Assignment 5

Mahin Bansal - CS21BTECH11034

May 26, 2022

Papoulis chap 4 Ex 4.8

Problem

Example 4-8: Suppose the random variable X is such that $X(\xi) = 1$ if $\xi \in A$ and zero otherwise. Find $F(x)$

Solution

Let $\Pr(A) = p$, denote the probability of the event A happening and 'S' denote the universal set.

Given,

$$X(\xi) = \begin{cases} 1 & \text{if } \xi \in A \\ 0 & \text{else} \end{cases}$$

For $x < 0$, $\{X(\xi) \leq x\} = \{\emptyset\}$, so that $F(x) = 0$

For $0 \leq x < 1$, $\{X(\xi) \leq x\} = \{A'\}$, so that $F(x) = \Pr(A') = 1 - p = q$

For $x \geq 1$, $\{X(\xi) \leq x\} = S$, so that $F(x) = 1$ (see figure).

Solution(Contd.)

Hnece, $F(x)$ can be written as :-

$$F(x) = \begin{cases} 0 & , x \in (-\infty, 0) \\ 1 - p & , x \in [\, 0, 1) \\ 1 & , x \in [\, 1, \infty) \end{cases}$$

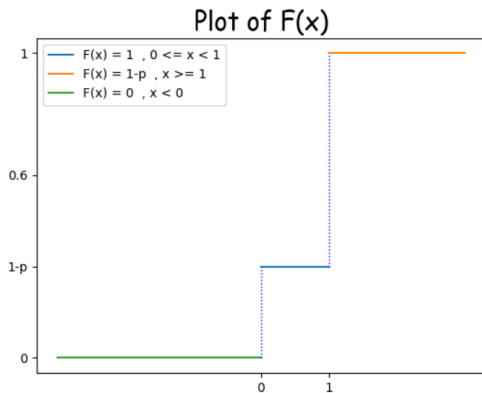The corresponding graph of $F(x)$ is plotted next page :-

CDF Graph

Figure: CDF function