ARTIFICIAL INTELLIGENCE AND MACHINE LEARNING (IT3201)

(Sem: 6th (Section B))

Lecture 29-32

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Tautology

- A tautology is a statement that is always true, regardless of the truth values of its individual components.
- In other words, the statement is true under every possible combination of truth values for its variables.
- A common example is the statement "A or not A," where A is any proposition. This statement is always true because either A is true or its negation (not A) is true.
- Example:

$A \vee \neg A$ (A or not A)

Tautology

- **Tautology:**
- **Example:** Consider the statement "The weather is either sunny or not sunny." Here, if we let A represent "the weather is sunny," then the statement can be written as $A \vee \neg A$, which is a tautology. This is because the weather must either be sunny (A) or not sunny ($\neg A$).
- **Tautology:**
- **Example:** Consider the statement "Every even number is either divisible by 2 or not divisible by 3." Let A be "the number is even." The statement can be expressed as $A \vee \neg A$, which is a tautology because every number is either even (A) or not even ($\neg A$).

Equivalence

- **Equivalence:**
- Two statements are said to be equivalent if they have the same truth values for all possible combinations of truth values of their variables.
- In other words, if Statement A and Statement B are equivalent, then A is true if and only if B is true.
- Example:
  - $A \wedge (B \vee C)$ is equivalent to $(A \wedge B) \vee (A \wedge C)$
  - In this case, both statements have the same truth values for all possible combinations of truth values for A, B, and C.

Equivalence

- **Example:** Let's take the statements:
- $P \land (Q \lor R)$
- $P \land Q) \lor (P \land R)$
- These two statements are equivalent. For instance, if P represents "It is raining," Q represents "I have an umbrella," and R represents "I am wearing a raincoat," then both statements express the idea that either it is raining and I have an umbrella, or it is raining and I am wearing a raincoat.

- **Example:** Let's take the statements:
- $P \rightarrow Q$ (If it's raining, then the ground is wet.)
- $\neg P \lor Q$ (It's not raining, or the ground is wet.)
- These two statements are equivalent. In real-world terms, they both convey the idea that the ground is wet, either because it's currently raining or because it's not raining but the ground is wet for some other reason.

Equivalence

Commutative Laws:

1. $P \wedge Q \equiv Q \wedge P$
2. $P \vee Q \equiv Q \vee P$

These laws state that the order of conjunction (AND) and disjunction (OR) does not affect the truth value.

Associative Laws:

1. $(P \wedge Q) \wedge R \equiv P \wedge (Q \wedge R)$
2. $(P \vee Q) \vee R \equiv P \vee (Q \vee R)$

These laws state that the grouping of conjunction and disjunction does not affect the truth value.

Distributive Laws:

1. $P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$
2. $P \vee (Q \wedge R) \equiv (P \vee Q) \wedge (P \vee R)$

These laws describe how conjunction and disjunction interact with each other.

Equivalence

Identity Laws:

1. $P \land \mathrm{T} \equiv P$
2. $P \lor \mathrm{F} \equiv P$

These laws indicate that the conjunction of any proposition with true (T) and the disjunction with false (F) are equivalent to the original proposition.

Negation Laws:

1. $\neg\neg P \equiv P$
2. $P \land \neg P \equiv \mathrm{F}$
3. $P \lor \neg P \equiv \mathrm{T}$

These laws govern the negation of propositions and the relationships between a proposition and its negation.

Double Negation:

1. $\neg\neg P \equiv P$

This rule states that negating a proposition twice is equivalent to the original proposition.

Equivalence

1.De Morgan's Laws:

1. $\neg (P \wedge Q) \equiv \neg P \vee \neg Q$
2. $\neg (P \vee Q) \equiv \neg P \wedge \neg Q$

These laws describe how negation interacts with conjunction and disjunction.

1.Implication Equivalence:

1. $P \rightarrow Q \equiv \neg P \vee Q$

This rule expresses the equivalence between implication and a combination of negation and disjunction.

Contradiction

Contradiction:

- A contradiction is a statement that is always false, regardless of the truth values of its individual components.
- It arises when a statement and its negation are combined.
- Example:
  - $P \land \neg P$
  - This statement asserts that both P and not P are true at the same time, which is impossible. Therefore, this statement is always false.

Contradiction

Contradiction:

•**Example:** Consider the statement "The car is both in the garage and not in the garage at the same time." Let P represent "the car is in the garage." The statement can be written as $P \wedge \neg P$, which is a contradiction. It asserts that the car is simultaneously in the garage (P) and not in the garage ($\neg P$), which is logically impossible.

•**Example:** Consider the statement "The light is both on and off simultaneously." Let L represent "the light is on." The statement can be written as $L \wedge \neg L$, which is a contradiction. It asserts that the light is both on (L) and not on ($\neg L$) at the same time, which is logically impossible.

Conclusion

In summary:

- A tautology is always true.
- Two statements are equivalent if they have the same truth values.
- A contradiction is always false.