AI1110
Assignment 1

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1 ICSE 2018 GRADE 10 QUESTION 11(A)

The 4^{th} term of an A.P. is 22 and 15^{th} term is 66. Find the first term and the common difference.Hence find the sum of series upto 8^{th} term.

2 SOLUTION

Symbol	Value	Description
m	4	a is $1^{st} term of A.P$
n	11	d is common difference of A.P
a_m	22	a_m is $m^{th} term of A.P$
a_n	66	a_m is $m^{th} term of A.P$

Let mth and nth term of an A.P be a_m and a_n respectively
Let first term of A.P. be a
Let common difference of A.P. be d
The xth (a_x) term of any Arithmetic progression is given by
$a_x = a + (x-1)d$
So,

$$a_m = a + (m-1)d \tag{2.0.1}$$
$$a_n = a + (n-1)d \tag{2.0.2}$$

Subtracting (2.0.1) and (2.0.2) we get,

$$(m-n)d = a_m - a_n \tag{2.0.3}$$
$$d = \frac{a_m - a_n}{m-n} \tag{2.0.4}$$

Putting the value of d in (2.0.1) equation

$$a + (m-1)\left(\frac{a_m - a_n}{m-n}\right) = a_m \tag{2.0.5}$$
$$a = a_m - (m-1)\left(\frac{a_m - a_n}{m-n}\right) \tag{2.0.6}$$

Given the 4^{th}term of the A.P is 22
15^{th} term of A.P. is 66
Putting the values of m , n ,a_m and a_n from the table in (2.0.4) and (2.0.6)
We get 1st term of the A.P. is 10 and common difference is 4.
Sum of an A.P till x terms is given by

$$S_x = \left(\frac{x}{2}\right)(2a + (x-1)d) \tag{2.0.7}$$

Putting the values of a and d from (2.0.4) and (2.0.6), we get

$$S_x = \left(\frac{x}{2}\right)\left(2\left(a_m - (m-1)\left(\frac{a_m - a_n}{m-n}\right)\right)\right) + \left(\frac{x}{2}\right)\left((x-1)\left(\frac{a_m - a_n}{m-n}\right)\right) \tag{2.0.8}$$

$$S_x = \left(\frac{x(a_m(m-n) - (m-1)(a_m - a_n))}{m-n}\right) + \left(\frac{x(x-1)(a_m - a_n)}{2(m-n)}\right) \tag{2.0.9}$$

So, putting the values of
m ,n , a_m and a_n from the (2) and $x = 8$
in (2.0.9) we get sum of A.P till 8^{th} term is

$$S_8 = \left(\frac{(8)(22(-11) - (3)(-44))}{-11}\right) + \left(\frac{8(7)(-44)}{2(-11)}\right) \tag{2.0.10}$$
$$S_8 = 80 + 112 \tag{2.0.11}$$
$$S_8 = 192 \tag{2.0.12}$$