Ant Colony Optimization: A New Meta-heuristic

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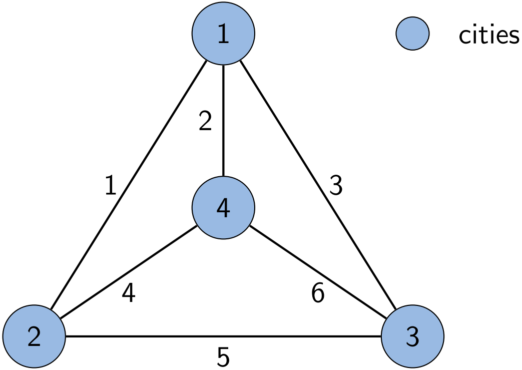
Problem Definition

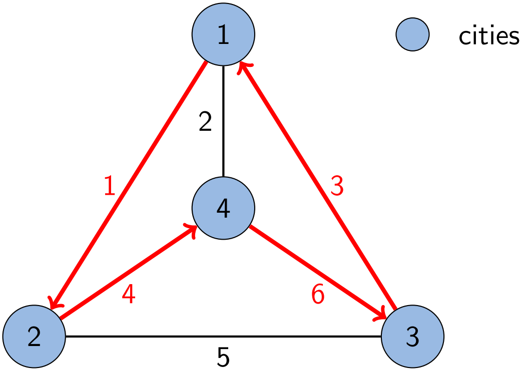
The Traveling Salesman Problem

A salesman needs to visit a number of customers located in different cities and return to the starting city using the shortest route.

Input:



Output:



Known Methods

- Backtracking

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Motivation

We will use Ant Colony Optimization (ACO) to solve TSP more efficiently.

Ants collecting food-1

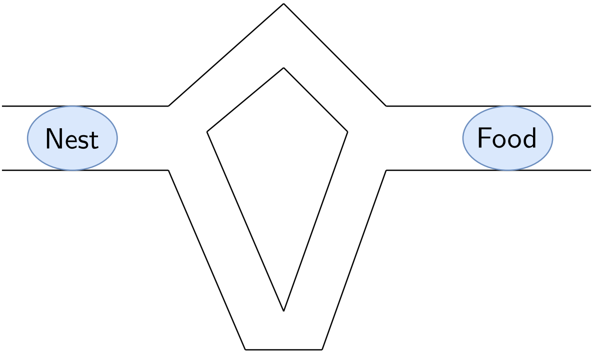

Figure: Paths From Food to Ants' Nest

Ants collecting food-2

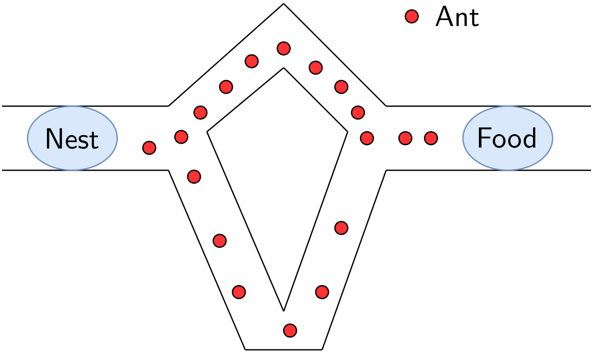

Figure: Ants Searching for Food

Ants collecting food-3

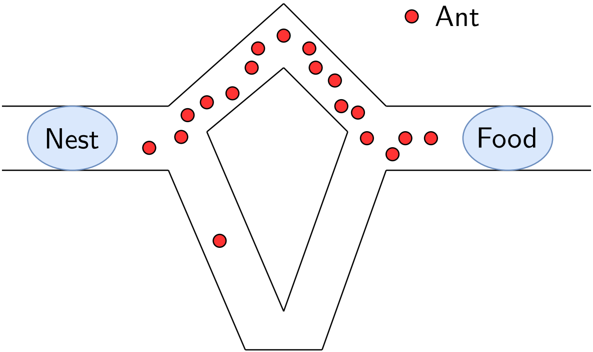

Figure: Ants Following An Optimal Path

So how do they communicate??

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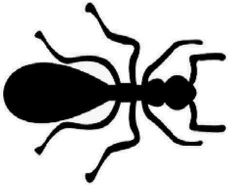
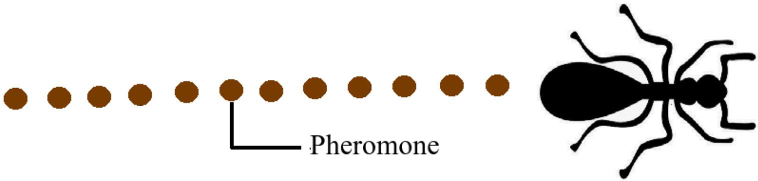


So how do they communicate??

Pheromone



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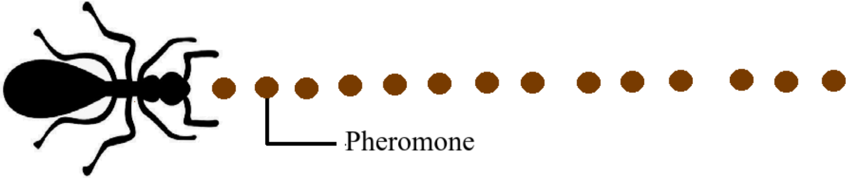

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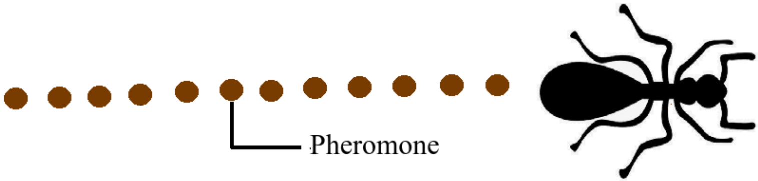


Previous Works

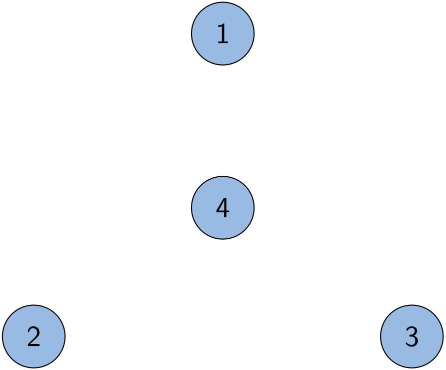
- In the year 1991, Marco Dorigo proposed an algorithm called "Ant System".
- AS was first applied to the Traveling Salesman Problem.

Figure: Marco Dorigo

Results

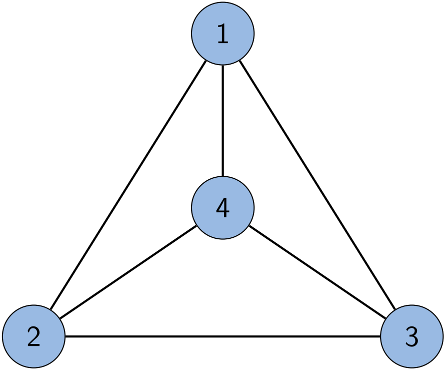
A more efficient algorithm to solve TSP is the ACO meta-heuristic.

Notations



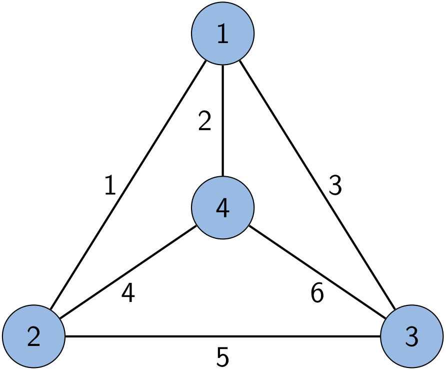
$C = \{c_1, c_2, ..., c_{N_C}\}$ is the set of *cities*.

Notations Continued..



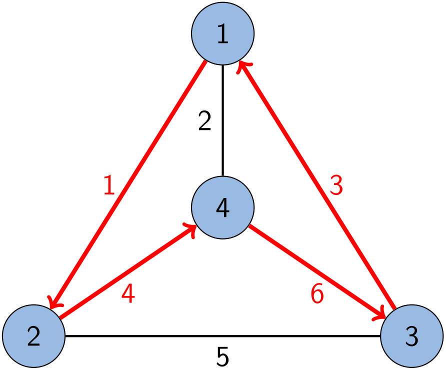
$L = \{l_{c_i c_j} \mid (c_i, c_j) \in \tilde{C}\}, \; |L| \leq N_C^2$ is the set of *connections* between *cities*.

Notations Continued..



$J_{c_i c_j} \equiv J(l_{c_i c_j})$ is a *cost* function associated with each *connection* $l_{c_i c_j} \in L$.

Notations Continued..

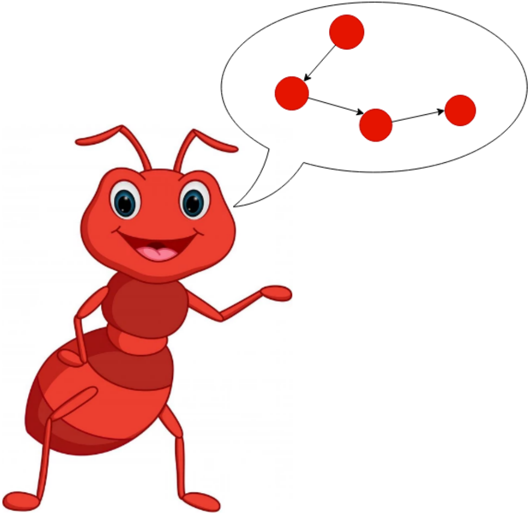


$J_\psi(L)$ is the *cost function* associated to each solution ψ. It is the summation of all the costs $J(c_i, c_j)$ of all the connections belonging to the solution ψ .

The optimal cost here is $1 + 4 + 6 + 3 = 14$

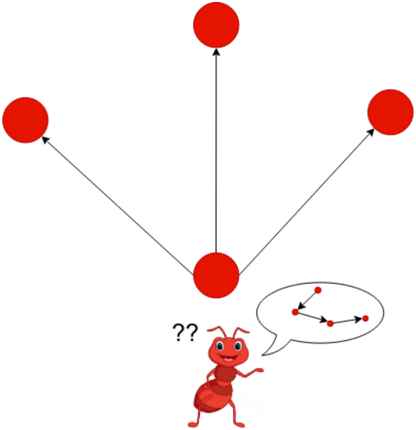
Ant Properties

- Every ant has its own memory.



Ant Properties Continued

- An ant chooses the next node to visit from its memory and the ant-routing table



Ant Properties Continued

$$\begin{pmatrix} 0 & 5 & 3 & 1 \\ 4 & 0 & 1 & 6 \\ 9 & 3 & 0 & 9 \\ 12 & 4 & 15 & 0 \end{pmatrix}$$

Ant-routing table, a

Here, a_{ij} is a measurement of the quality of the edge from node i to node j.

Formula for ant-routing table

The formula for updating the ant-routing table is:

$$a_{ij} = \frac{[\tau_{ij}(t)]^{\alpha}[\eta_{ij}]^{\beta}}{\sum_{l \in \mathcal{N}_i} [\tau_{il}(t)]^{\alpha}[\eta_{il}]^{\beta}} \qquad \forall j \in \mathcal{N}_i$$

- τ_{ij} is the intensity of pheromone trail of the edge l_{ij}
- η_{ij} is the heuristic value of the edge between i and j.

$$\eta_{ij} = \frac{1}{J_{c_i c_j}}$$

- α and β are two parameters that control the relative weight of pheromone trail and heuristic value.

Formula for ant-routing table Continued

The probability $p_{ij}^k(t)$ with which an ant k located in city i chooses the city $j \in \mathcal{N}_i^k$ to move to at the t-th iteration is:

$$p_{ij}^k(t) = \frac{a_{ij}(t)}{\displaystyle\sum_{l \in \mathcal{N}_i^k} a_{il}(t)}$$

where $\mathcal{N}_i^k \subseteq \mathcal{N}_i$ is the feasible neighborhood of node i for ant k.

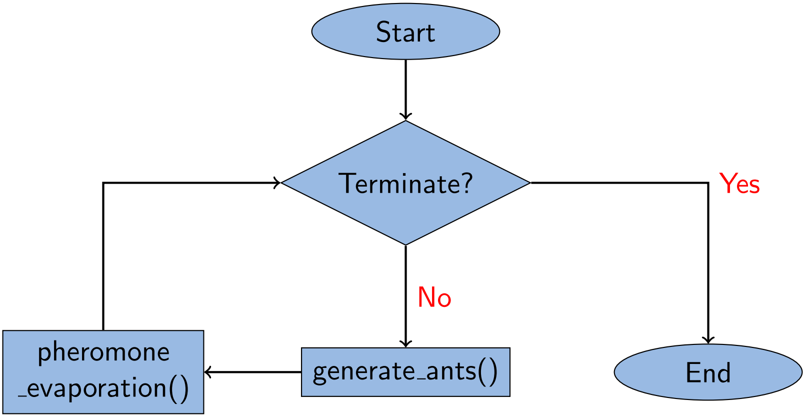
Pheromone Trail Evaporation

After pheromone updating has been performed by the ants, pheromone evaporation is triggered: the following rule is applied to all the edges l_{ij} of the graph G

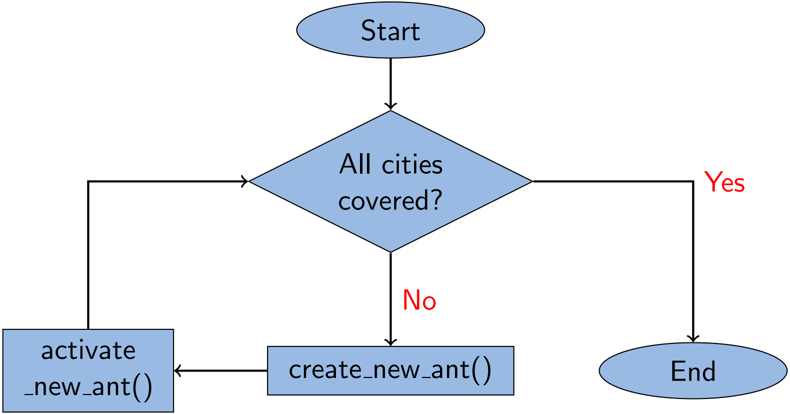
$$\tau_{ij}(t) \leftarrow (1-\rho)\tau_{ij}(t)$$

where $\rho \in (0,1]$ is the pheromone trail decay coefficient.

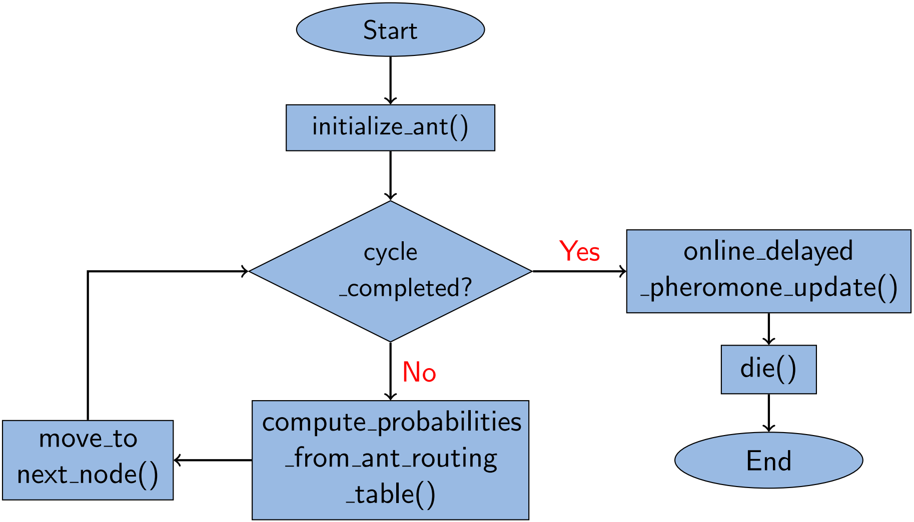
procedure ACO_meta-heuristic()



procedure generate_ants()



procedure activate_new_ant() {Ant lifecycle}



Conclusions

ACO is a new weapon to attack NP-hard graph problem!!