

Figure 6.1 AES Encryption Process

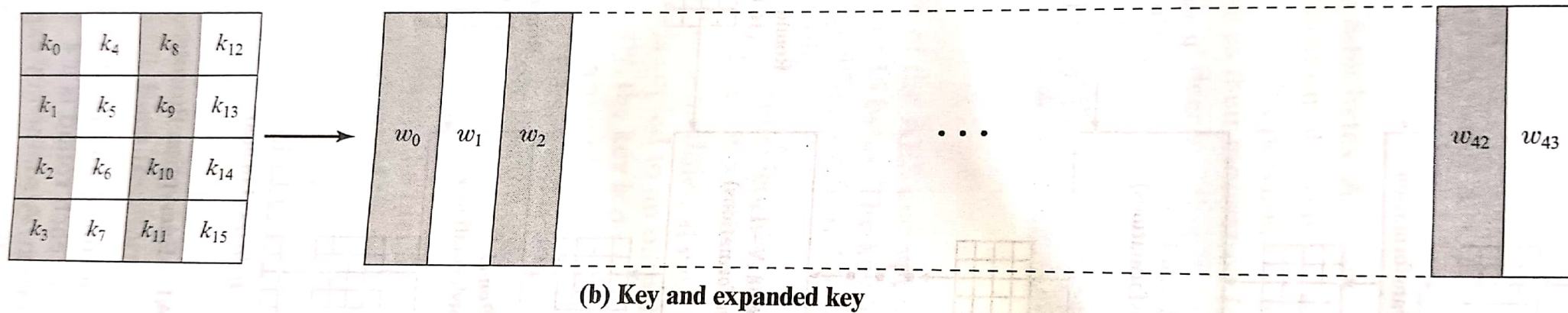


Figure 6.2 AES Data Structures

Table 6.1 AES Parameters

Key Size (words/bytes/bits)	4/16/128	6/24/192	8/32/256
Plaintext Block Size (words/bytes/bits)	4/16/128	4/16/128	4/16/128
Number of Rounds	10	12	14
Round Key Size (words/bytes/bits)	4/16/128	4/16/128	4/16/128
Expanded Key Size (words/bytes)	44/176	52/208	60/240



Table 6.2 AES S-Boxes

x	y															
	0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F
0	63	7C	77	7B	F2	6B	6F	C5	30	01	67	2B	FE	D7	AB	76
1	CA	82	C9	7D	FA	59	47	F0	AD	D4	A2	AF	9C	A4	72	C0
2	B7	FD	93	26	36	3F	F7	CC	34	A5	F5	F1	71	D8	31	15
3	04	C7	23	C3	18	96	05	9A	07	12	80	F2	EB	27	B2	75
4	09	83	2C	1A	1B	6E	5A	A0	52	3B	D6	B3	29	E3	2F	84
5	53	D1	00	ED	20	FC	B1	5B	6A	CB	BE	39	4A	4C	58	CF
6	D0	FF	AA	FB	43	4D	33	85	45	F9	02	7F	50	3C	9F	A8
7	51	A3	40	8F	92	9D	38	F5	BC	B6	DA	21	10	FF	F3	D2
8	CD	0C	13	EC	5F	97	44	17	C4	A7	7E	3D	64	5D	19	73
9	60	81	4F	DC	22	2A	90	88	46	EE	B8	14	DE	5E	0B	DB
A	F0	32	3A	0A	49	06	24	5C	C2	D3	AC	62	91	95	E4	79
B	E7	C8	37	6D	8D	D5	4E	A9	6C	56	F4	EA	65	7A	AE	08
C	BA	78	25	2E	1C	A6	B4	C6	E8	DD	74	1F	4B	BD	8B	8A
D	70	3E	B5	66	48	03	F6	0E	61	35	57	B9	86	C1	1D	9E
E	E1	F8	98	11	69	D9	8E	94	9B	1E	87	E9	CE	55	28	DF
F	8C	A1	89	0D	BF	E6	42	68	41	99	2D	0F	B0	54	BB	16

(a) S-box

x	y															
	0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F
0	52	09	6A	D5	30	36	A5	38	BF	40	A3	9E	81	F3	D7	FB
1	7C	E3	39	82	9B	2F	FF	87	34	8E	43	44	C4	DE	E9	CB
2	54	7B	94	32	A6	C2	23	3D	EE	4C	95	0B	42	FA	C3	4E
3	08	2E	A1	66	28	D9	24	B2	76	5B	A2	49	6D	8B	D1	25
4	72	F8	F6	64	86	68	98	16	D4	A4	5C	CC	5D	65	B6	92
5	6C	70	48	50	FD	ED	B9	DA	5E	15	46	57	A7	8D	9D	84
6	90	D8	AB	00	8C	BC	D3	0A	F7	E4	58	05	B8	B3	45	06
7	D0	2C	1E	8F	CA	3F	0F	02	C1	AF	BD	03	01	13	8A	6B
8	3A	91	11	41	4F	67	DC	EA	97	F2	CF	CE	F0	B4	E6	73
9	96	AC	74	22	E7	AD	35	85	E2	F9	37	E8	1C	75	DF	6E
A	47	F1	1A	71	1D	29	C5	89	6F	B7	62	0E	AA	18	BE	1B
B	FC	56	3E	4B	C6	D2	79	20	9A	DB	C0	FE	78	CD	5A	F4
C	1F	DD	A8	33	88	07	C7	31	B1	12	10	59	27	80	EC	5F
D	60	51	7F	A9	19	B5	4A	0D	2D	E5	7A	9F	93	C9	9C	FF
E	A0	F0	3B	4D	AE	2A	F5	B0	C8	EB	BB	3C	83	53	99	61
F	17	2B	04	7E	BA	77	D6	26	E1	69	14	63	55	21	0C	7D

(b) Inverse S-box

Here is an example of the SubBytes transformation:

EA	04	65	85
83	45	5D	96
5C	33	98	B0
F0	2D	AD	C5

→

87	F2	4D	97
EC	6E	4C	90
4A	C3	46	E7
8C	D8	95	A6

The S-box is constructed in the following fashion (Figure 6.6a).

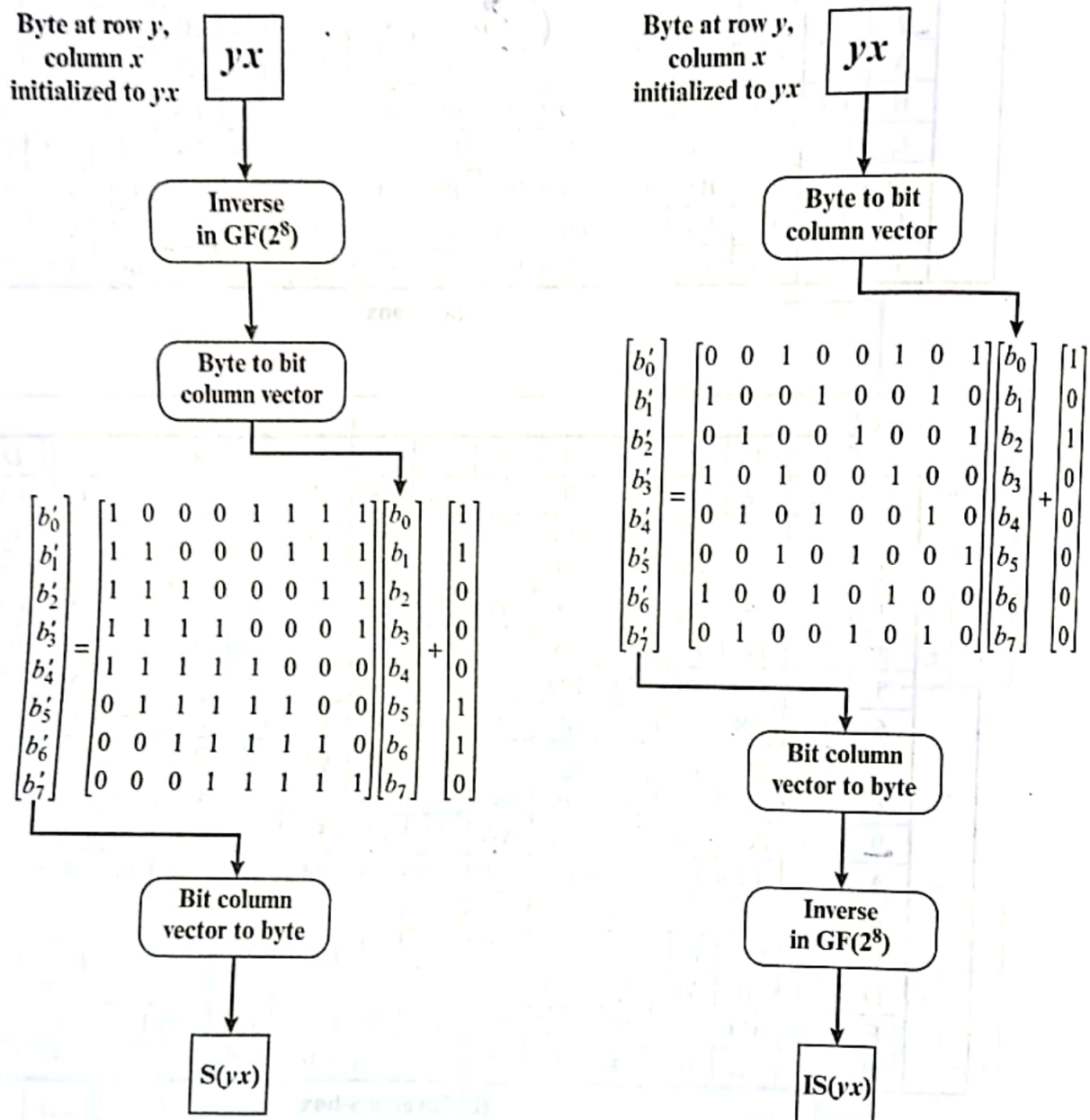


Figure 6.6 Constuction of S-Box and IS-Box



1. Initialize the S-box with the byte values in ascending sequence row by row. The first row contains {00}, {01}, {02}, ..., {0F}; the second row contains {10}, {11}, etc.; and so on. Thus, the value of the byte at row  $y$ , column  $x$  is  $\{yx\}$ .
2. Map each byte in the S-box to its multiplicative inverse in the finite field  $GF(2^8)$ ; the value {00} is mapped to itself.
3. Consider that each byte in the S-box consists of 8 bits labeled  $(b_7, b_6, b_5, b_4, b_3, b_2, b_1, b_0)$ . Apply the following transformation to each bit of each byte in the S-box:

$$b'_i = b_i \oplus b_{(i+4) \bmod 8} \oplus b_{(i+5) \bmod 8} \oplus b_{(i+6) \bmod 8} \oplus b_{(i+7) \bmod 8} \oplus c_i \quad (6.1)$$

where  $c_i$  is the  $i$ th bit of byte  $c$  with the value {63}; that is,  $(c_7c_6c_5c_4c_3c_2c_1c_0) = (01100011)$ . The prime (') indicates that the variable is to be updated by the value on the right. The AES standard depicts this transformation in matrix form as follows.

$$\begin{bmatrix} b'_0 \\ b'_1 \\ b'_2 \\ b'_3 \\ b'_4 \\ b'_5 \\ b'_6 \\ b'_7 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} b_0 \\ b_1 \\ b_2 \\ b_3 \\ b_4 \\ b_5 \\ b_6 \\ b_7 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 1 \\ 1 \\ 0 \end{bmatrix} \quad (6.2)$$

Equation (6.2) has to be interpreted carefully. In ordinary matrix multiplication,<sup>4</sup> each element in the product matrix is the sum of products of the elements of one row and one column. In this case, each element in the product matrix is the bitwise XOR of products of elements of one row and one column. Furthermore, the final addition shown in Equation (6.2) is a bitwise XOR. Recall from Section 5.6 that the bitwise XOR is addition in  $GF(2^8)$ .

As an example, consider the input value {95}. The multiplicative inverse in  $GF(2^8)$  is  $\{95\}^{-1} = \{8A\}$ , which is 10001010 in binary. Using Equation (6.2),

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \oplus \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \oplus \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

<sup>4</sup>For a brief review of the rules of matrix and vector multiplication, refer to Appendix E.

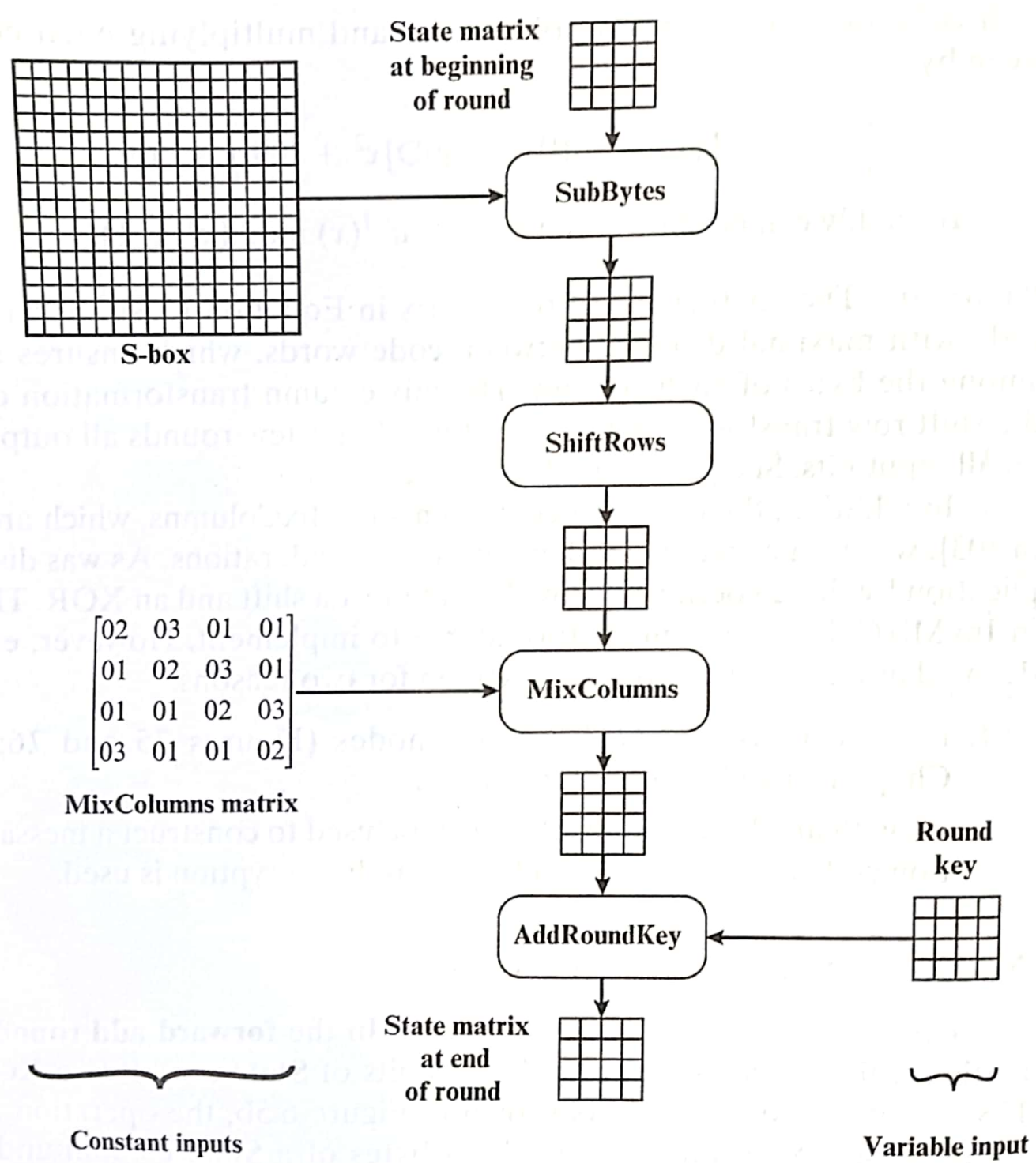
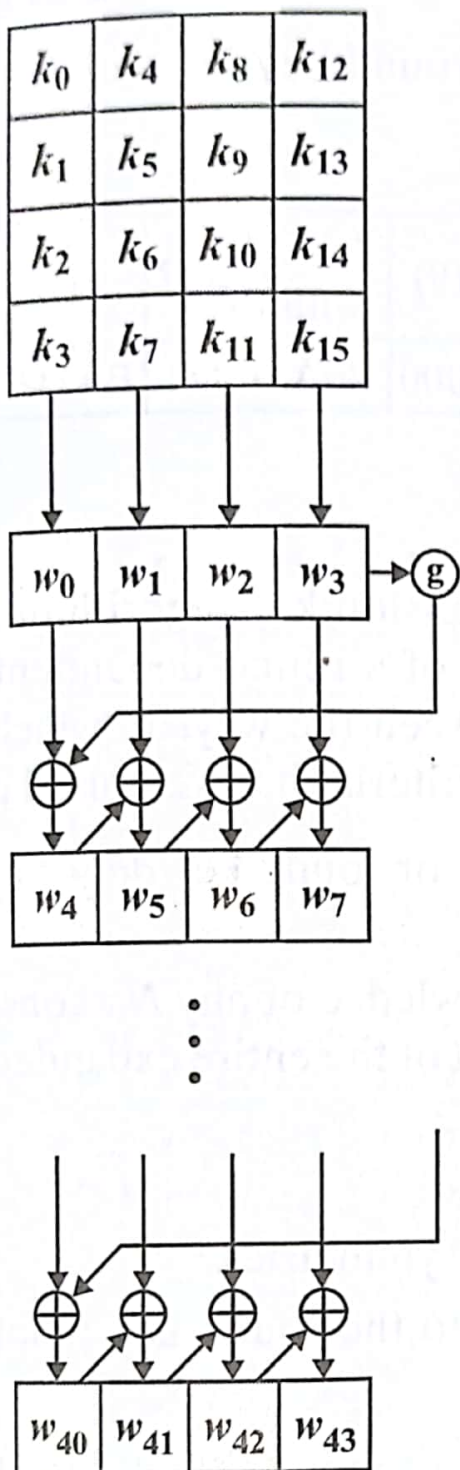
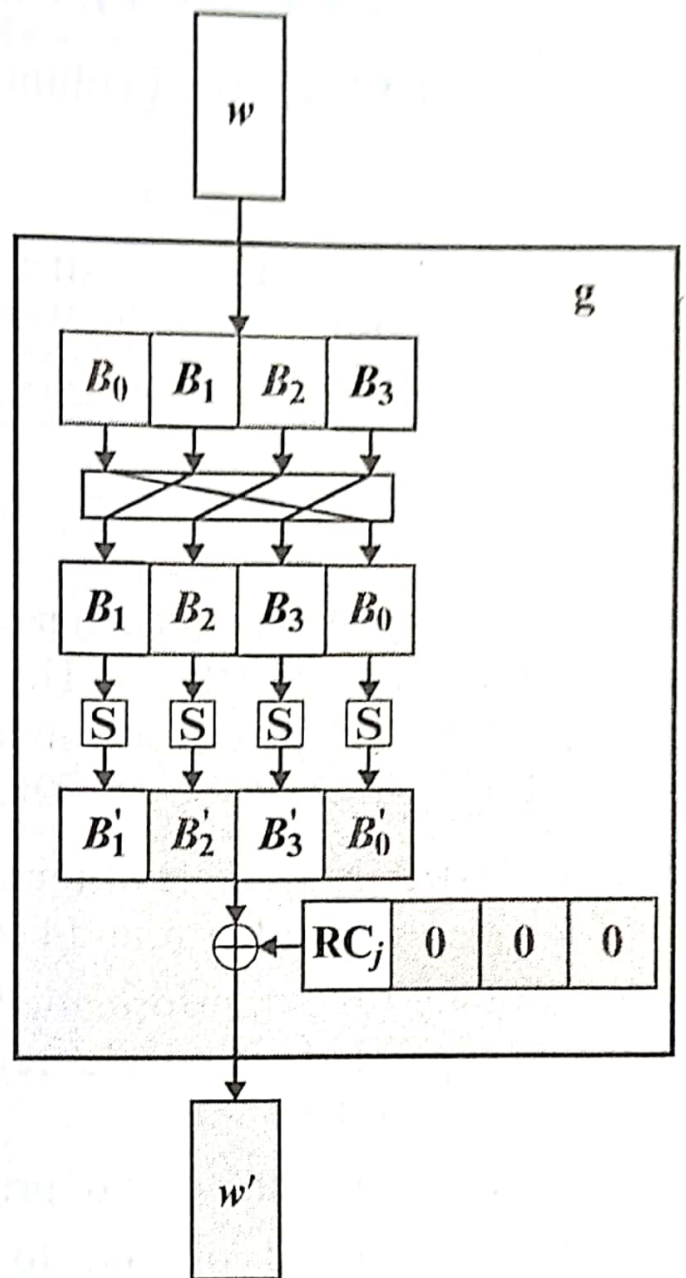


Figure 6.8 Inputs for Single AES Round



(a) Overall algorithm



(b) Function  $g$

Figure 6.9 AES Key Expansion



Table 6.3 Key Expansion for AES Example

Key Words	Auxiliary Function
$w_0 = 0f\ 15\ 71\ c9$ $w_1 = 47\ d9\ e8\ 59$ $w_2 = 0c\ b7\ ad\ d6$ $w_3 = af\ 7f\ 67\ 98$	$RotWord\ (w_3) = 7f\ 67\ 98\ af = x_1$ $SubWord\ (x_1) = d2\ 85\ 46\ 79 = y_1$ $Rcon\ (1) = 01\ 00\ 00\ 00$ $y_1 \oplus Rcon\ (1) = d3\ 85\ 46\ 79 = z_1$
$w_4 = w_0 \oplus z_1 = dc\ 90\ 37\ b0$ $w_5 = w_4 \oplus w_1 = 9b\ 49\ df\ e9$ $w_6 = w_5 \oplus w_2 = 97\ fe\ 72\ 3f$ $w_7 = w_6 \oplus w_3 = 38\ 81\ 15\ a7$	$RotWord\ (w_7) = 81\ 15\ a7\ 38 = x_2$ $SubWord\ (x_2) = 0c\ 59\ 5c\ 07 = y_2$ $Rcon\ (2) = 02\ 00\ 00\ 00$ $y_2 \oplus Rcon\ (2) = 0e\ 59\ 5c\ 07 = z_2$
$w_8 = w_4 \oplus z_2 = d2\ c9\ 6b\ b7$ $w_9 = w_8 \oplus w_5 = 49\ 80\ b4\ 5e$ $w_{10} = w_9 \oplus w_6 = de\ 7e\ c6\ 61$ $w_{11} = w_{10} \oplus w_7 = e6\ ff\ d3\ c6$	$RotWord\ (w_{11}) = ff\ d3\ c6\ e6 = x_3$ $SubWord\ (x_3) = 16\ 66\ b4\ 83 = y_3$ $Rcon\ (3) = 04\ 00\ 00\ 00$ $y_3 \oplus Rcon\ (3) = 12\ 66\ b4\ 8e = z_3$
$w_{12} = w_8 \oplus z_3 = c0\ af\ df\ 39$ $w_{13} = w_{12} \oplus w_9 = 89\ 2f\ 6b\ 67$ $w_{14} = w_{13} \oplus w_{10} = 57\ 51\ ad\ 06$ $w_{15} = w_{14} \oplus w_{11} = b1\ ae\ 7e\ c0$	$RotWord\ (w_{15}) = ae\ 7e\ c0\ b1 = x_4$ $SubWord\ (x_4) = e4\ f3\ ba\ c8 = y_4$ $Rcon\ (4) = 08\ 00\ 00\ 00$ $y_4 \oplus Rcon\ (4) = ec\ f3\ ba\ c8 = z_4$

(Continued)

Table 6.3 Continued

Key Words	Auxiliary Function
$w_{16} = w_{12} \oplus z_4 = 2c\ 5c\ 65\ f1$ $w_{17} = w_{16} \oplus w_{13} = a5\ 73\ 0e\ 96$ $w_{18} = w_{17} \oplus w_{14} = f2\ 22\ a3\ 90$ $w_{19} = w_{18} \oplus w_{15} = 43\ 8c\ dd\ 50$	$RotWord\ (w_{19}) = 8c\ dd\ 50\ 43 = x_5$ $SubWord\ (x_5) = 64\ c1\ 53\ 1a = y_5$ $Rcon(5) = 10\ 00\ 00\ 00$ $y_5 \oplus Rcon\ (5) = 74\ c1\ 53\ 1a = z_5$
$w_{20} = w_{16} \oplus z_5 = 58\ 9d\ 36\ eb$ $w_{21} = w_{20} \oplus w_{17} = fd\ ee\ 38\ 7d$ $w_{22} = w_{21} \oplus w_{18} = 0f\ cc\ 9b\ ed$ $w_{23} = w_{22} \oplus w_{19} = 4c\ 40\ 46\ bd$	$RotWord\ (w_{23}) = 40\ 46\ bd\ 4c = x_6$ $SubWord\ (x_6) = 09\ 5a\ 7a\ 29 = y_6$ $Rcon(6) = 20\ 00\ 00\ 00$ $y_6 \oplus Rcon(6) = 29\ 5a\ 7a\ 29 = z_6$
$w_{24} = w_{20} \oplus z_6 = 71\ c7\ 4c\ c2$ $w_{25} = w_{24} \oplus w_{21} = 8c\ 29\ 74\ bf$ $w_{26} = w_{25} \oplus w_{22} = 83\ e5\ ef\ 52$ $w_{27} = w_{26} \oplus w_{23} = cf\ a5\ a9\ ef$	$RotWord\ (w_{27}) = a5\ a9\ ef\ cf = x_7$ $SubWord\ (x_7) = 06\ d3\ bf\ 8a = y_7$ $Rcon\ (7) = 40\ 00\ 00\ 00$ $y_7 \oplus Rcon(7) = 46\ d3\ df\ 8a = z_7$
$w_{28} = w_{24} \oplus z_7 = 37\ 14\ 93\ 48$ $w_{29} = w_{28} \oplus w_{25} = bb\ 3d\ e7\ f7$ $w_{30} = w_{29} \oplus w_{26} = 38\ d8\ 08\ a5$ $w_{31} = w_{30} \oplus w_{27} = f7\ 7d\ a1\ 4a$	$RotWord\ (w_{31}) = 7d\ a1\ 4a\ f7 = x_8$ $SubWord\ (x_8) = ff\ 32\ d6\ 68 = y_8$ $Rcon\ (8) = 80\ 00\ 00\ 00$ $y_8 \oplus Rcon(8) = 7f\ 32\ d6\ 68 = z_8$
$w_{32} = w_{28} \oplus z_8 = 48\ 26\ 45\ 20$ $w_{33} = w_{32} \oplus w_{29} = f3\ 1b\ a2\ d7$ $w_{34} = w_{33} \oplus w_{30} = cb\ c3\ aa\ 72$ $w_{35} = w_{34} \oplus w_{32} = 3c\ be\ 0b\ 3$	$RotWord\ (w_{35}) = be\ 0b\ 38\ 3c = x_9$ $SubWord\ (x_9) = ae\ 2b\ 07\ eb = y_9$ $Rcon\ (9) = 1B\ 00\ 00\ 00$ $y_9 \oplus Rcon\ (9) = b5\ 2b\ 07\ eb = z_9$
$w_{36} = w_{32} \oplus z_9 = fd\ 0d\ 42\ cb$ $w_{37} = w_{36} \oplus w_{33} = 0e\ 16\ e0\ 1c$ $w_{38} = w_{37} \oplus w_{34} = c5\ d5\ 4a\ 6e$ $w_{39} = w_{38} \oplus w_{35} = f9\ 6b\ 41\ 56$	$RotWord\ (w_{39}) = 6b\ 41\ 56\ f9 = x_{10}$ $SubWord\ (x_{10}) = 7f\ 83\ b1\ 99 = y_{10}$ $Rcon\ (10) = 36\ 00\ 00\ 00$ $y_{10} \oplus Rcon\ (10) = 49\ 83\ b1\ 99 = z_{10}$
$w_{40} = w_{36} \oplus z_{10} = b4\ 8e\ f3\ 52$ $w_{41} = w_{40} \oplus w_{37} = ba\ 98\ 13\ 4e$ $w_{42} = w_{41} \oplus w_{38} = 7f\ 4d\ 59\ 20$ $w_{43} = w_{42} \oplus w_{39} = 86\ 26\ 18\ 76$	



Start of Round	After SubBytes	After ShiftRows	After MixColumns	Round Key
01 89 fe 76 23 ab dc 54 45 cd ba 32 67 ef 98 10				0f 47 0c af 15 d9 b7 7f 71 e8 ad 67 c9 59 d6 98
0e ce f2 d9 36 72 6b 2b 34 25 17 55 ae b6 4e 88	ab 8b 89 35 05 40 7f f1 18 3f f0 fc e4 4e 2f c4	ab 8b 89 35 40 7f f1 05 f0 fc 18 3f c4 e4 4e 2f	b9 94 57 75 e4 8e 16 51 47 20 9a 3f c5 d6 f5 3b	dc 9b 97 38 90 49 fe 81 37 df 72 15 b0 e9 3f a7
65 0f c0 4d 74 c7 e8 d0 70 ff e8 2a 75 3f ca 9c	4d 76 ba e3 92 c6 9b 70 51 16 9b e5 9d 75 74 de	4d 76 ba e3 c6 9b 70 92 9b e5 51 16 de 9d 75 74	8e 22 db 12 b2 f2 dc 92 df 80 f7 c1 2d c5 1e 52	d2 49 de e6 c9 80 7e ff 6b b4 c6 d3 b7 5e 61 c6
5c 6b 05 f4 7b 72 a2 6d b4 34 31 12 9a 9b 7f 94	4a 7f 6b bf 21 40 3a 3c 8d 18 c7 c9 b8 14 d2 22	4a 7f 6b bf 40 3a 3c 21 c7 c9 8d 18 22 b8 14 d2	b1 c1 0b cc ba f3 8b 07 f9 1f 6a c3 1d 19 24 5c	c0 89 57 b1 af 2f 51 ae df 6b ad 7e 39 67 06 c0
71 48 5c 7d 15 dc da a9 26 74 c7 bd 24 7e 22 9c	a3 52 4a ff 59 86 57 d3 f7 92 c6 7a 36 f3 93 de	a3 52 4a ff 86 57 d3 59 c6 7a f7 92 de 36 f3 93	d4 11 fe 0f 3b 44 06 73 cb ab 62 37 19 b7 07 ec	2c a5 f2 43 5c 73 22 8c 65 0e a3 dd f1 96 90 50
f8 b4 0c 4c 67 37 24 ff ae a5 c1 ea e8 21 97 bc	41 8d fe 29 85 9a 36 16 e4 06 78 87 9b fd 88 65	41 8d fe 29 9a 36 16 85 78 87 e4 06 65 9b fd 88	2a 47 c4 48 83 e8 18 ba 84 18 27 23 eb 10 0a f3	58 fd 0f 4c 9d ee cc 40 36 38 9b 46 eb 7d ed bd
72 ba cb 04 1e 06 d4 fa b2 20 bc 65 00 6d e7 4e	40 f4 1f f2 72 6f 48 2d 37 b7 65 4d 63 3c 94 2f	40 f4 1f f2 6f 48 2d 72 65 4d 37 b7 2f 63 3c 94	7b 05 42 4a 1e d0 20 40 94 83 18 52 94 c4 43 fb	71 8c 83 cf c7 29 e5 a5 4c 74 ef a9 c2 bf 52 ef
0a 89 c1 85 d9 f9 c5 e5 d8 f7 f7 fb 56 7b 11 14	67 a7 78 97 35 99 a6 d9 61 68 68 0f b1 21 82 fa	67 a7 78 97 99 a6 d9 35 68 0f 61 68 fa b1 21 82	ec 1a c0 80 0c 50 53 c7 3b d7 00 ef b7 22 72 e0	37 bb 38 f7 14 3d d8 7d 93 e7 08 a1 48 f7 a5 4a
db a1 f8 77 18 6d 8b ba a8 30 08 4e ff d5 d7 aa	b9 32 41 f5 ad 3c 3d f4 c2 04 30 2f 16 03 0e ac	b9 32 41 f5 3c 3d f4 ad 30 2f c2 04 ac 16 03 0e	b1 1a 44 17 3d 2f ec b6 0a 6b 2f 42 9f 68 f3 b1	48 f3 cb 3c 26 1b c3 be 45 a2 aa 0b 20 d7 72 38
f9 e9 8f 2b 1b 34 2f 08 4f c9 85 49 bf bf 81 89	99 1e 73 f1 af 18 15 30 84 dd 97 3b 08 08 0c a7	99 1e 73 f1 18 15 30 af 97 3b 84 dd a7 08 08 0c	31 30 3a c2 ac 71 8c c4 46 65 48 eb 6a 1c 31 62	fd 0e c5 f9 0d 16 d5 6b 42 e0 4a 41 cb 1c 6e 56
cc 3e ff 3b a1 67 59 af 04 85 02 aa a1 00 5f 34	4b b2 16 e2 32 85 cb 79 f2 97 77 ac 32 63 cf 18	4b b2 16 e2 85 cb 79 32 77 ac f2 97 18 32 63 cf		b4 ba 7f 86 8e 98 4d 26 f3 13 59 18 52 4e 20 76
ff 08 69 64 0b 53 34 14 84 bf ab 8f 4a 7c 43 b9				