DEPARTMENT OF INFORMATION TECHNOLOGY

Practice Set -1

Automata and Compiler Design (IT3202)

B. Tech (IT), VI sem., Even 2023-24

Due Date: Complete withing 5 days of release.

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1. Prove or disprove that a set of numbers divisible by n is closed under addition, subtraction, multiplication and division operations.
2. Prove that: A U (B ∩ C) = (A U B) ∩ (A U C).
3. Prove that: (AUB) ∩ (B U C) ∩ (C U A) = (A ∩ B) U (B ∩ C) U (C ∩ A).
4. If A={a, b} and B={b, c}, find:
   a. $(A \cup B)^*$
   b. $(A \cap B)^*$
   c. $A^* \cup B^*$
   d. $A^* \cap B^*$
   e. $(A - B)^*$
   f. $(B - A)^*$
5. Let $S = \{a, b\}^*$. For x, y ϵ S , define x o y = xy, i.e. x o y is obtained by concatenating x and y.
   a. Is S closed under o ?
   b. Is o associative?
   c. Does S have identity element with respect to o ?
   d. Is o commutative?
6. Let $S = 2^X$, where X is any non-empty set. For A, B ⊆ X, let A o B = A U B.
   a. Is o commutative and associative?
   b. Does S have the identity element with respect to o ?
   c. If A o B = A o C , does it imply that B = C ?
7. Test whether the following statements are true or false. Justify your answer.
   a. The set of all odd integers is a monoid under multiplication.
   b. The set of all complex numbers is a group under multiplication.
   c. The set of all integers under the operation o given by a o b = a + b – ab is a monoid.
8. Let R = {(1,2), (2,3), (1,4),(4,2),(3,4)}. Find R^+ and R^*.
9. Construct a DFA with minimum number of states, accepting all strings over {a, b} such that the number of a’s is divisible by two and the number of b’s is divisible by three.

10. Construct a DFA with minimum number of states, accepting all strings over {a, b} such that the number of a's is divisible by three and the number of b's is divisible by two.
11. Draw a DFA which accepts all strings over {a, b} such that no string has three consecutive occurrences of the letter b.
12. Design a DFA to recognize all strings over {a,b} such that L={awa : w ϵ $\{a,b\}^*$}.
13. Design a DFA to accept all strings over {a,b} such that L={aw_1aaw_2a : w_1 , w_2 ϵ $\{a,b\}^*$.
14. Design a DFA for $\Sigma = \{a, b\}$ that can accept all strings with no more than three a's.
15. Find DFA for the following languages on $\Sigma = \{a, b\}$:
    a. L = {w : |w| mod 3 = 0
    b. L = {w : $n_a(w)$ mod 3 > $n_b(w)$ mod 3}
16. Consider the set of strings on {0,1}. Design a DFA to accept all strings where every 00 is followed immediately by a 1.
17. Draw a DFA for all binary strings divisible by 3.
18. Draw a DFA for all binary strings divisible by 2.
19. Suppose $\Sigma = \{0, 1, 2\}$. Draw a DFA for the language L = { w | the sum of digits in w is divisible by 5}.
20. Draw a DFA for $L = \{0^n1^m \mid m \geq 1 , n \geq 0 ; (n+m)$ is divisible by $3\}$.
21. Suppose $\Sigma = \{a, b\}$. Find a DFA for the set of strings w such that the number of occurrences of the substring ab in w equals the number of occurrences of the substring ba in w.
22. Construct a DFA accepting all strings w over $\Sigma = \{0, 1\}$ such that the number of 1's in w is 3 mod 4.
23. Construct a DFA to accept all strings over $\Sigma = \{a, b\}$ that ends in bb.
24. Design a DFA to accept all strings over $\Sigma = \{a, b\}$ with even number of a's and even number of b's.
25. Define DFA and NDFA and differentiate between them.