GOVERNMENT CITY COLLEGE, NAYAPUL
DEPARTMENT OF MATHEMATICS

PROJECT TITLE

“A WORLD OF CHECK DIGITS”

Submitted in partial fulfillment of
Bachelor of Science (2019 – 2022)

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Introduction

Numbers are integral part of our daily life. This world is filled with numbers .1f we buy any product from supermarket it is identified by a twelve digit number. If we issue a bank cheque to someone it contains a Nine digit number at bottom of the cheque If we purchase a book it has a ten digit number called ISBN number our Aadhaar card contains a twelve digit number there are so many other things on which we see a string of digits these things are identified by Unique numbers. The last digit of these big numbers are called check digit. Generally we did not give much attention to it this digit has specific purpose while feeding these numbers into computers through keyboard some typing mistakes may occur these mistakes are like single digit errors to multi digit errors happen which leads to a wrong check hence operator can easily identify the mistake and he or she can re-enter that number

In this project we are going to see different schemes or methods to find out cheque digit in Postal money order, Air tickets, Universal product code, International standard book number ,Cheque number ,Bank note serial numbers and we will be deriving conditions when error go un detected. Most of these methods are based on modular arithmetic which is mainly dependent on division algorithm. We have been using this division algorithm in our daily life for example if it is now Wednesday we know that in 23 days it will be Friday because 23 = 7.3 + 2 so we have added 2 days to the Wednesday instead of counting 23 days. Surprisingly, this simple idea has numerous important applications in mathematics and computer science. There is a method which was developed by J.Verhoeff a German software developer this is relied on dihedral group ü5.This method detects all single digit errors and all transposition errors. Aadhaar a Unique identity number is a 12 digit number based on biometric related information.a digit will be appended to a I l digit number as a check which can be calculated by Verhoeff scheme.Using these methods we found out check digits of various identifiers .At the end we form a table of formulas of check digits and condition when errors detection is not caught

Objectives

- To Understand Division algorithm and its applications ➢ To
- Understand Various schemes for finding check digits
- Finding check digits of UPC, ISBN etc
- To understand Dihedral groups
- To understand how Algebraic structures like groups can be used in real life

Division Algorithm

Let □ and □ be integers with□ > then there exist unique integers and with the property that

□ = □ □ + □ Where 0 < □ < □

Example

□ = 18, □ = 4 Then 18=4.4 + 2
□ = -25, □ = 7 Then -25 =7(-40) + 3

Modular Arithmetic

This is an application of Division Algorithm. Knowingly or un knowingly we use this in our day to day life.

For example

If it is now October, what month it will be 27 months from now the answer is January

Logic behind this answer is D.A Since 27 2.12 + 3 just add 3 months to October

If □ = □□ + □
□ = □□□□□□□□□□
□ = □□□□□□□□□□□□□□□□□□□□□□□

We write it as □□□□□ = □ or □ = □□□□□

Thus 3 □□□ 2 = 1

11□□□ 3 = 2□.□ 11 = 3.3 + 2

62□□□ 85 = 62□ .□ 62 = 0.85 + 62

Simple Properties

(□□) □□□□ = ((□□□□)(□□□□□)) □□□□

(□ + □) □□□□ = ((□□□□)+(□□□□□)) □□□□

Applications of Modular Arithmetic

Identification of numbers for the purpose of detecting forgery or errors

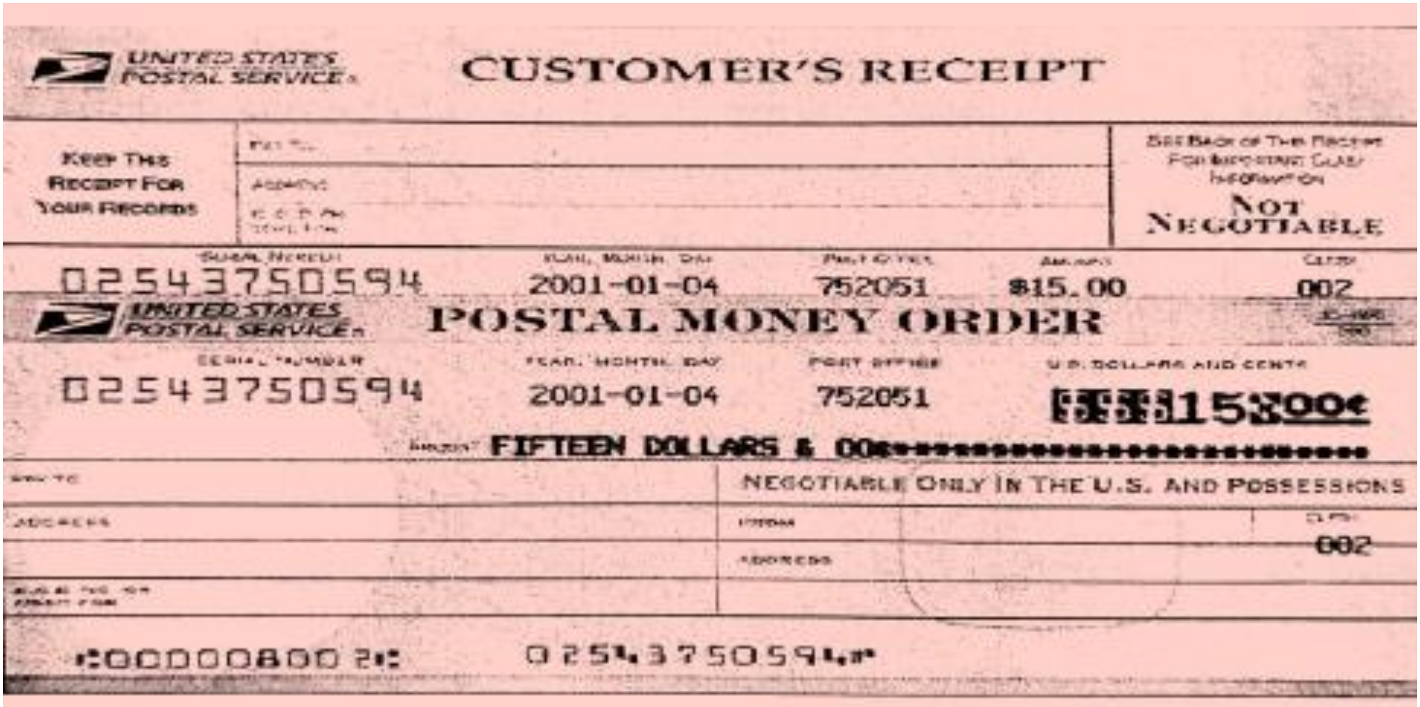
When human beings entering identification numbers such as Money orders, ISBN, UPC, Air ticket, Cheque, Aadhaar numbers into computers for specific purposes some errors may occur they are of following type

- ➢ Single digit errors, such as 2 → 3
- ➢ Transposition errors, such as 32 → 23
- ➢ Twin errors, such as 22 → 33
- ➢ Jump transpositions errors, such as 134 ->431
- ➢ Jump twin errors, such as 131 → 232
- ➢ Phonetic errors, such as 60 → 16 ("sixty" to "sixteen")

For simplicity, of above we confine ourselves to Single digit errors and Transposition errors to avoid these exams one digit is appended to identification number. for different identifications different formulas an used. In this project we will see various check digit formulas, how they works, merits and demerits **POSTAL SERVICE MONEY ORDER**

The United States Postal Service money order shown in the following figure has an identification number consists of 10 digits together with an extra digit called CHECK. The check digit is the sum of the digits modulo 9. Thus the number □□□□□□□□□□□□ has check digit 4 since

$0 + 2 + 5 + 4 + 3 + 7 + 5+0+5 + 9 = 4$



If the number 0254375059 were incorrectly entered into a computer as say 02 4375059 the computer would calculate the check as 2 since

$0 + 2 + \square + 4 + 3 + 7 + 5 + 0 + 5 + 9 = 38 \square\square\square\ 9 = 2$

Where as the check digit would be 4. Thus the error would be detected

But is it detects all single digit errors?

If the number 024375059 were incorrectly entered into a computer as say 0234375050 the computer would calculate the check as 4 since

$0 + 2 + 5 + 4 + 3 + 7 + 5 + 0 + 5 + 0 = 31\ \square\square\square\ 9 = 4$

But error is not detected because 0 $\square\square\square$ 9 = 9 $\square\square\square$ 9

Let us formulate above concept

The last digit is the check digit given by the following formula

$$\square_{11} = (\square_1 + \square_2 + \square_3 + \square_4 + \square_5 + \square_6 + \square_7 + \square_8 + \square_9 + \square_{10})\ \square\square\square\ 9$$

If the Entered incorrect number is $\square_1 + \square_2 + \square_3 + \square_4 + \square_5 + \square_6 + \square_7 + \square_8 + \square_9 + \square_{10}$ error would go undetected if

$(\square_1 + \square_2 + \square_3 + \square_4 + \square_5 + \square_6 + \square_7 + \square_8 + \square_9 + \square_{10})\ \square\square\square\ 9 =$
$(\square_1 + \square_2 + \square_3 + \square_4 + \square_5 + \square_6 + \square_7 + \square_8 + \square_9 + \square_{10})\ \square\square\square\ 9$

⇨ $\square_{10}\ \square\square\square\ 9 = \square_{10}\ \square\square\square\ 9$ In General $\square_{10}\ \square\square\square\ 9 = \square_{10}\ \square\square\square\ 9$

It detects the error only if $\square_{10}\ \square\square\square\ 9 \neq \square_{10}\ \square\square\square\ 9$

If the number 0254375059 were incorrectly entered into a computer as say 0524375059 the computer would calculate the check as 4 only since

$0 + 5 + 2 + 4 + 3 + 7 + 5 + 0 + 5 + 9 = 40\ \square\square\square\ 9 = 4$

In this case computer cannot detect this error because addition is commutative. in general It detects error if only

$(\square_1 + \square_2 + \square_3 + \square_4 + \square_5 + \square_6 + \square_7 + \square_8 + \square_9 + \square_{10})\ \square\square\square\ 9 \neq$
$(\square_1 + \square_2 + \square_3 + \square_4 + \square_5 + \square_6 + \square_7 + \square_8 + \square_9 + \square_{10})\ \square\square\square\ 9$

AIR TICKET

Some Airline companies use Modulo 7 to assign check digit of identification of their tickets.

For example following Airline Ticket identification number □ □□□ □□□□□□□□□□ □ □ has the check digit □ appended to it.

Because 2001602780871□□□ 7= I since □□□□□□□□□□□□□□□□ = □□ □□□□□□□□□□□□□ *7 + □

If the number 2 001 6027808715 were incorrectly entered into a computer as say 2001□027808715

the computer would calculate the check as□ since 2001□027808715 = 2859289686959 x 7+2

Where as the check digit would be 3. Thus the error would be detected

The formula of last digit is □□□ = (□□□□□□□□□□□□□□□□□□□□□□□□□□□□) □□ □□

If the number 2 001 6027808715 were incorrectly entered into a computer as say 20016097808715 the computer would calculate the check as□since 20016097808715 = 2859442544102 x 7 + 1

In this case we cannot detect the error if

(□$_1$□$_2$□ $_3$□ $_4$□$_5$□ $_6$□ $_7$□ $_8$□ $_9$□ $_{10}$□ $_{11}$□ $_{12}$□ $_{13}$□ $_{14}$) □□□ 7 = (□$_1$□$_2$□ $_3$□ $_4$□ $_5$□ $_6$□ $_7$□ $_8$□ $_9$□ $_{10}$□ $_{11}$□ $_{12}$□ $_{13}$□ $_{14}$) □□□ 7

⇨ □ $_7$□□□ 7 = □ $_7$□□□ 7

In general single digit errors cannot be detected if □□□□□ 7 = □□□□□ 7

7 here 9□□□7=2□□□7 hence unable to detect the error

If the Ticket identification number □ □□□ □□□□□□□□□□ □ □ has the check digit □ appended to it because

20016297808715□□□7 = 5 Since □□□□□□□□□□□□□□□□ = □□□□ □□□□□□□□□□□ *7 + □

If the number 20016297808715 were incorrectly entered into a computer as say 20016□□7808715 the computer would calculate the check as 5 since 20016297808715 = 2859561115530 x 7 + 5

It is clear that check digit is same system cannot detect this transition error in general transposition error cannot be detected if □□□□□ 7 = □□-1□□□ 7

UPC (UNIVERSAL PRODUCT CODE)

Universal Product Code is printed on package items it has 12 digits (Shown below). The first six digits identify the manufacture; the next five identify the product and last is a check

The check digit is calculated by following formula

(9,8,7,6,5,4,3,2,1,0,9,8). (3,1,3,1,3,1,3,1,3,1,3,1,3) □□□10= 0

□.□.□ = 9.3+8.1+7.3+6.1+5.3+4.1+3.3+2.1+1.3+0.1+9.3+8.1

= 27 +8+21 +6 + 15 +4+9+2+3 + 27 +8

= 130

130□□□10=0

UPC should satisfy the following formula

(□1□2□ 3□ 4□5□ 6□ 7□ 8□ 9□ 10□ 11□ 12). (3,1,3,1,3,1,3,1,3,1,3,1,3) □□□10

The fixed 12-tuple used in the calculation of check digits is called the weighting vector

Suppose a single error is made in entering the number into computer. Say for instance that 98765 □ 321098 is entered. Then computer (Which is programmed to calculate check) calculates

(9,8,7,6,5,4,3,2,1,0,9,8). (3,1,3,1,3,1,3,1,3,1,3,1,3) □□□10

=9.3+8.1+7.3+6.1+5.3+4.1+3.3+2.1+1.3+0.1+9.3+8.1

= 27 +8+21 +6 + 15 +3+9+2+3 + 27 +8

= 129

But 129 □□□10 = 9 not equals 0

Therefore entered number cannot be correct

In general any single error will result in a sum modulo 10 not equal to zero

Suppose a transposition error is made in entering the number into computer. Say for instance that 987653321089 is entered. Then computer calculates

(9,8,7,6,5,4,3,2,1,0,9,8). (3,1,3,1,3,1,3,1,3,1,3,1,3) □□□10

= 9.3 + 8.1 +7.3 + 6.1 + 5.3 + 3.1 +3.3 +2.1 + 1.3 + 0.1+ 8.3 + 9.1

=27 + 8+21 + 6 + 15+3+9+2+3+24+9

= 135

But 135 □□□10 = 5 not equals 0

Therefore entered number cannot be correct

If the following UPC is wrongly entered in the computer as 892680501003.Then computer calculates

(8,9,6,8,0,5,0,1,0,0,3). (3,1,3,1,3,1,3,1,3,1,3,1) □□□10

8.3+9.1+6.3+8.1+0.3+5.1+0.3+1.0+0.3+0.1+0.3+3

=24+9+6+6+24+0+15+0+3+0+0+3

=90

Hence 90 □□□10 = 0

8 92685 00100 3

To verify this, we observe that a transposition error of the form

$\square_1\square_2\square_3\square_4\square_5\square_6\square_7\square_8\square_9\square_{10}\square_{11}\square_{12} \dashrightarrow \square_1\square_2\square_3\square_4\square_5\square_6\square_7\square_8\square_9\square_{10}\square_{11}\square_{12}$ is undetected if

$(\square_1\square_2\square_3\square_4\square_5\square_6\square_7\square_8\square_9\square_{10}\square_{11}\square_{12}).(3,1,3,1,3,1,3,1,3,1,3,1,3)\ \square\square\square 10$

$=0(\square_1\square_2\ldots\ldots\square_6\ldots\ldots\square_8\square_{11}\square_{12}).(3,1,3,1,3,1,3,1,3,1,3,1,3)\ \square\square\square 10$

$= (\square_1\square_2\ldots\ldots_6\square_7\ldots\ldots\square_{10}\square_{11}\square_{12}).(3,1,3,1,3,1,3,1,3,1,3,1,3)\ \square\square\square 10$

⇨ $(3\square_5\square_6)\ \square 10= (3\square_5\square_6)\ \square\square\square 10$

⇨ $(2\square_5 - 2\square_6)\ \square 10= 0$

⇨ $\square_5 - \square_6 =5$

In general this scheme cannot detect transposition error if $|\square\square - \square\square\text{-}1| = 5$

Advantage of UPC Method

Detects nearly all errors involving the transposition of two adjacent digits as well as errors involving one digit

ISBN

Stands for International Standard Book number identifier. It has 10 digits First two digits represents Group, Second four digits indicates Publisher, third three digits denotes title and last is check

ISBN 1-55615-678-2

9 781556 156786

The check digit is calculated by the following formula

$(\square_1.10 + \square_2.9 + \square_3.8 + \square_4.7 + \square_5.6 + \square_6.5 + \square_7.4 + \square_8.3 + \square_9.2 + \square_{10}.1) \square\square\square\ 11 = 0$

That is (1,5,5,6,1,5,6,7,8,2). (10,9,8,7,6,5,4,3,2,1)=0

$\square\square\square$ = 10.1+9.5+8.5+7.6+6.1+5.5+4.6+3.7+2.8+1.2

=10 + 45 + 40 + 42 + 06 + 25 + 24 + 21 + 16 + 2

=231

23$\square\square\square$11 =0

Here weighing vector is (10,9,8,7,6,5,4,3,2,1)

Suppose a single error is made in entering the number into computer. Say for instance that 15541567S2 is entered. Then computer calculates

(1,5,5,4,1,5,6,7,8,2). (10,9,8,7,6,5,4,3,2,1) $\square\square\square$11

= 10.1 +9.5 + 8.5 + 7.4 + 6.1+5.5+4.6+ 3.7 + 2.8 + 1.2

= 10+ 45 +40 + 28 +06 + 25 + 24 +21 + 16+2

=217

217$\square\square\square$11 = 8

Therefore entered number cannot be correct

We observe that a single digit error of the form

$\square_1\square_2\square_3\square_4\square_5\square_6\square_7\square_8\square_9\square_{10} \dashrightarrow \square_1\square_2\square_3\square_4$
$\square_5\square_6\square_7\square_8\square_9\square_{10}$

$(10\square_1 + 9\square_2 + 8\square_3 + 7\square_4 + 6\square_5 + 5\square_6 + 4\square_7 + 3\square_8 + 2\square_9 + 1\square_{10}) \square\square\square 11 = (10\square_1 + 9\square_2 + 8\square_3 + 7\square_4 + 6\square_5 + 5\square_6 + 4\square_7 + 3\square_8 + 2\square_9 + 1\square_{10}) \square 11$

- ⇨ $6\square_5 \square\square\square 11 = 6\square_5 \square\square\square 11$
- ⇨ $6\square_5 \square 11$
- ⇨ $\square_5 - \square_5 = 11$

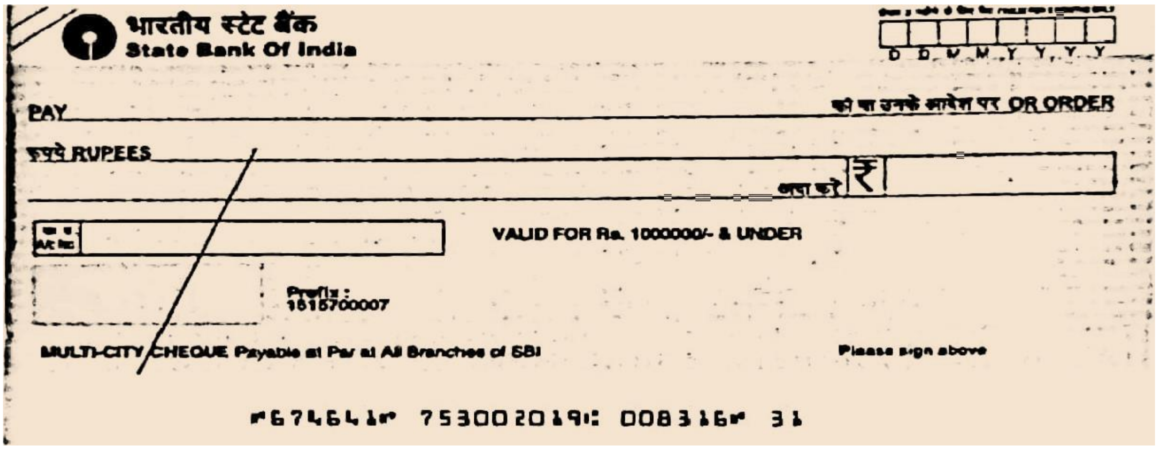
As it is not possible because all the digits are less than 10 so it detects all the single digit errors always

Similarly we can show all transposition errors can be detected

Advantage

ISBN method is capable of detecting all single digit errors and all transposition errors involving adjacent di2its

BANK CHEQUE NUMBER



Bank cheque is identified by a number printed between two colons consists of an eight digit number and a check digit

The check digit is calculated by the following formula

$(\square_1 .7 + \square_2 .3 + \square_3 .9 + \square_4 .7 + \square_5 .3 + \square_6 .9 + \square_7 .7 + \square_8 .3 + \square_9 .9) \square\square\square 10 = 0$

That is

$(7,5,3,0,0,2,0,1,9).(7,3,9,7,3,9,7,3,9) \square\square\square 10 = 0$

$\square\square\square = 7.7 + 5.3 + 3.9 + 0.7 + 0.3 + 2.9 + 0.7 + 1.3 + 9.9$

$= 49 + 15 + 24 + 18 + 3 + 81$

=190

190□□□10 =0

Here weighing vector is (7,3,9,7,3,9,7,3,9)

BANK NOTES

Now we see a check digit system based on Dihedral group .□5.1t is a very sophisticated scheme to append check digit to identification number among these schemes only the ISBN method is capable of detecting all single digit errors and all transposition errors involving adjacent digits. It was found by J.Verhoeff .□5 consists of 10 elements five elements from rotations and five elements from reflexions namely 0, ,2,3,4,5,6,7,8,9 this group is represented by following composition table

□	0	1	2	3	4	5	6	7	8	9
0	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
1	**1**	**2**	**3**	**4**	**0**	**6**	**7**	**8**	**9**	**5**
2	**2**	**3**	**4**	**0**	**1**	**7**	**8**	**9**	**5**	**6**
3	**3**	**4**	**0**	**1**	**2**	**8**	**9**	**5**	**6**	**7**
4	**4**	**0**	**1**	**2**	**3**	**9**	**5**	**6**	**7**	**8**
5	**5**	**9**	**8**	**7**	**1**	**0**	**4**	**3**	**2**	**1**
6	**6**	**5**	**9**	**8**	**7**	**1**	**0**	**4**	**3**	**2**
7	**7**	**6**	**5**	**9**	**8**	**2**	**1**	**0**	**4**	**3**
8	**8**	**7**	**6**	**5**	**9**	**3**	**2**	**1**	**0**	**4**
9	**9**	**8**	**7**	**6**	**5**	**4**	**3**	**2**	**1**	**0**

Check digit $\square\square$ is appended to $\square_1\square_2$..................$\square\square$-1 such that

$(\square_1)\square\square^2(\square_2)\square$....... ......$\square\square^{-1}(\square\square\text{-}1)\square\square\square = 0$

Here $\square = (01589427)(36) = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 1 & 5 & 7 & 6 & 2 & 8 & 3 & 0 & 9 & 4 \end{pmatrix}$

And $(\square) \neq \square(\square)\square\square\square \neq \square$

$\square\square\square\ (\square) \neq \square\square\square(\square)\square\square\square \neq \square$

$$\square^2 = \square\square\square = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 1 & 5 & 7 & 6 & 2 & 8 & 3 & 0 & 9 & 4 \end{pmatrix} \square \begin{pmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 1 & 5 & 7 & 6 & 2 & 8 & 3 & 0 & 9 & 4 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 5 & 8 & 0 & 3 & 7 & 9 & 6 & 1 & 4 & 2 \end{pmatrix}$$

$$\square^{11} = \square^{10}\square\square = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 5 & 8 & 0 & 3 & 7 & 9 & 6 & 1 & 4 & 2 \end{pmatrix} \square \begin{pmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 1 & 5 & 7 & 6 & 2 & 8 & 3 & 0 & 9 & 4 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 8 & 9 & 1 & 6 & 0 & 4 & 3 & 5 & 2 & 7 \end{pmatrix}$$

$$\square^{12} = \square^{11}\square\square = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 8 & 9 & 1 & 6 & 0 & 4 & 3 & 5 & 2 & 7 \end{pmatrix} \square \begin{pmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 1 & 5 & 7 & 6 & 2 & 8 & 3 & 0 & 9 & 4 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 9 & 4 & 5 & 3 & 1 & 2 & 6 & 8 & 7 & 0 \end{pmatrix}$$

Similarly we can write other exponents of $\square$

The following table gives the values of the functions $\square$, $\square^2$,…., $\square^{10}$ needed för computations

$\square$	0	1	2	3	4	5	6	7	8	9
$\square$	1	5	7	6	2	8	3	0	9	4
$\square^{2}$	5	8	0	3	7	9	6	1	4	2
$\square^{3}$	8	9	1	6	0	4	3	5	2	7
$\square^{4}$	9	4	5	3	1	2	6	8	7	0
$\square^{5}$	4	2	8	6	5	7	3	9	0	1
$\square^{6}$	2	7	9	3	8	0	6	4	1	5
$\square^{7}$	7	0	4	6	9	1	3	2	5	8
$\square^{8}$	0	1	2	3	4	5	6	7	8	9
$\square^{9}$	1	5	7	6	2	8	3	0	9	4
$\square^{10}$	5	8	0	3	7	9	6	1	4	2
$\square^{11}$	8	9	1	6	0	4	3	5	2	7
$\square^{12}$	9	4	5	3	1	2	6	8	7	0

To any strings of digits $\square_1\square_2.........\square_{10}$ Check digit $\square_{11}$ is chosen so that

Let us verity above formula with following bank note bearing the number DL2496197N7

Let $\square$ be required check digit satisfying the following expression

$\square(\square)\square\square^{2}(\square)\square\square^{3}(2)\square\square^{4}(4)\square\square^{5}(9)\square\square^{6}(6)\square\square^{7}(1)\square\square^{8}(9)\square\square^{9}(7)\square\square^{10}(5)\square\square=0$

$5\square7\square1\square1\square1\square6\square0\square9\square0\square9=0$ [using above table]

$7\square\square= 0$ [from the above table]

$\square=7$

So7 is check digit hence verified

Here serial numbers on the bank notes are alphanumeric hence it is necessary to assign numerical values to alphabets to compute check digit this assignment is shown in the following table

A	D	G	K	L	N	S	U	Y	Z
0	1	2	3	4	5	6	7	8	9

Suppose a single error is made in entering the number into computer. Say for instance that □□ 249619□□7 is entered. The scheme calculates

□(□)□□^{2}(□)□□^{3}(2)□□^{4}(4)□□^{5}(9)□□^{6} (6)□□^{7}(1)□□ ^{8}(9)□□ ^{9}(7)□□^{10}(5)□7□h□□□□□□□□□

5□7□1□1□1□ 6□ 0□ 9□ 9□ 9□ 7=7

Hence error can be found

Draw backs

This scheme does not distinguish between a letter and its assigned value Thus a substitution of 2 for G or G for 2 becomes an error.

Does not detect all transpositions of adjacent character involving check digit itself.

Both theses defects can be overcome by Verhoeff method with 18 which is dihedral group of order 36 using this scheme all single position errors and transposition errors involving adjacent digits can be detected

Check Digit Tables

PMO

Number	Check
0421300001	2
0771330536	8
7404348478	4
0254375059	4

AIR TICKET

Number	Check
0021373147367	3
10745778782465	6
10017048327873	6
12174851913640	3
10010424308333	1

ISBN

Number	Check
812190306	8
817525766	0
155615678	2
812192661	0
007048298	5

CHEQUES

Number	Check
09190204	8
82700200	0
40024000	2

UPC

Number	Check
02035712268	2
02233454545	3
12222233344	5
89268500100	3
57241200001	5

BANK NOTES

Number	Check
GA9556216U	7
DL2496197N	7

ITEM	NUMBER OF DIGITS	FORMULA OF CHECK	SINGLE DIGIT ERROR	TRANSPOSITION ON ERROR
POSTAL MONEY ORDER	**11**	$\square_{11} = (\square_1 + \square_2 + \square_3 + \square_4 + \square_5 + \square_6 + \square_7 + \square_8 + \square_9 + \square_{10})\ \square\ \square\ \square\ 9$	**Expect** $\square\square\square\square\ \square\ 9 = \square\square\ \square\square\square\ 9$	Cannot detects
AIR TICKET	**15**	$\square_{15} = (\square_1\square_2\square_3\square_4\square_5\square_6\square_7\square_8\square_9\square_{10}\square_{11}\square_{12}\square_{13}\square_{14})\ \square\square\square\ 7$	Expect $\square\square\square\square\ \square\ 7 = \square\ \square\text{-}1\ \square\square\ \square\ 7$	**Expect** $\square\square\square\square\square\ 7 = \square\square\text{-}1\square\square\square\ 7$
UPC	**12**	(9,8,7,6,5,4,3,2,1,0,9,8).(3,1,3,1,3,1,3,1,3,1,3,1,3) $\square\square\square 10 = 0$	All can be detected	**Except** $\lvert \square\square - \square\square\text{-}1 \rvert = 5$
ISBN	**10**	$(\square_1.10 + \square_2.9 + \square_3.8 + \square_4.7 + \square_5.6 + \square_6.5 + \square_7.4 + \square_8.3 + \square_9.2 + \square_{10}.1)\square\ \square\ \square\ 11 = 0$	All can be detected	All can be detected
CHEQUE	**9**	$(\square_1 .7 + \square_2.3 + \square_3.9 + \square_4.7 + \square_5.3 + \square_6.9 + \square_7.7 + \square_8.3 + \square_9.9)\ \square\square\square 10 = 0$	All can be detected	**Except** $\lvert \square\square - \square\square\text{-}1 \rvert = 5$
BANK NOTE	**11(NOT ALL) CONTAINS ALPHA NUMERIC**	$\square(\square_1)\square\square^2(\square_2)\square \ldots \square\square^{10}(\square_{10})\square\square_{11} = 0$, $\square = (01589427)(36)$	Cannot differentiate alphabets and numerals	Adjacent characters Involving the check digit itself

References

- Fraleigh. J.B a first course in Abstract Algebra
- Herstcin, I.N Topic in Algebra
- Robert B. Ash, Basic Abstract Algebra
- I 'Martin Isaacs, Finite Group theory
- Joseph J Rotman, Advanced Modern Algebra
- Bhattacharya, P.B Jain, S.K; and Nagpaul, S.R, Basic Abstract Algebra
- Joseph A Gallian, Contemporary Abstract algebra (9th edition)
- Google

Software

- Microsoft Calculator