AI1110
Assignment 2

Mahin Bansal
cs21btech11034

1 ICSE 2018 GRADE 12 QUESTION 8(B)

Water is dripping out of a conical funnel of semi verticle angle $\frac{\pi}{4}$ at the uniform rate of 2 cm^2/sec in the surface ,through a tiny hole at the vertex of the bottom.When slant height of the water level is 4 cm ,find the rate of decrease of slant height of water .

2 SOLUTION

Let r be the radius , h be the height and V be the Volume of the funnel at any time t.

$$V = \frac{1}{3}\pi r^2 h \qquad (2.0.1)$$

Let l be the slant height of the funnel.
Semi-Verical angle = $45°$
So

$$h = l \sin 45 = \frac{l}{\sqrt{2}} \qquad (2.0.2)$$

$$r = l \cos 45 = \frac{l}{\sqrt{2}} \qquad (2.0.3)$$

So , equation (2.0.1) becomes

$$V = \frac{1}{3}\pi \frac{l}{\sqrt{2}}^2 \frac{l}{\sqrt{2}} \qquad (2.0.4)$$

$$V = \frac{\pi}{6\sqrt{2}}l^3 \qquad (2.0.5)$$

Differentiating it w.r.t t will give us rate of change of volume

$$\frac{dV}{dt} = \frac{\pi}{6\sqrt{2}}3l^2\frac{dl}{dt} \qquad (2.0.6)$$

$$\frac{dV}{dt} = \frac{\pi}{2\sqrt{2}}l^2\frac{dl}{dt} \qquad (2.0.7)$$

$$\frac{dl}{dt} = \frac{2\sqrt{2}dV}{\pi l^2 dt} \qquad (2.0.8)$$

As it is given that rate of change of water w.r.t to t is

$$\frac{dV}{dt} = -2\frac{cm^3}{sec} \qquad (2.0.9)$$

Therefore

$$\frac{dl}{dt} = \frac{4\sqrt{2}}{\pi l^2} \qquad (2.0.10)$$

$$\frac{dl}{dt} = \frac{\sqrt{2}cm}{4\pi sec} \qquad (2.0.11)$$