{dt} = \frac{4\sqrt{2}}{\pi l^2} \quad (2.0.8)$$

$$\frac{dl}{dt} = \frac{\sqrt{2} \text{cm}}{4\pi \text{sec}} \quad (2.0.9)$$

So, rate of decrease of slant height is

$$\frac{\sqrt{2} \text{cm}}{4\pi \text{sec}}$$