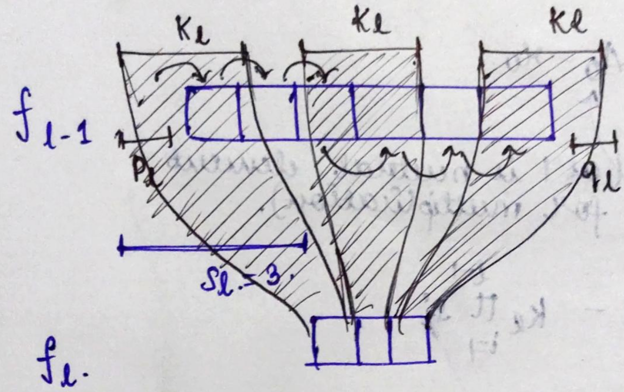
fully-Convolutional Network with L layers ($l = 1, 2, \ldots L$).
feature map $f_l \in \mathbb{R}^{h_l \times w_l \times d_l}$ to denote output of l^{th} layer of dimension ($h_l \times w_l \times d_l$).

Each layer l's spatial configuration is parametrised by 4 variables:

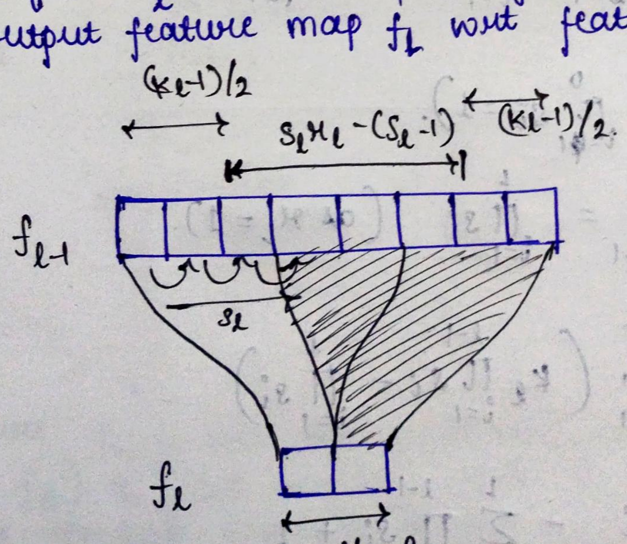
k_l = Kernel size ($k_l \geq 0$) ∪ ($k_l \in \mathbb{Z}$)
s_l = stride ($s_l \geq 0$) ∪ ($s_l \in \mathbb{Z}$).
p_l = padding to left side of input feature map.
q_l = " right side



let
$\begin{cases} k_l = 2 \\ p_l = q_l = 1 \\ s_l = 3 \end{cases}$

* k_l features from f_{l-1} can influence one feature from f_l

* Define r_l as the receptive feild size of the final output feature map f_L wrt feature map f_l.



let
$\begin{cases} k_l = 5 \\ p_l = q_l = 0 \\ s_l = 3 \end{cases}$

we obtain a general recurrence equation of first order, non-homogeneous with variable coefficients.

$$\boxed{r_{l-1} = s_l r_l + (k_l - s_l)}$$

Note $r_L = 1$.
compute $r_0 = ?$

$$\left\{ r_{l-1} \prod_{i=1}^{l-1} s_i = s_l \cdot r_l \prod_{i=1}^{l+1} s_i + (k_l - s_l) \prod_{i=1}^{l-1} s_i \right\}$$

$$= r_l \prod_{i=1}^{l} s_i + k_l \prod_{i=1}^{l-1} s_i - \prod_{i=1}^{l} s_i$$

let $A_l = r_l \prod_{i=1}^{l} s_i$, $A_0 = r_0$.

NOTE: $\prod_{i=1}^{0} s_i = 1$ (as 1 is neutral element for multiplication).

$$A_l - A_{l-1} = \prod_{i=1}^{l} s_i - k_l \prod_{i=1}^{l-1} s_i$$

$$\sum_{l=1}^{L} (A_l - A_{l-1}) = \{A_L - A_0\}$$

$$= \sum_{l=1}^{L} \left(\prod_{i=1}^{l} s_i - k_l \prod_{i=1}^{l-1} s_i \right)$$

substitute $A_0 = r_0$ ($\because \prod_{i=1}^{0} s_i = 1$).

$$A_L = r_L \prod_{i=1}^{L} = \prod_{i=1}^{L} s_i \quad (\text{as } r_L = 1).$$

$$r_0 = \prod_{i=1}^{L} s_i + \sum_{l=1}^{L} \left(k_l \prod_{i=1}^{l-1} s_i - \prod_{i=1}^{l} s_i \right)$$

$$= \sum_{l=1}^{L} k_l \prod_{i=1}^{l-1} s_i - \sum_{l=1}^{L} \prod_{i=1}^{l-1} s_i + 1$$

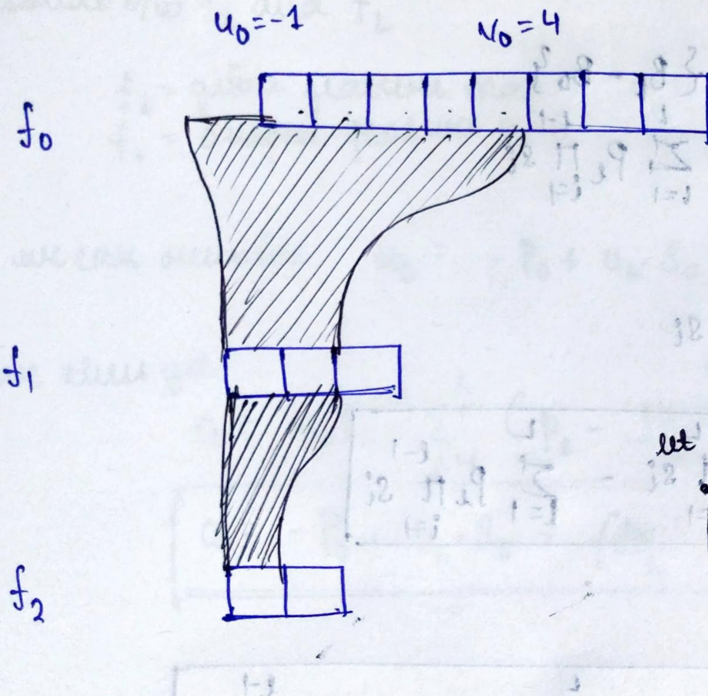
$$\boxed{r_0 = \sum_{l=1}^{L}\left((k_l-1)\prod_{i=1}^{l-1} s_i\right)+1}$$

defined jump (effective stride) that tells how far apart two adjacent units in layer L are,

$$\left[S_L = \prod_{i=1}^{L} s_l\right]$$

Assuming pixel coordinate start at 0, ~~$c_0 = \frac{1}{2}$~~ (c_0).

centre position $c_L(n)$ [input-space coordinate of the centre of the receptive field for unit n in layer L,



let
$$\begin{cases} k_l = 3 \\ p_l = q_l = 1 \\ s_l = 3 \end{cases}$$

let
$$\begin{cases} k_l = 2 \\ q_l = p_l = 0 \\ s_l = 1 \end{cases}$$

recurrence forms:

$$u_{l-1} = -p_l + u_l \cdot s_l$$

$$v_{l-1} = -p_l + v_l \cdot s_l + k_l - 1$$

where u_l, v_l are left-most and right-most coordinates (in f_l) of the region which is used to compute the desired feature in f_L.

$$u_{l-1} = \left(u_l \cdot s_l - p_l \right)$$

similar process:

$$u_{l-1} \prod_{i=1}^{l-1} s_i = u_l \cdot s_l \prod_{i=1}^{l-1} s_i - p_l \prod_{i=1}^{l-1} s_i$$

$$= u_l \prod_{i=1}^{l} s_i - p_l \prod_{i=1}^{l-1} s_i$$

let $B_l = u_l \prod_{i=1}^{l} s_i$

$$B_l - B_{l-1} = p_l \prod_{i=1}^{l-1} s_i$$

$$\sum_{l=1}^{L} \left(B_l - B_{l-1} \right) = \{ B_L - B_0 \}$$

$$= \sum_{i=1}^{L} p_l \prod_{i=1}^{l-1} s_i$$

note $B_0 = u_0$

$B_L = u_L \prod_{i=1}^{L} s_i$

$$\boxed{u_0 = u_L \prod_{i=1}^{L} s_i - \sum_{l=1}^{L} p_l \prod_{i=1}^{l-1} s_i}$$

similarly,

$$\boxed{v_0 = v_L \prod_{i=1}^{L} s_i - \sum_{l=1}^{L} \left(1 + p_l - k_l \right) \prod_{i=1}^{l-1} s_i}$$

centre of receptive field region:

$\cancel{c_0 = u_L}$ $\boxed{c_l = \frac{u_l + v_l}{2}}$ and $\boxed{v_0 = u_0 + r_0 - 1}$

intuitive intuitive

$$c_0 = u_L \prod_{i=1}^{L} s_i - \sum_{l=1}^{L} \left(p_l - \frac{k_l - 1}{2} \right) \prod_{i=1}^{l-1} s_i$$

define effective stride; $S_l = \prod_{i=l+1}^{L} s_i$

effective padding; $P_l = \sum_{m=l+1}^{L} p_m \prod_{i=l+1}^{m-1} s_i$

padding b/w f_l and f_L

stride b/w f_l and f_L

$$\left[\begin{array}{l} S_{l-1} = s_l S_l \\ P_{l-1} = s_l \cdot P_l + p_l \end{array} \right]$$

f_l = given feature map
f_L = output feature map.

we can rewrite $u_0 = -P_0 + u_L \cdot S_0$

we thus get,

$$c_0 = u_L S_0 - \sum_{l=1}^{L} \left(p_l - \frac{k_l - 1}{2} \right) \prod_{i=1}^{l-1} s_i.$$

$$\boxed{c_0 = -P_0 + u_L \cdot S_0 + \left(\frac{r_0 - 1}{2} \right)}$$