Attention U-Net:

- This architecture is a variation of the standard U-Net with the inclusion of an Attention Gate (AG).
- The AG model takes two inputs:
    1. Features from the previous layer (down-below), which have been through convolutions and ReLU activations.
    2. Features from the skip connection, which carry spatial information.
- The features from the skip connection and the output of the previous layer's convolutions are concatenated and then processed through a ReLU activation.
- The gating signal (g) in the AG model is derived from the features obtained after ReLU activation, and it is used to control the skip connections, effectively highlighting important features and suppressing less relevant ones.
- The order of operations in the blocks is convolution, batch normalization, activation, and optionally dropout.

Attention Filter Gate:

- Operates in the frequency domain using Fast Fourier Transform (FFT).
- The Attention Filter Gate (AFG) is positioned strategically to filter features in the frequency domain, potentially reducing computational costs compared to convolution operations in the spatial domain.
- Sigmoid and Softmax functions are used in the AFG to handle output classifications. Specifically, Sigmoid is used to control the gating mechanism by determining the importance of different frequency components. Softmax is then applied to normalize these components, ensuring that the most significant frequencies have a higher impact on the model's output.

Mathematical Representation:

- Sigmoid Activation Function: $g(z)=g\sigma_{real}(Re(z))+g\sigma_{imag}(Im(z))$
    1. This function controls the gating mechanism, determining the importance of different frequency components.
- Softmax Function: Applied to normalize the components, ensuring that the most significant frequencies have a higher impact on the model's output.

CVNNs and AFG:

- Complex-valued neural networks (CVNNs) are introduced with the AFG, which combines Softmax and Sigmoid functions.
- The AFG is tailored to handle frequencies in the frequency domain before executing the inverse FFT.
- The Sigmoid activation function is represented as a combination of real-valued activation functions on the real and imaginary parts of the input.

Global Filters Application:

- Global filters are used to emphasize or suppress certain frequency components.
- They decide which frequencies are most relevant to the segmentation task (using Sigmoid) and ensure that the model focuses on the most significant frequencies (using Softmax).

FFT Implementation:

- FFT calculates the contribution of each pixel at different frequency components.
- Utilizing FFT over convolution operations is advantageous because convolution in the time domain is equivalent to multiplication in the frequency domain.
- FFT, with its O(N log N) complexity, is implemented between up-sampling and down-sampling to reduce matrix multiplication costs.

Self-Attention Mechanism:

- Self-attention processing sequences assign higher probabilities to certain elements, allowing for parallel processing and inter-sequence interactions.
- Softmax normalizes these probabilities between 0 and 1.
- Attention networks for image captioning (e.g., "Show, Attend and Tell") emphasize informative features and suppress less informative ones.
- Soft-attention mechanisms are crucial for advanced processing and interpretation of frequency components of medical images.
- These notes reflect the intricacies of the Attention U-Net and its components, emphasizing the role of the Attention Gate in enhancing feature selection for image segmentation tasks.