

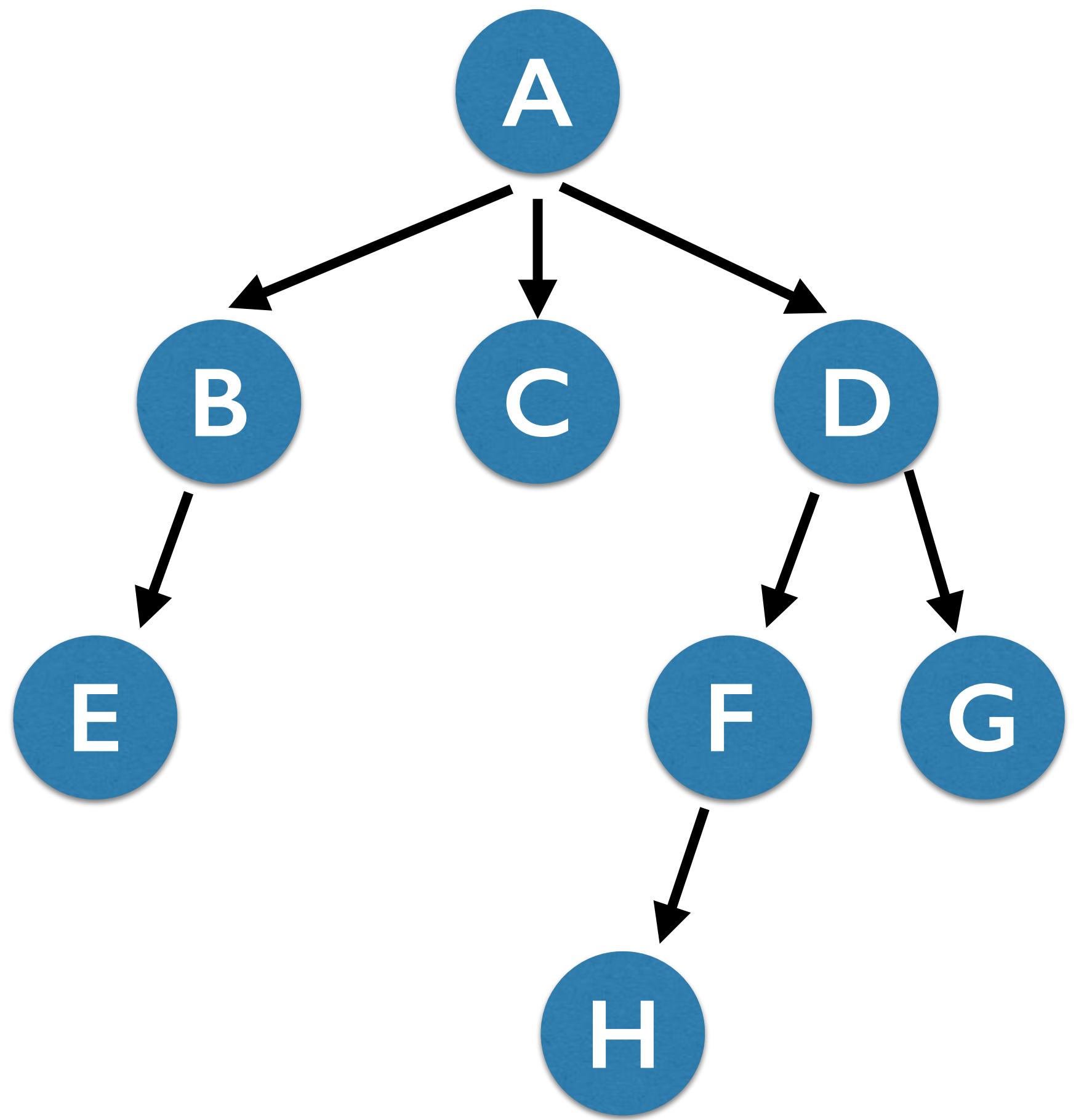


# Trees





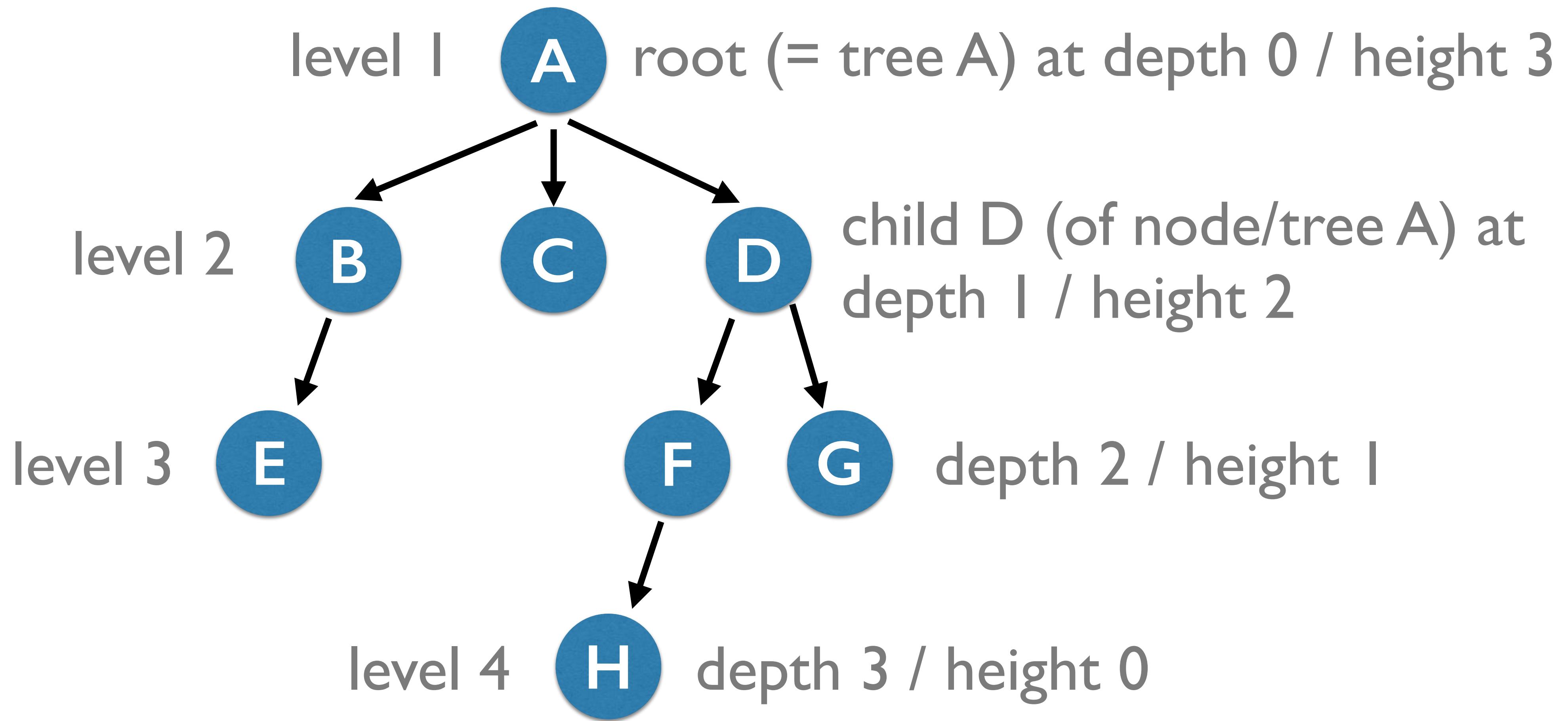
# The Tree ADT



- Nodes contain value(s)
- A primary "root" node
- Children are subtrees (recursive!)
- No duplicated children (cycles); trees can *branch* but never *converge*
- Final nodes called "leaves"
- Height of tree = longest path to leaf
- Level = 1 + number of jumps to root
- Depth = inverse of height

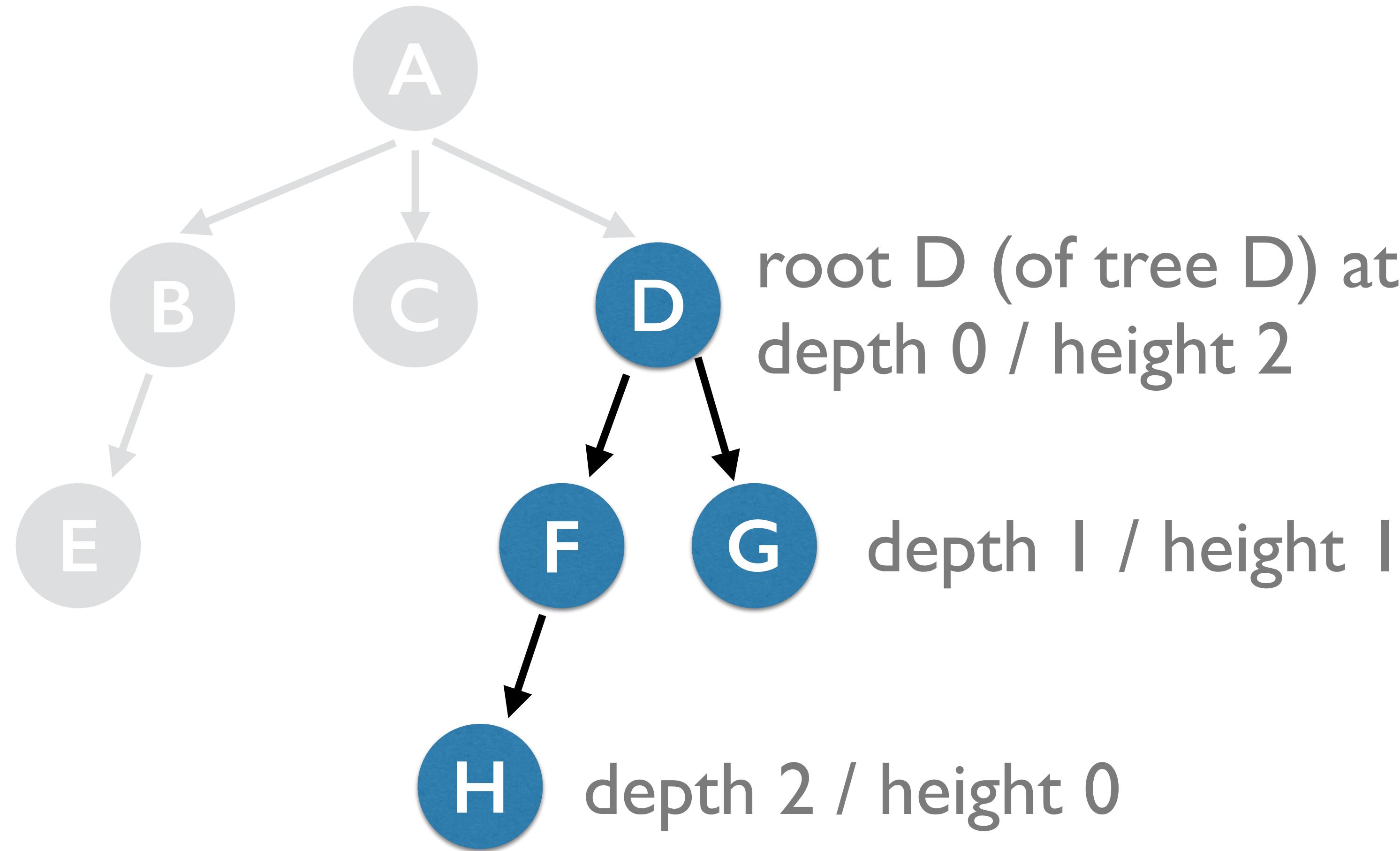


# The Tree ADT





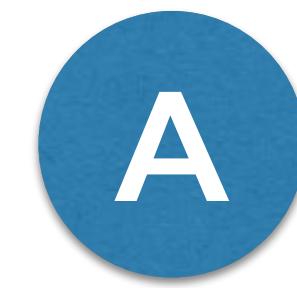
**Every node is the root of a tree.  
You might even say a node *represents* a tree.**



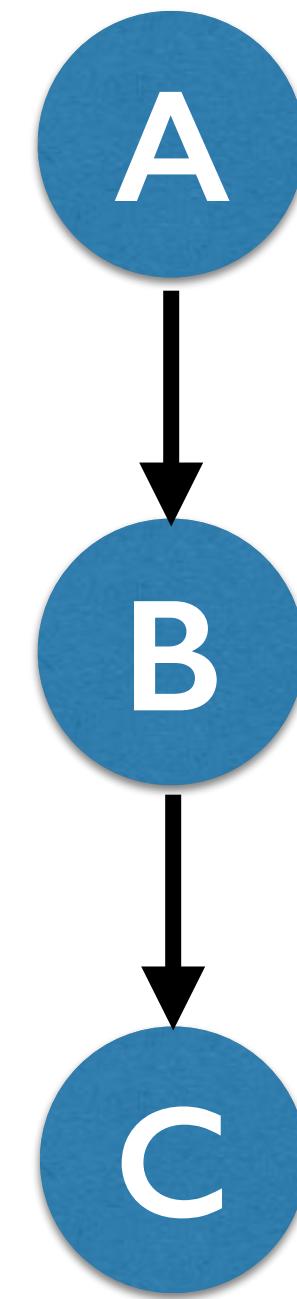


# "Degenerate" trees are still trees

A tree of one node

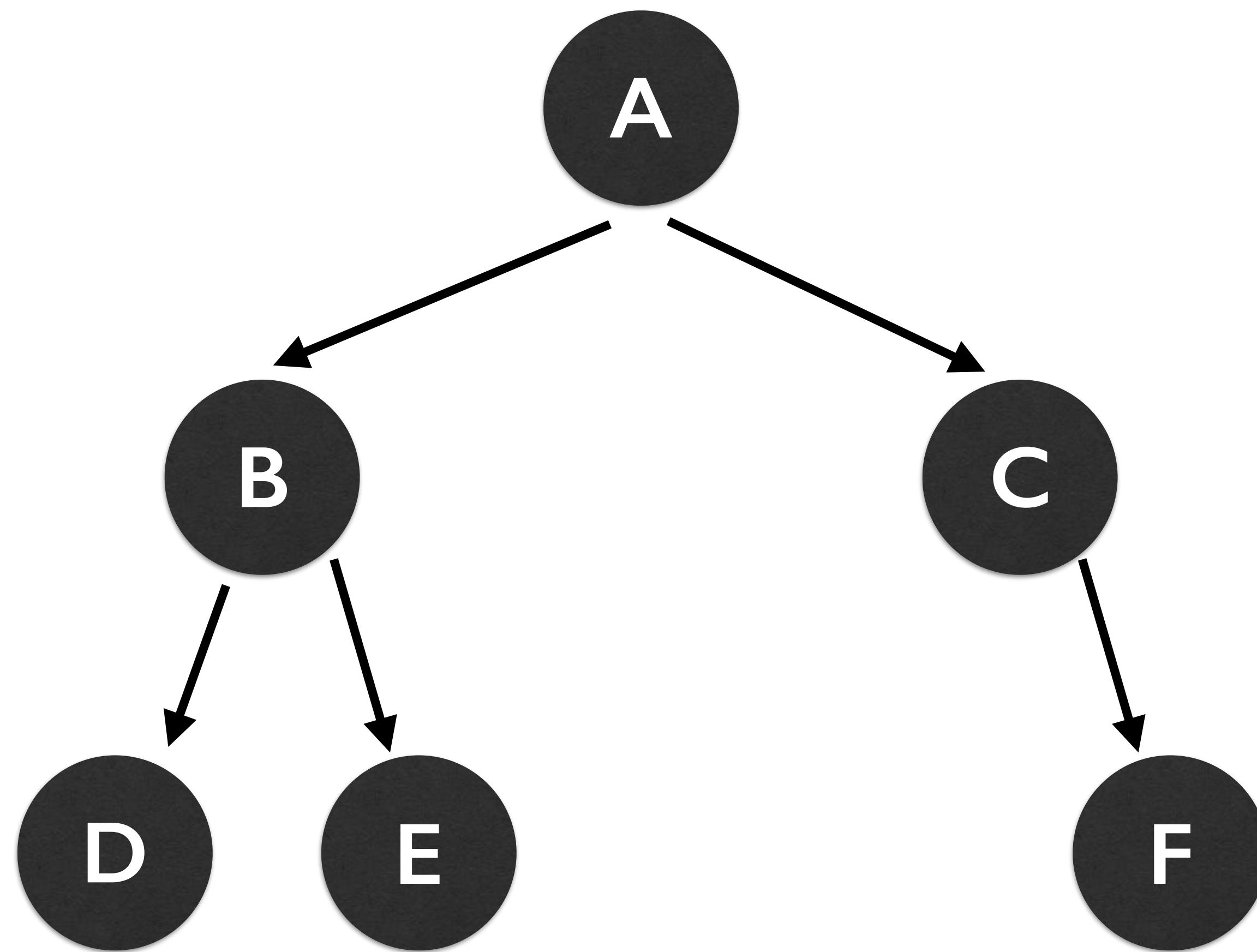


A tree of three nodes

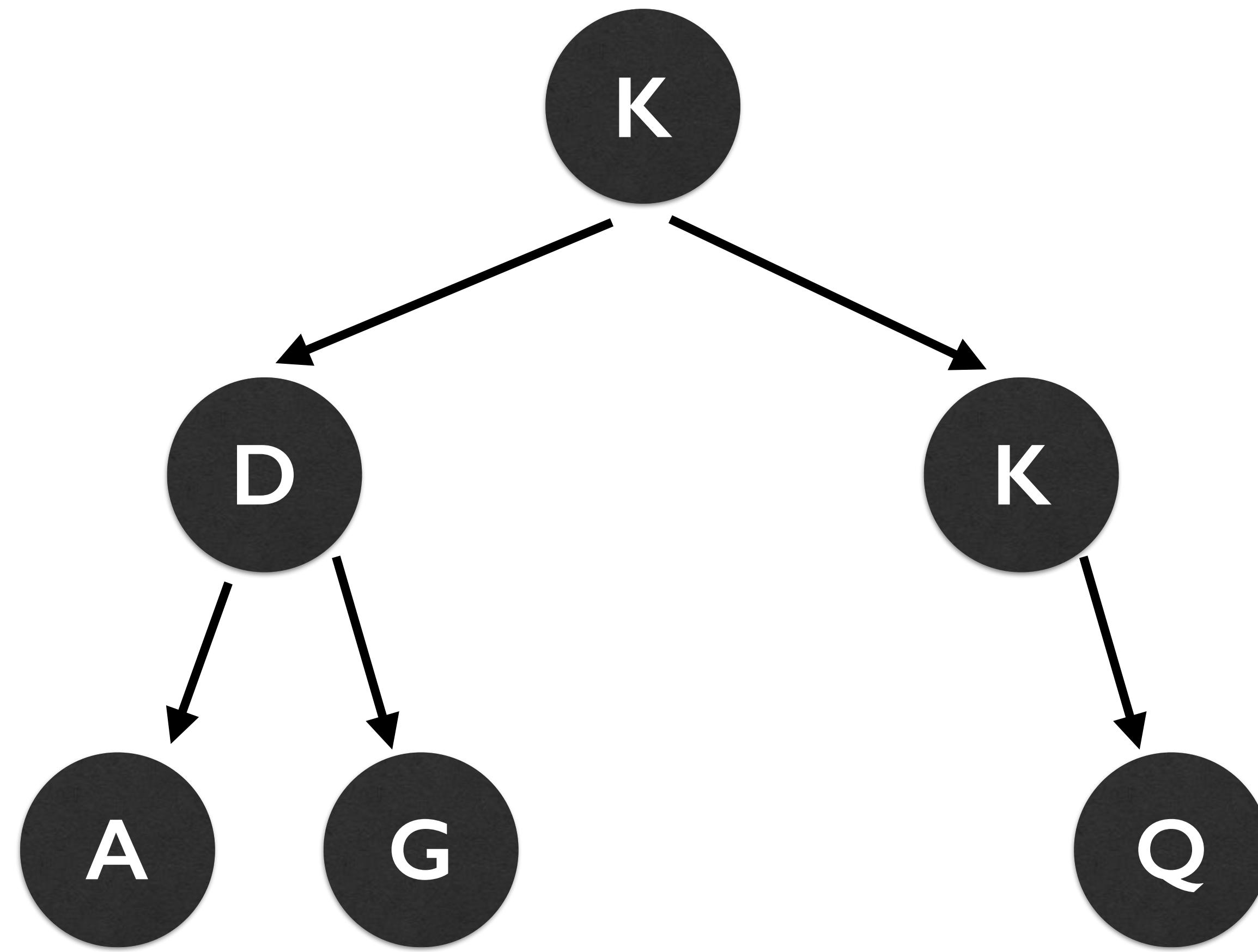




# Binary Tree

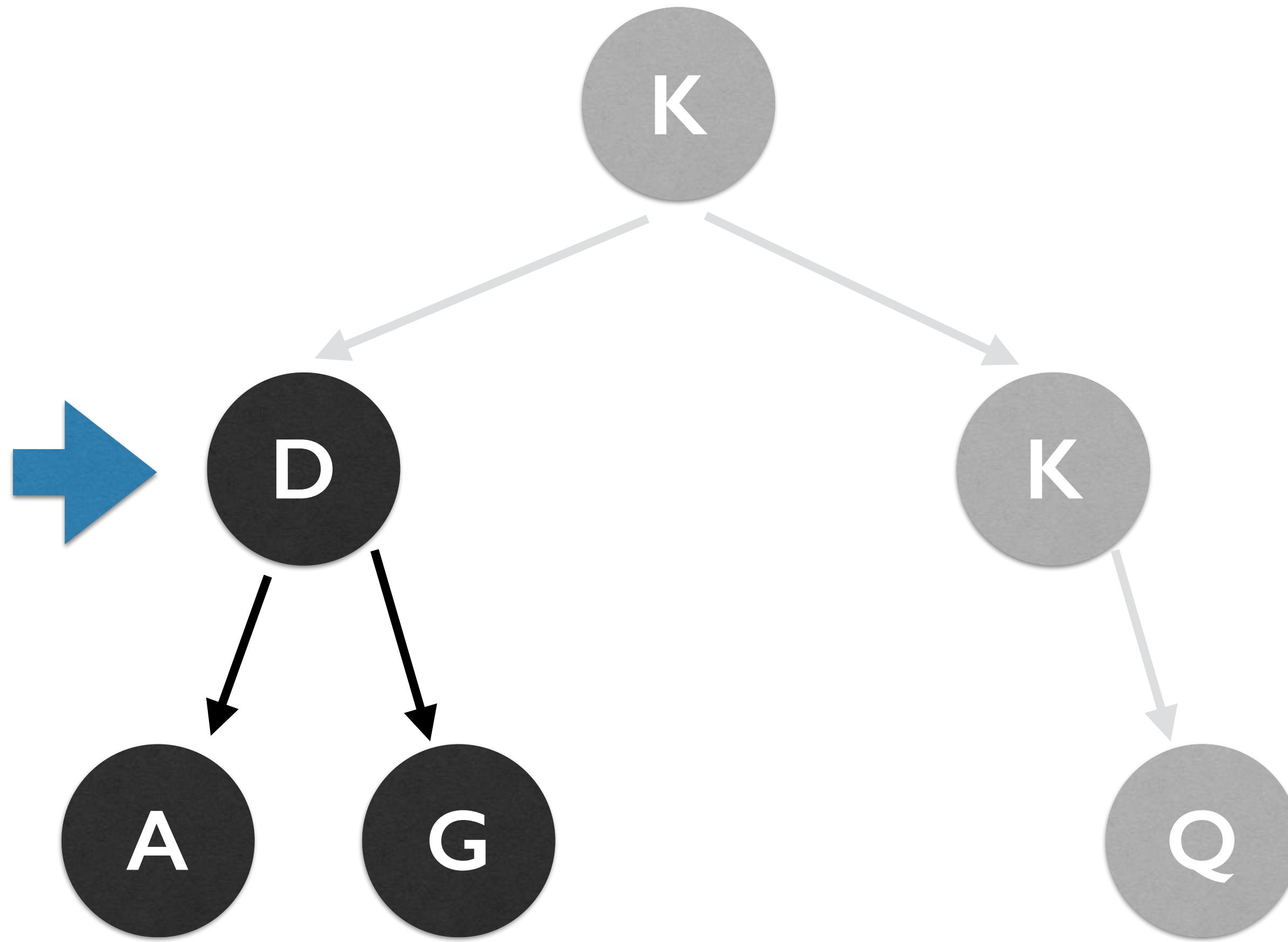


# Binary Search Tree

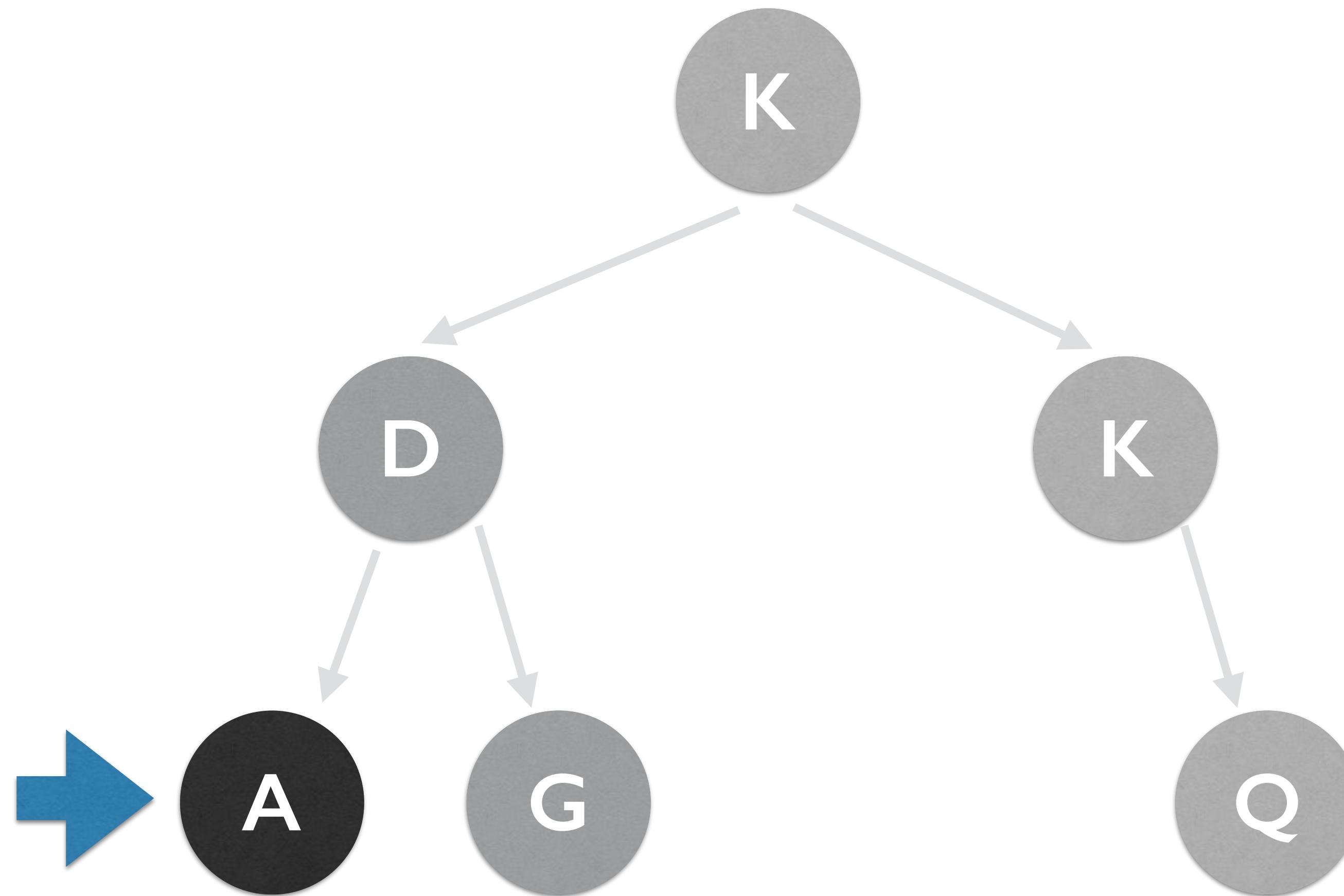




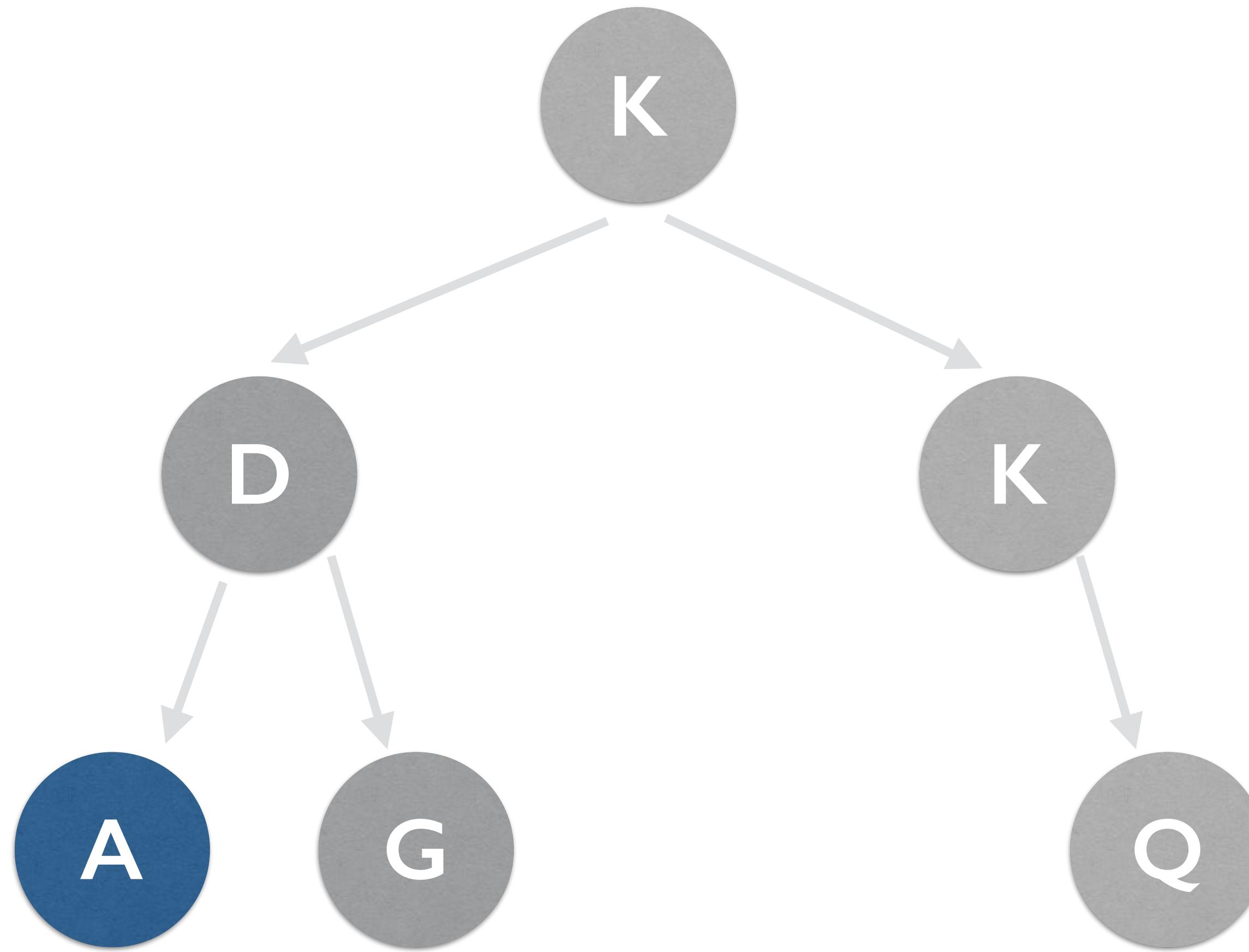
# Binary Search Tree: Min Value?



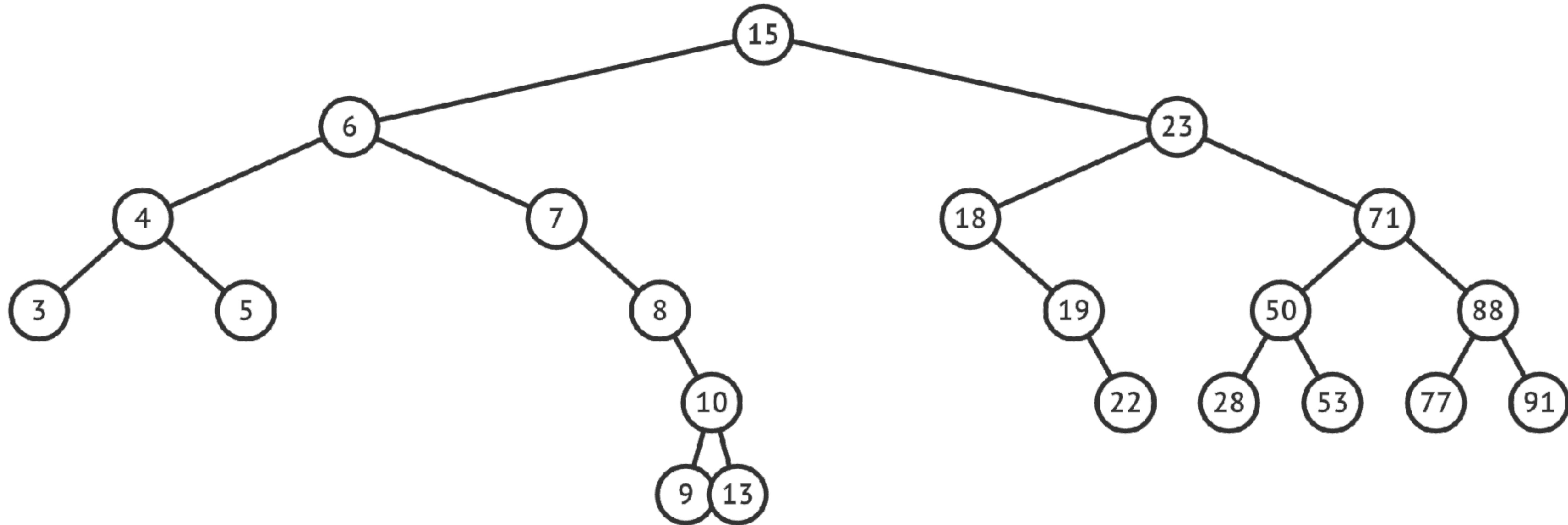
# Binary Search Tree: Min Value?



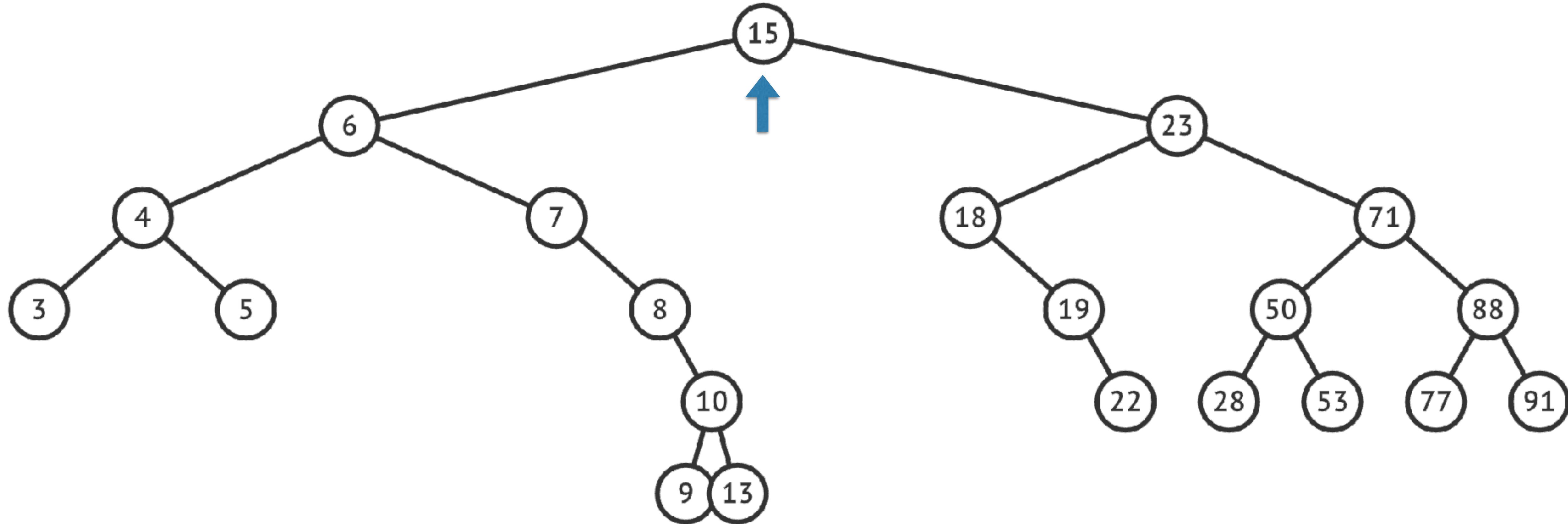
# Binary Search Tree: Min Value?



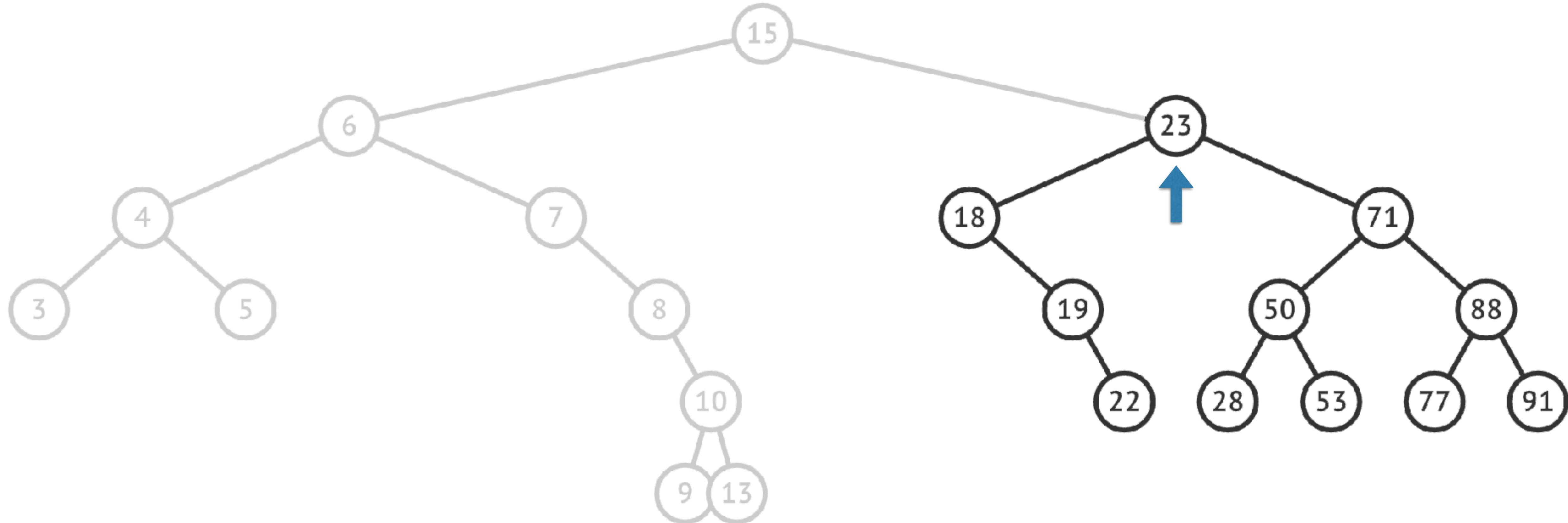
# Does this BST contain the value 28?



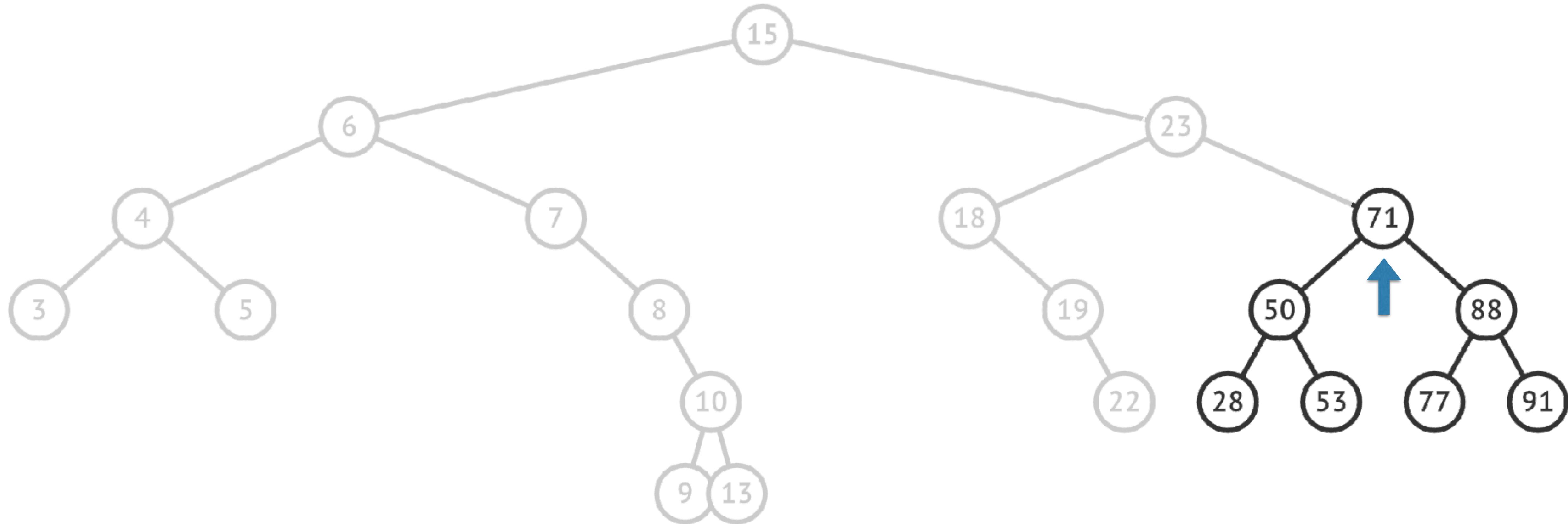
# Does this BST contain the value 28?



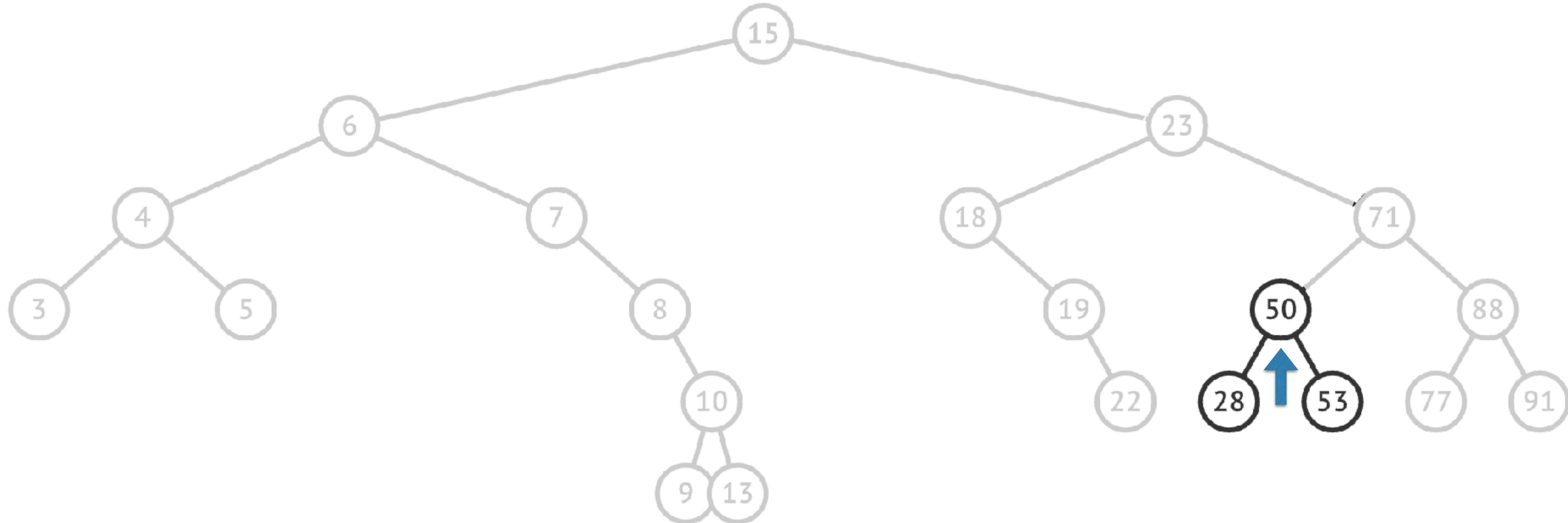
# Does this BST contain the value 28?



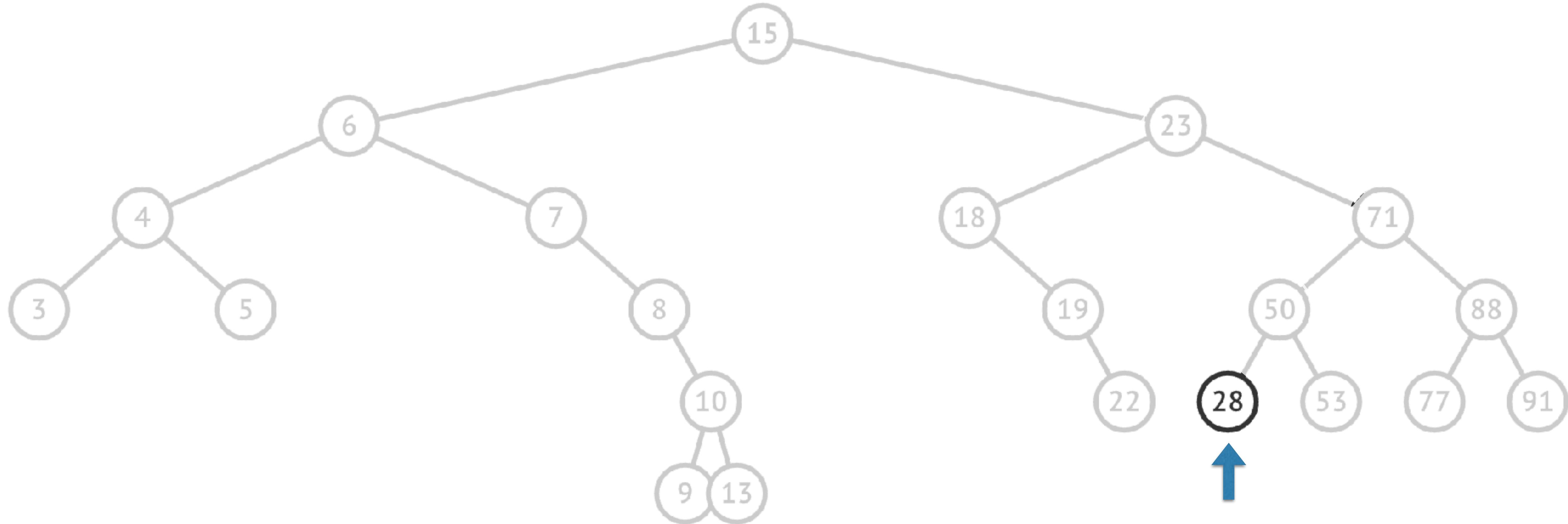
# Does this BST contain the value 28?



# Does this BST contain the value 28?



# Does this BST contain the value 28?



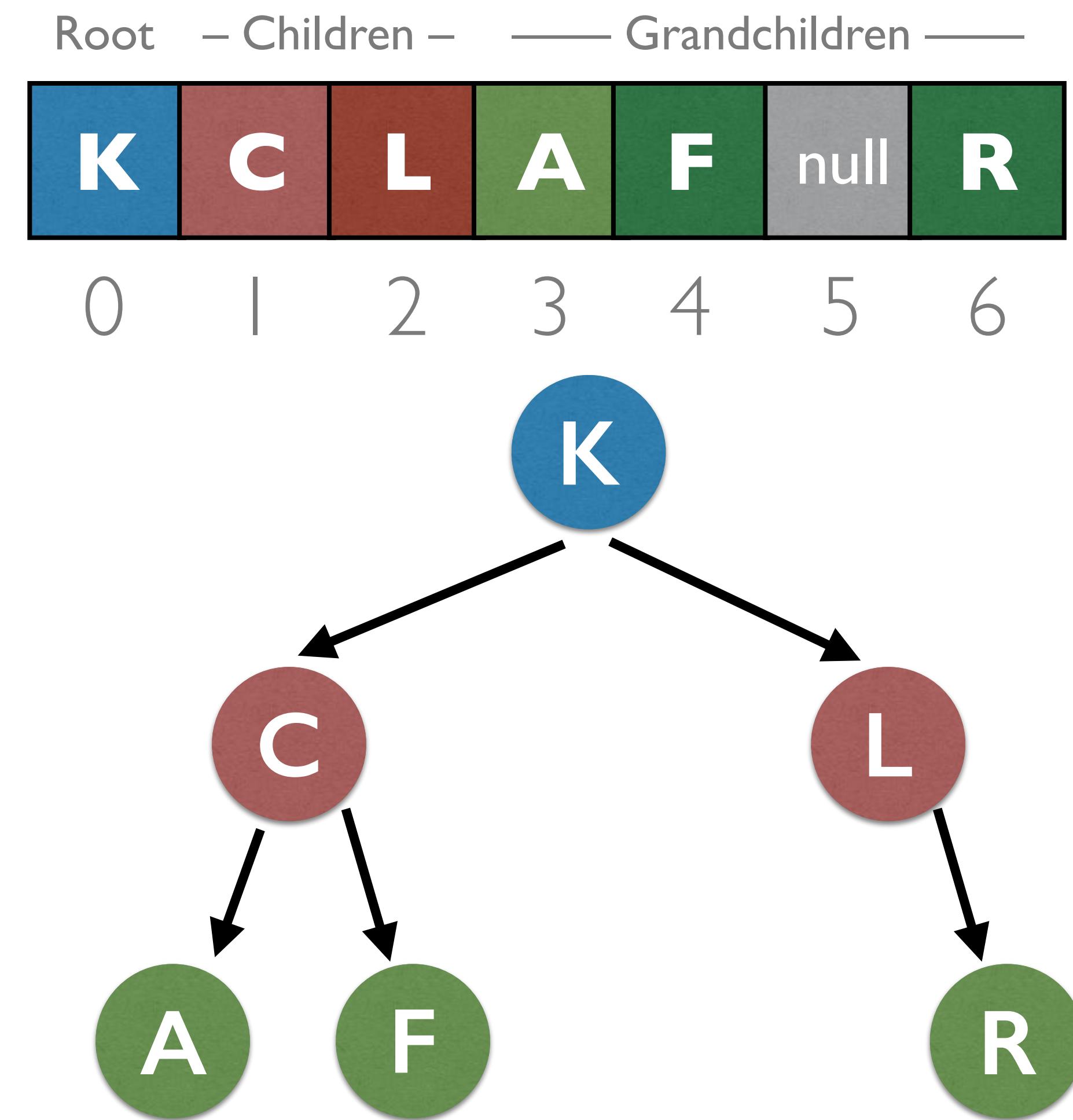
# BST ADT

- Root node satisfies ordering principle
  - Left descendants < root value  $\leq$  right descendants
- Both children are BSTs (recursive definition)
- Operations
  - **Insert** new values, respecting the ordering principle
  - **Find** existing values (takes advantage of ordering)
  - **Delete** values (tricky, skipped in workshop)

# How to implement this ADT?

# ...Maybe an array (seriously)?

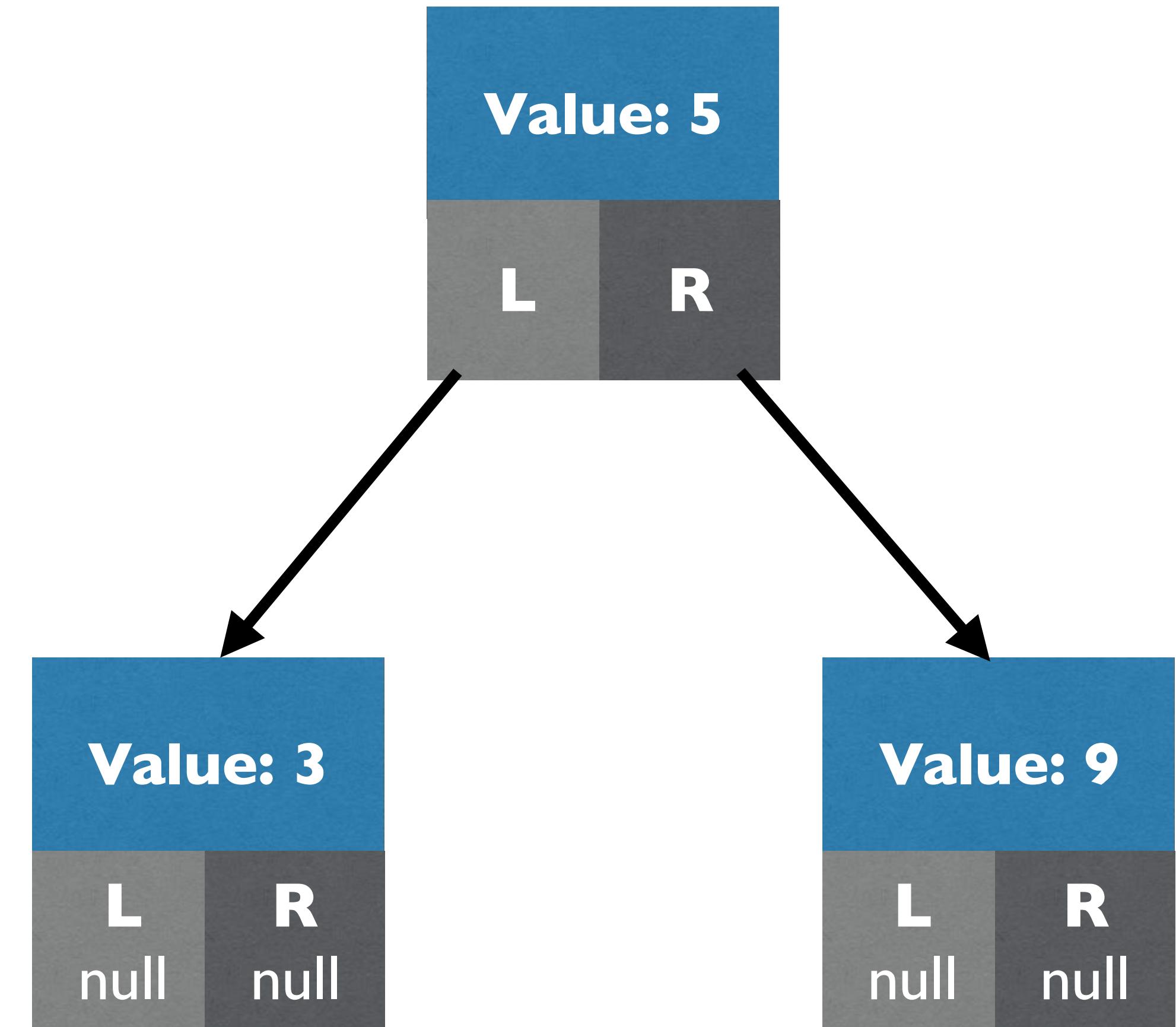
- The Tree ADT, with its talk of "nodes" and "references," seems so obviously to describe a data *structure* that it is perhaps confusing to tell the two apart.
- In fact, a tree can be stored in a few different ways. For example, if you knew your tree nodes always had at most two children, you could store the tree in an array!



# The Linked Tree Data Structure

- However, the concept of *nodes with values and children* maps so well to the concrete case of *memory structs with fields and references* that the most common DS used to implement the Tree ADT is...

...the Linked Tree DS.





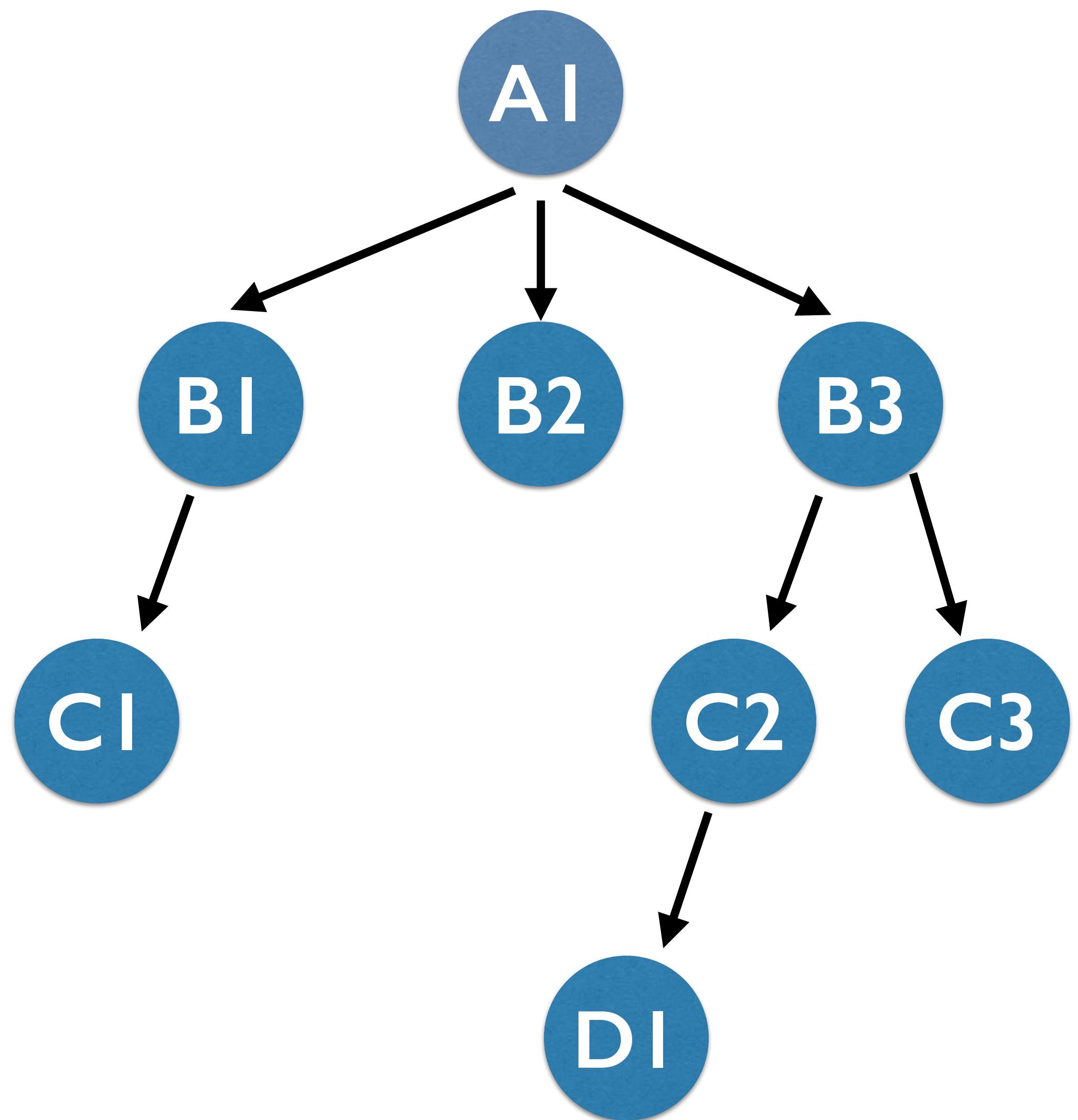
# Tree traversal

# Traversal: visiting every node

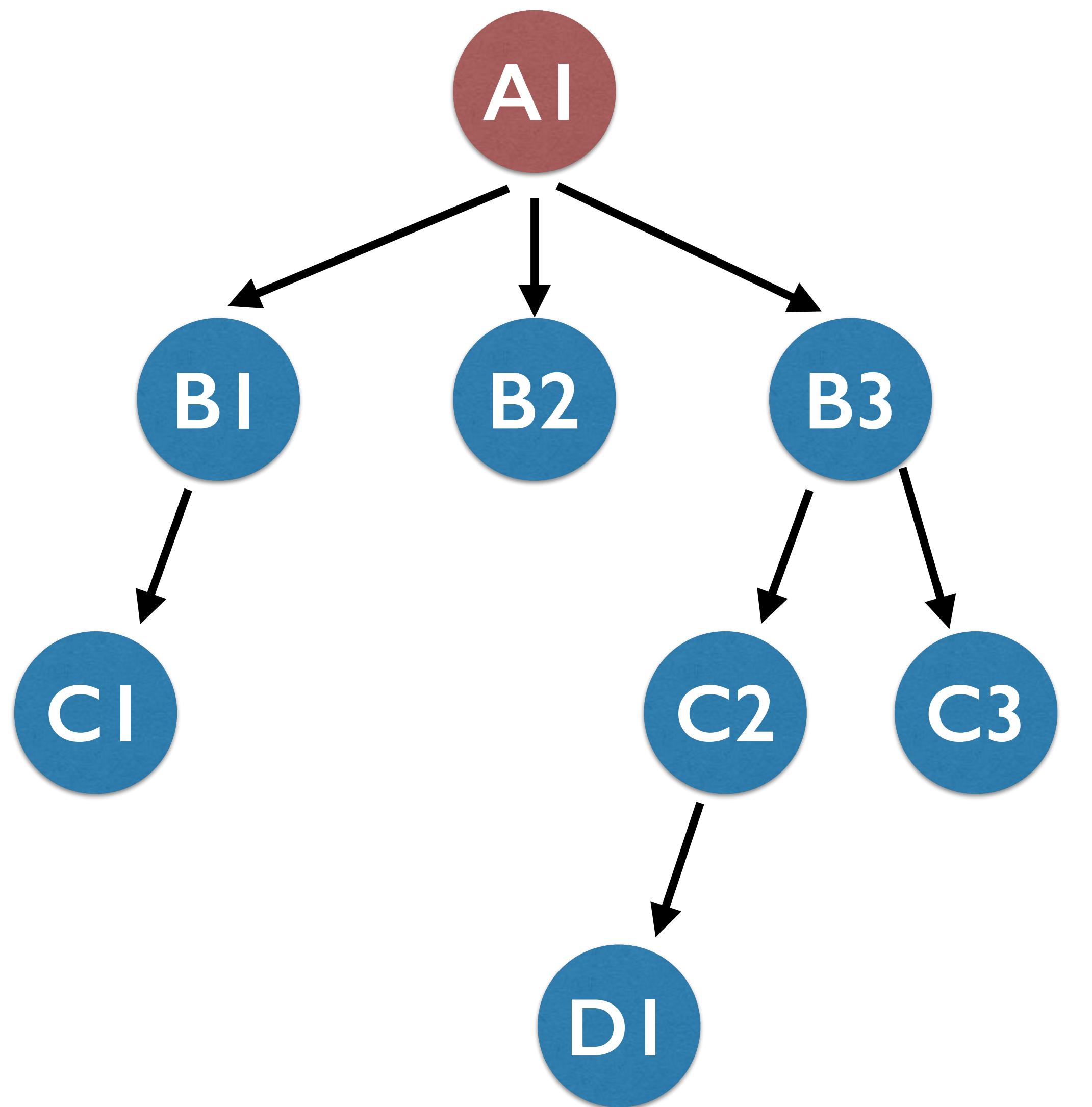
- Breadth-first search (level by level)
- Depth-first search (branch by branch)
  - Pre-order: process **root node**, process **left subtree**, process **right subtree**
  - In-order: process **left subtree**, process **root node**, process **right subtree**
  - Post-order: process **left subtree**, process **right subtree**, process **root node**

# Breadth-First

# BFS

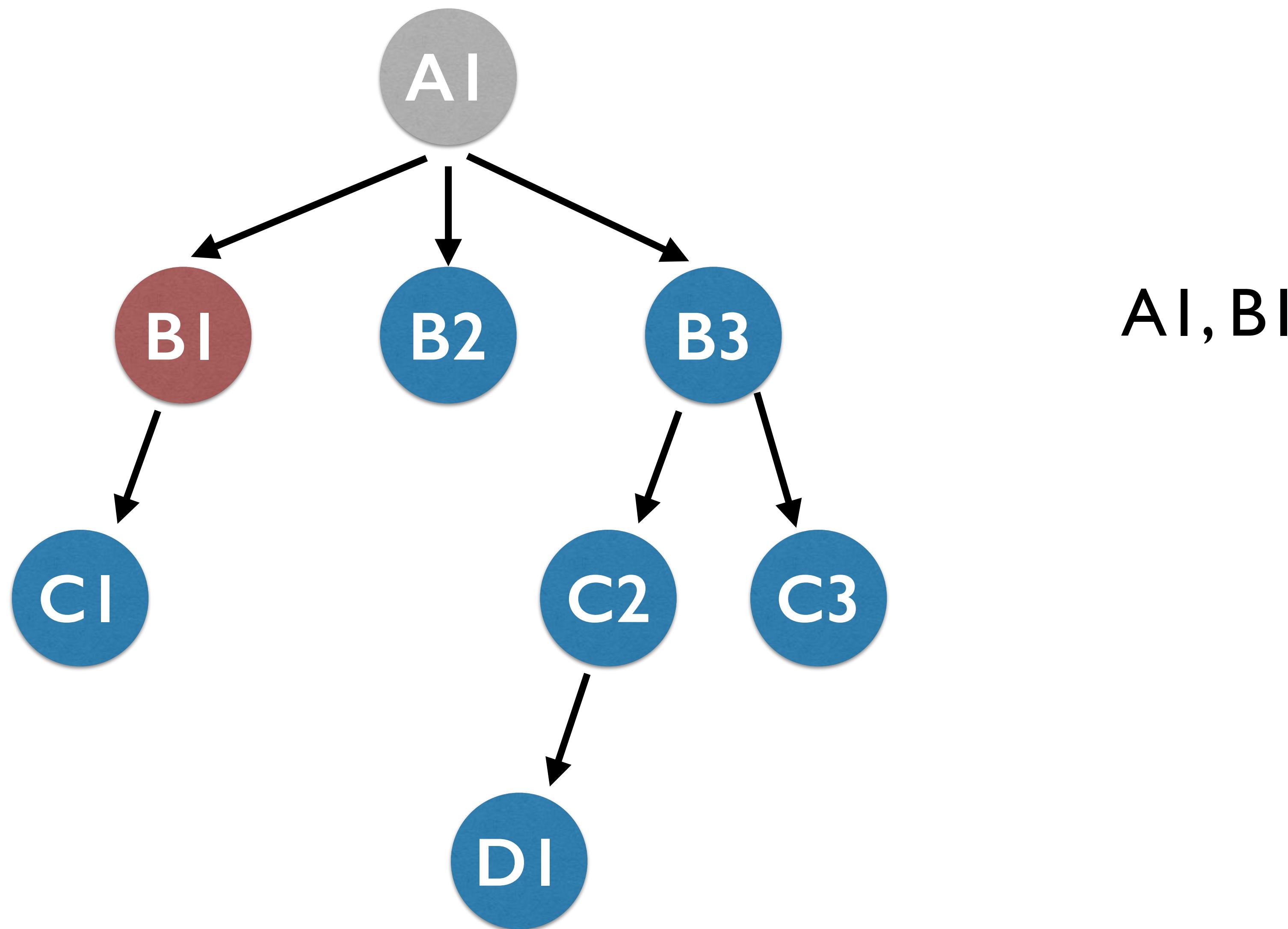


BFS



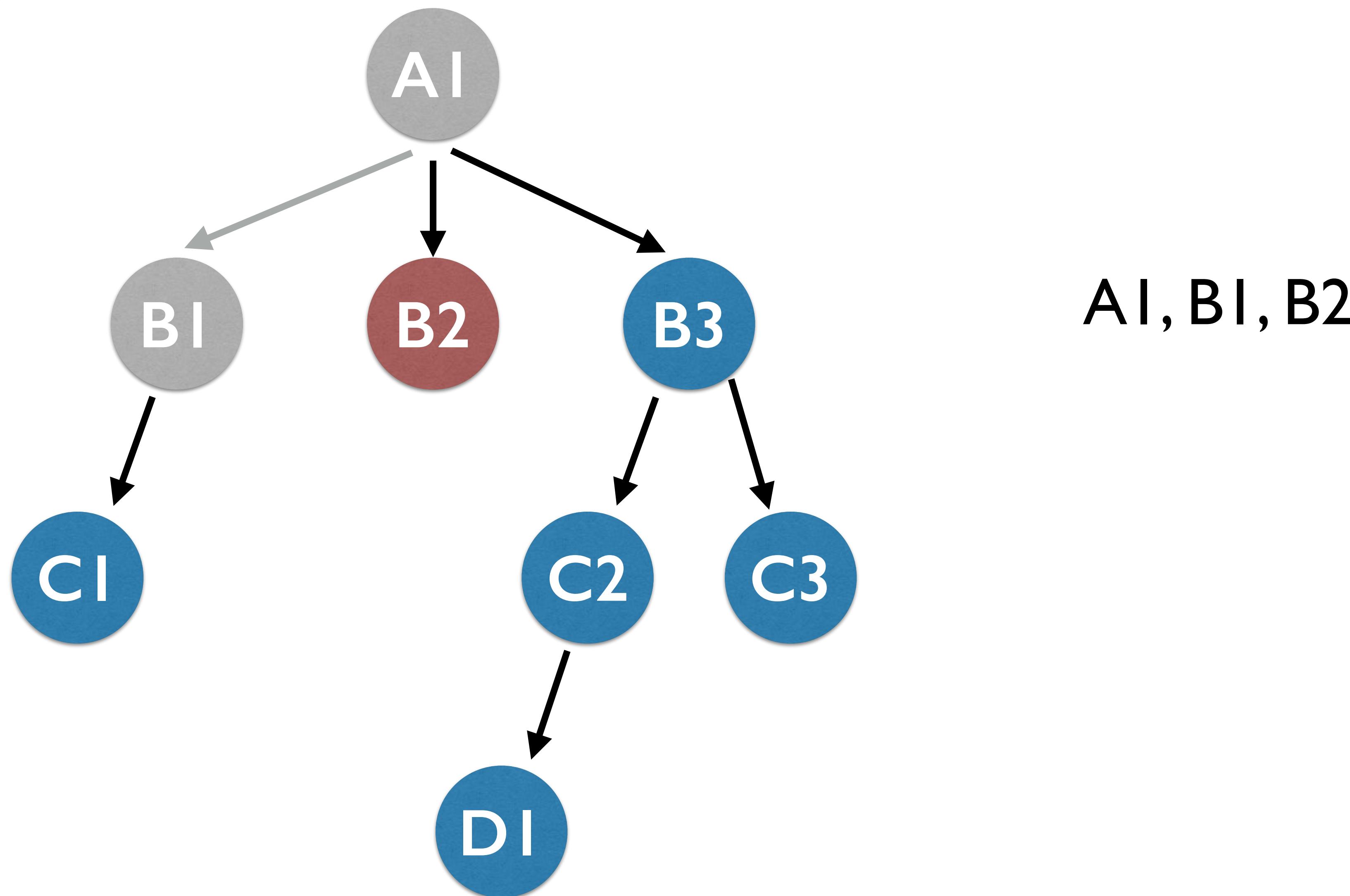
AI

BFS



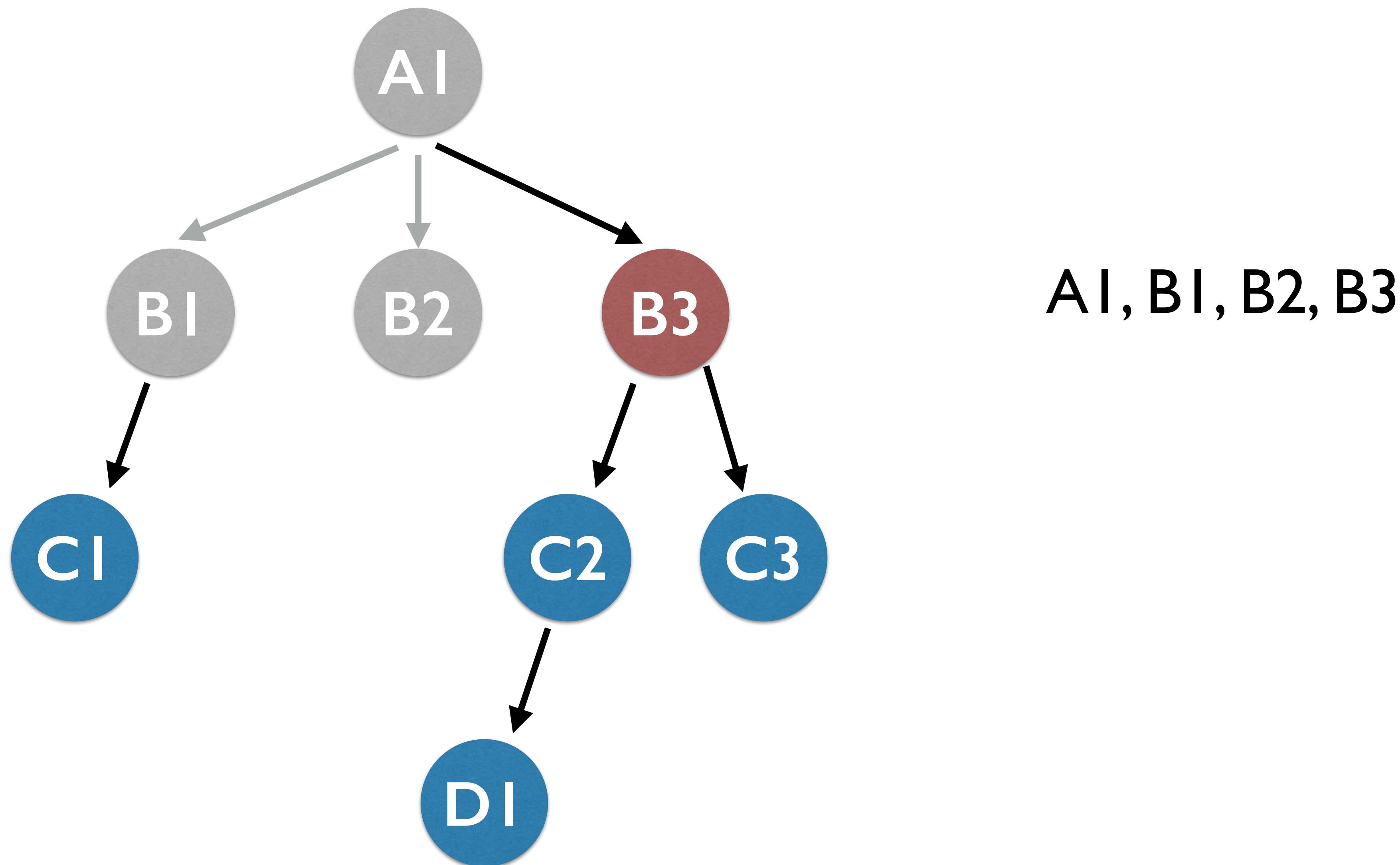
A1, B1

# BFS

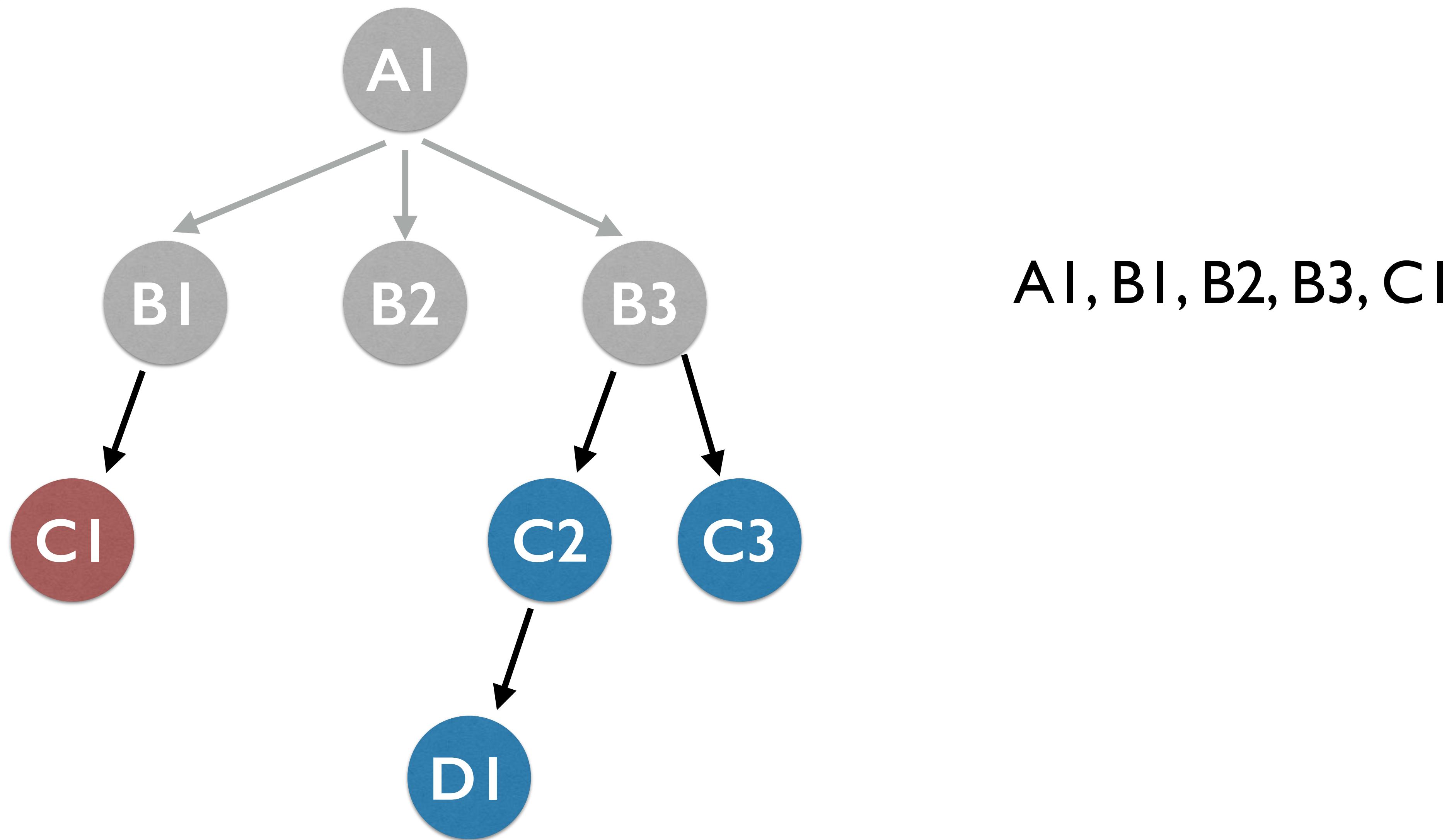


A1, B1, B2

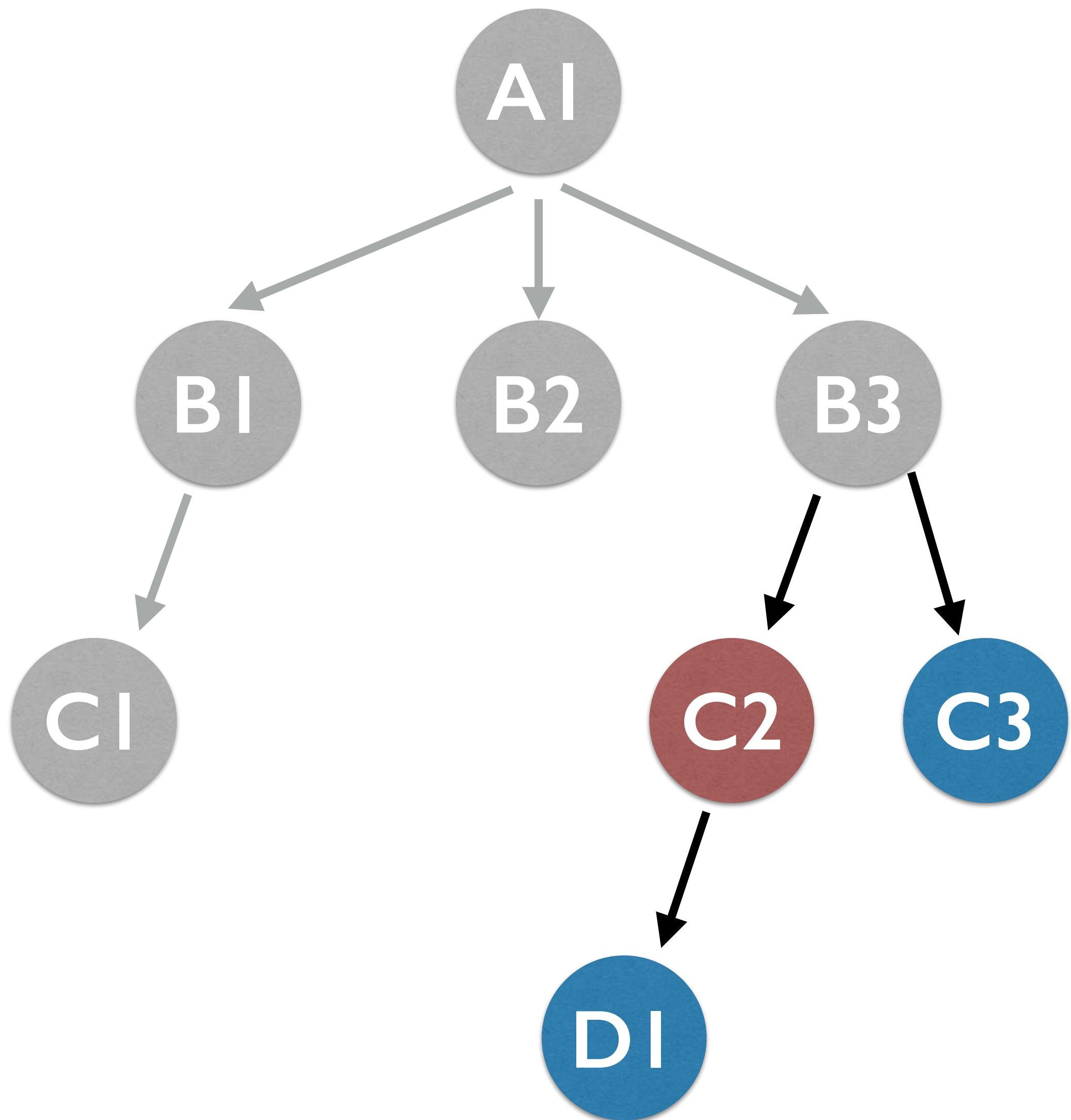
# BFS



# BFS

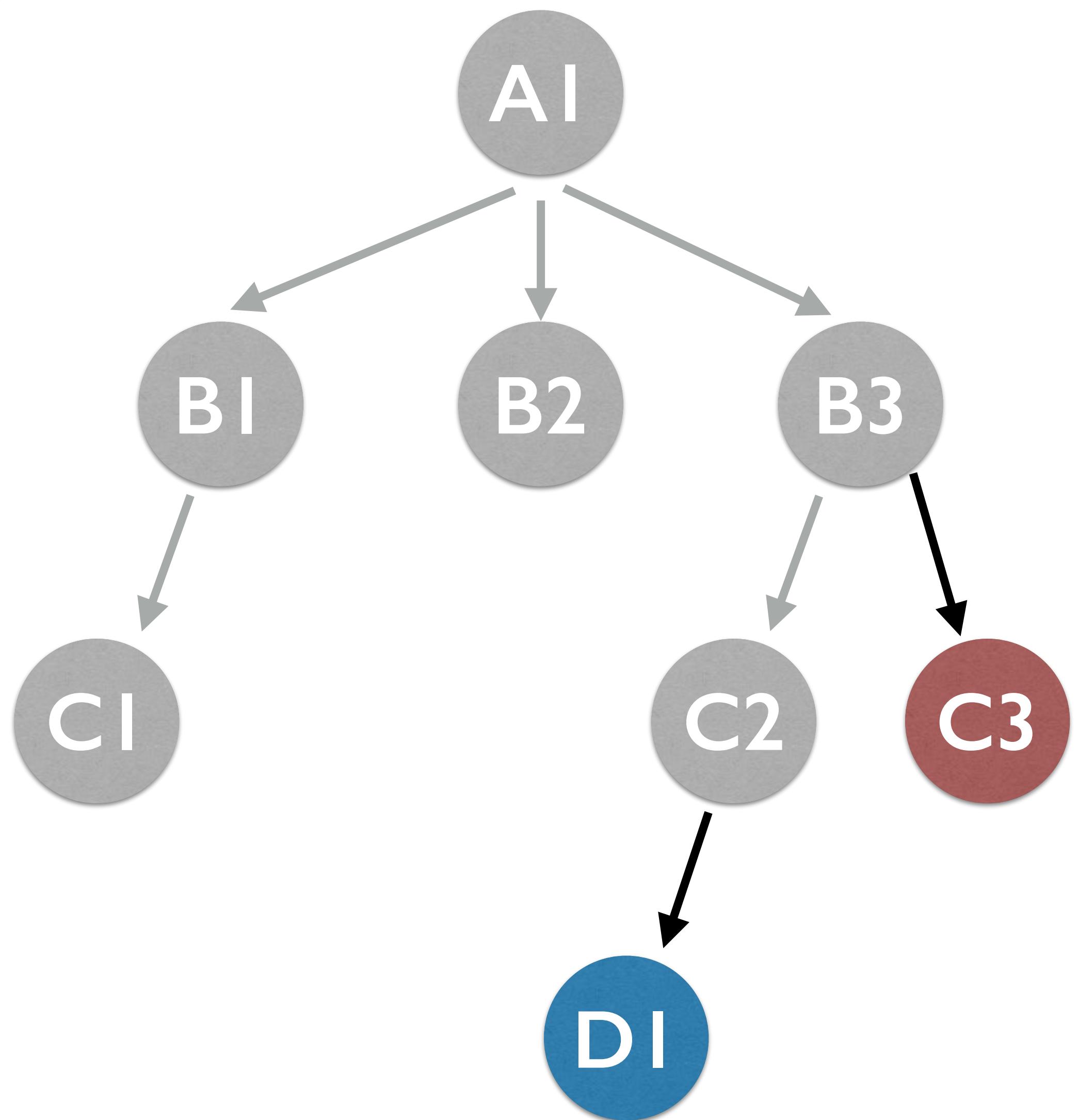


BFS



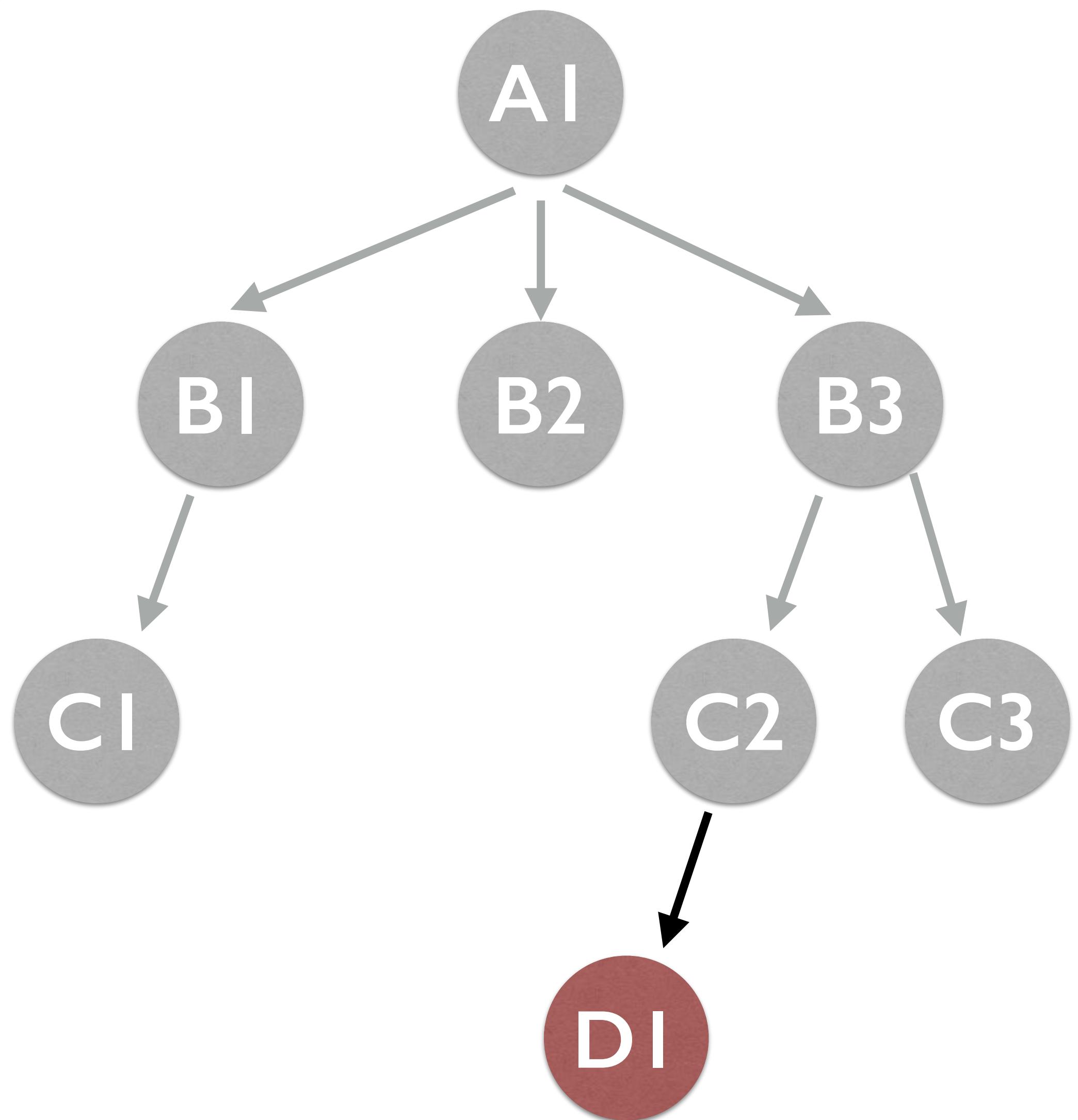
A1, B1, B2, B3, C1, C2

BFS



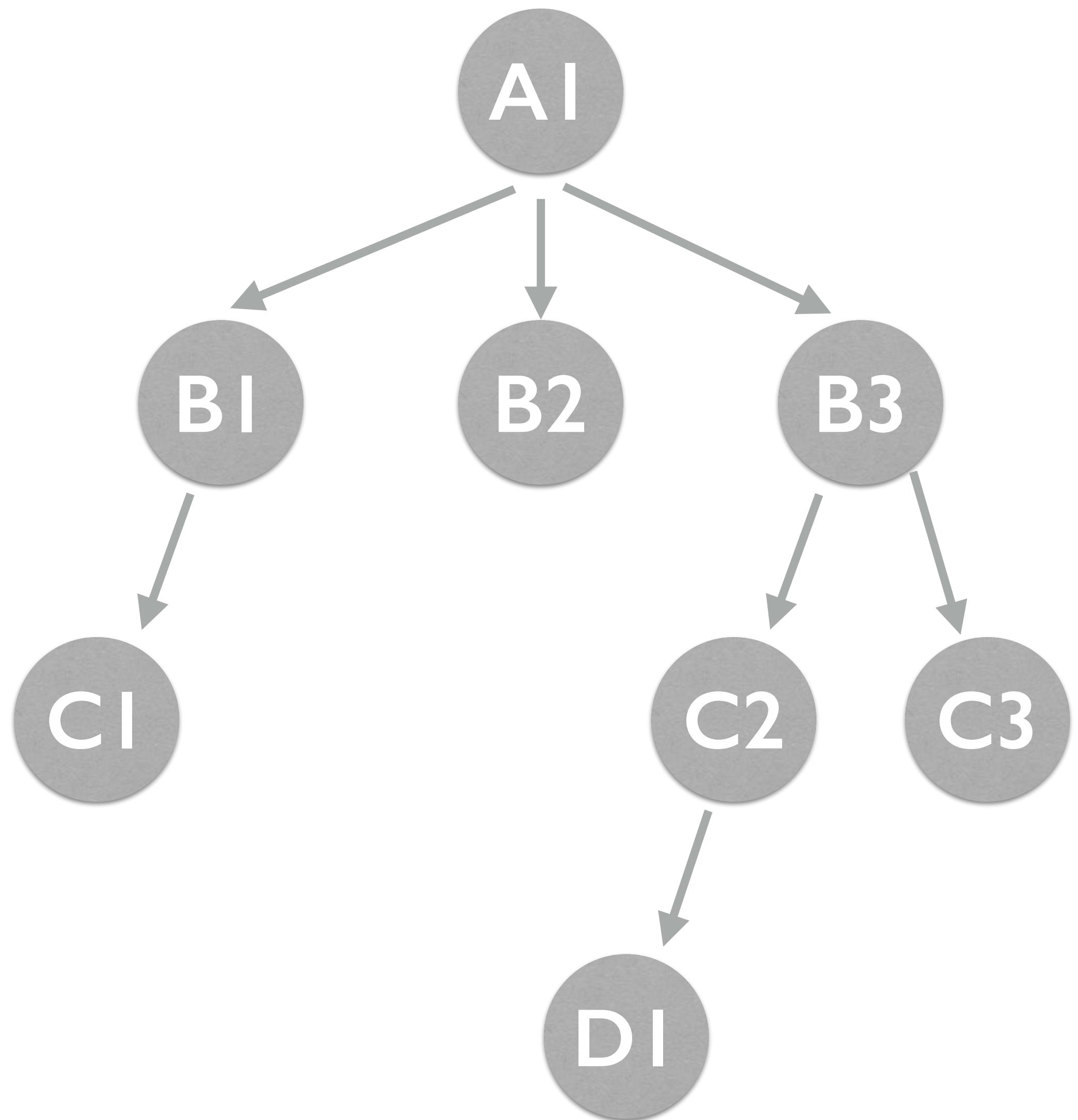
A1, B1, B2, B3, C1, C2, C3

BFS



A1, B1, B2, B3, C1, C2, C3, D1

# BFS

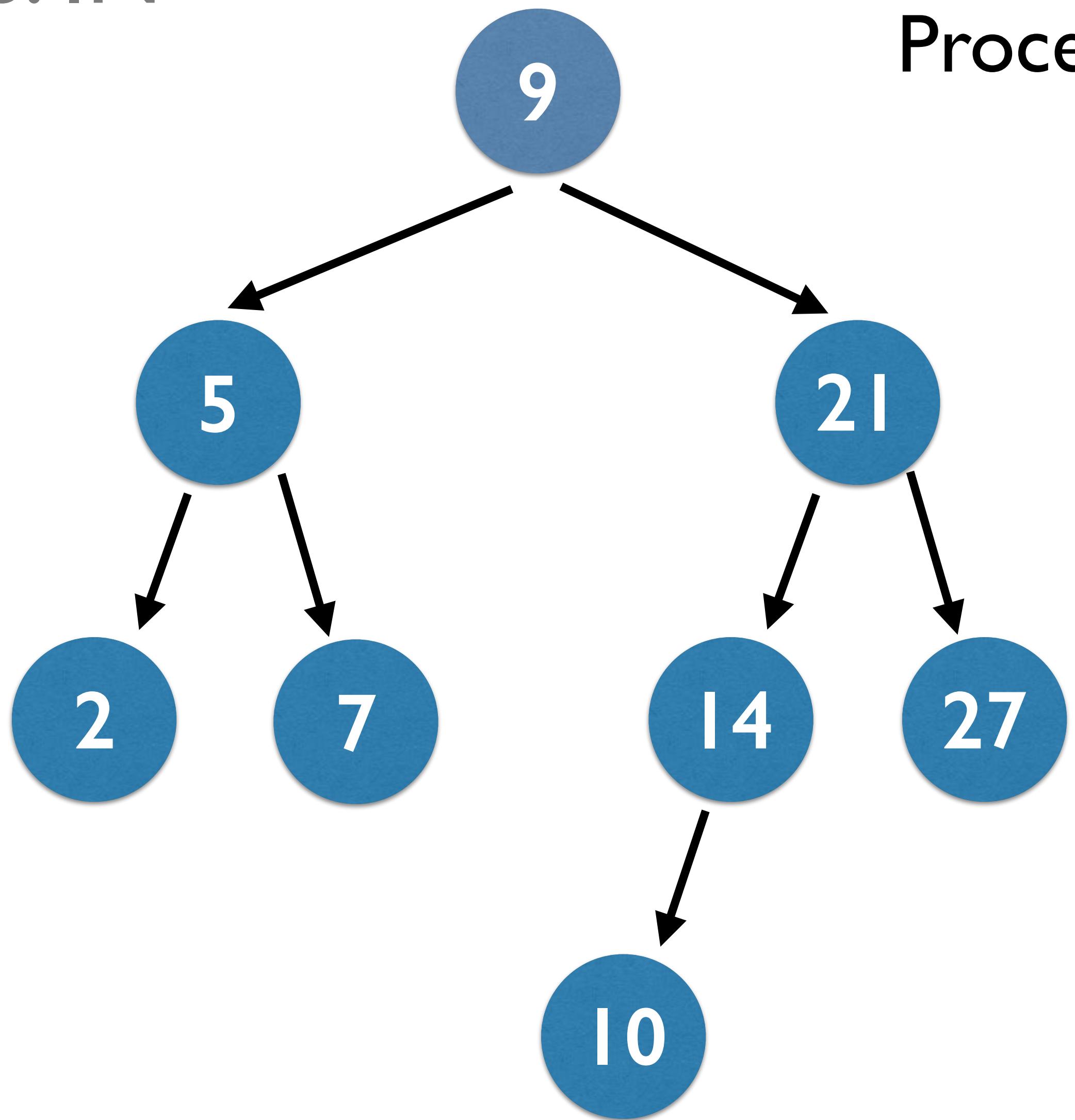


A1, B1, B2, B3, C1, C2, C3, D1

- Looks easy... but with a tree data structure it actually requires an elegant trick to do correctly.
- No full solution here, but a BIG hint: you'll need a queue!

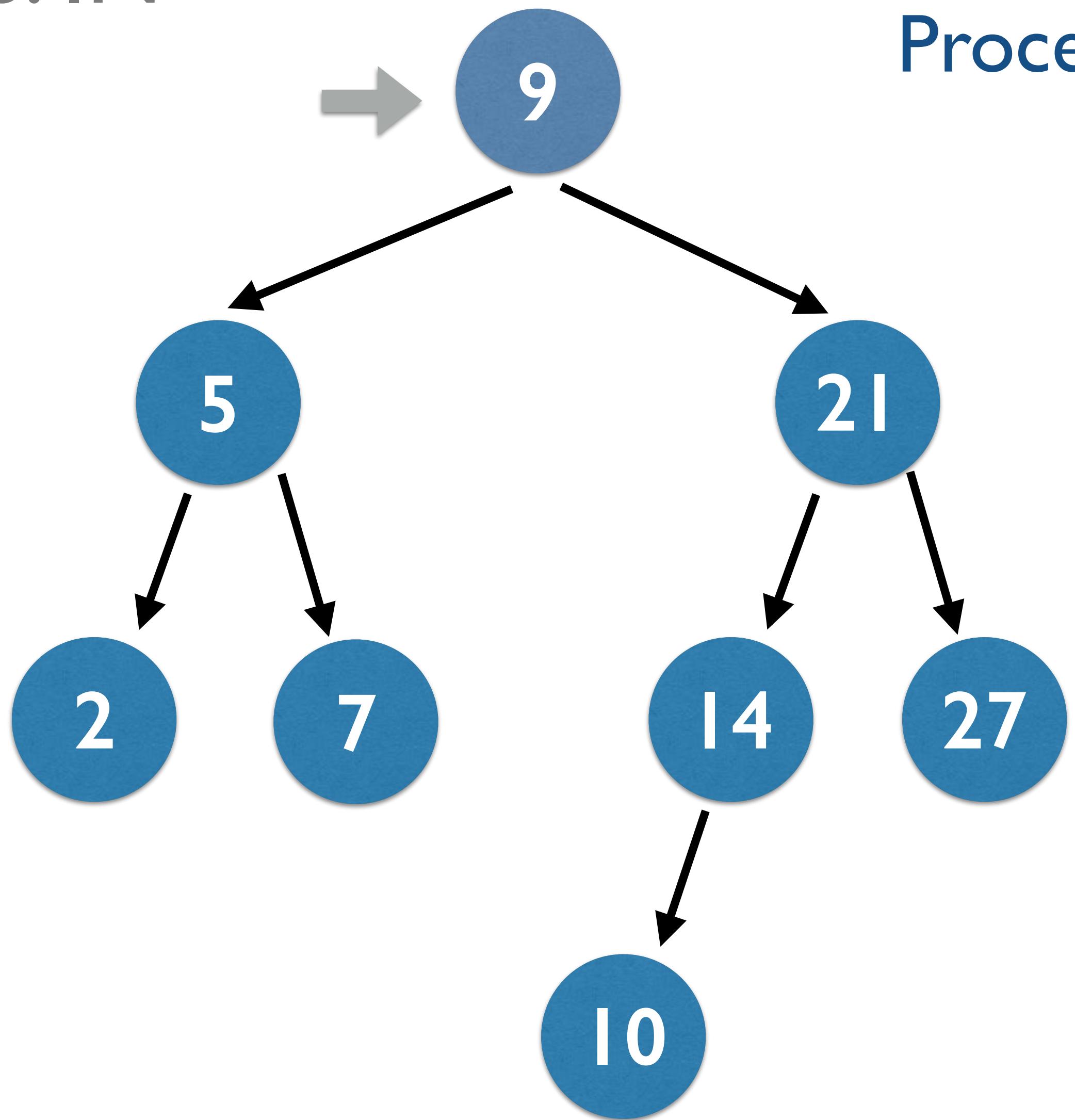
# Depth First: In-Order

DFS: IN



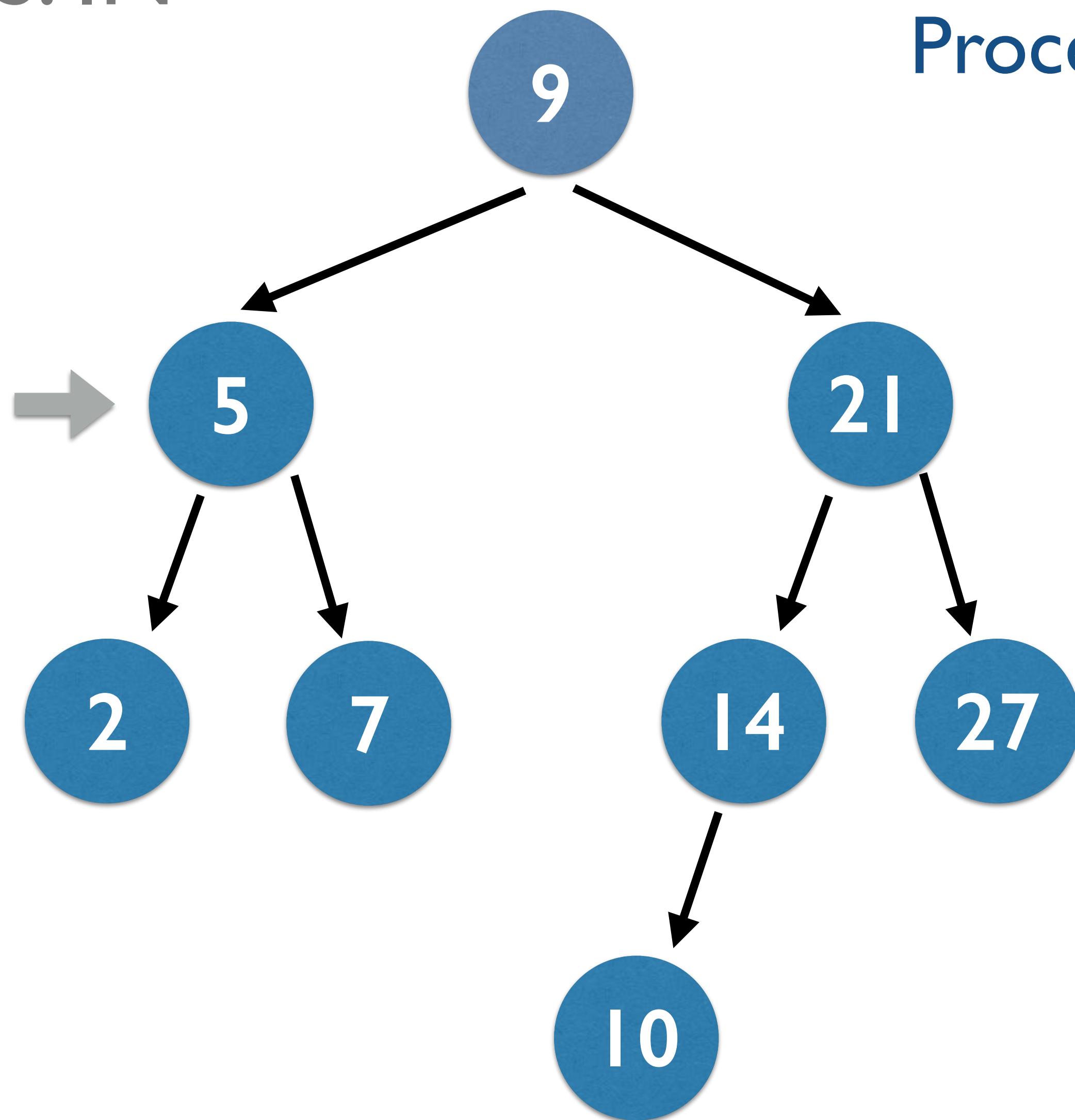
Process left · Process root · Process right

DFS: IN



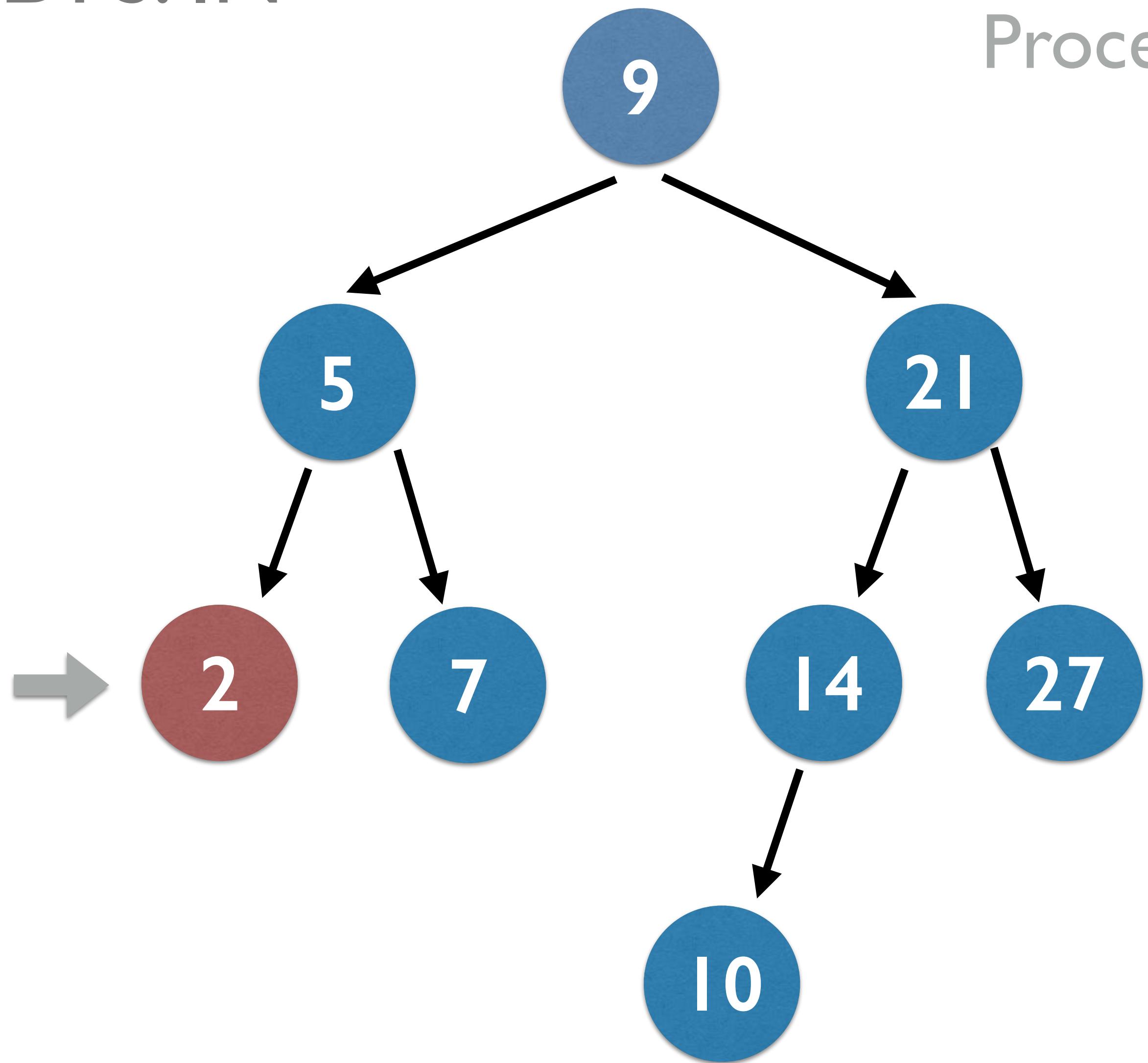
Process left · Process root · Process right

DFS: IN



Process left · Process root · Process right

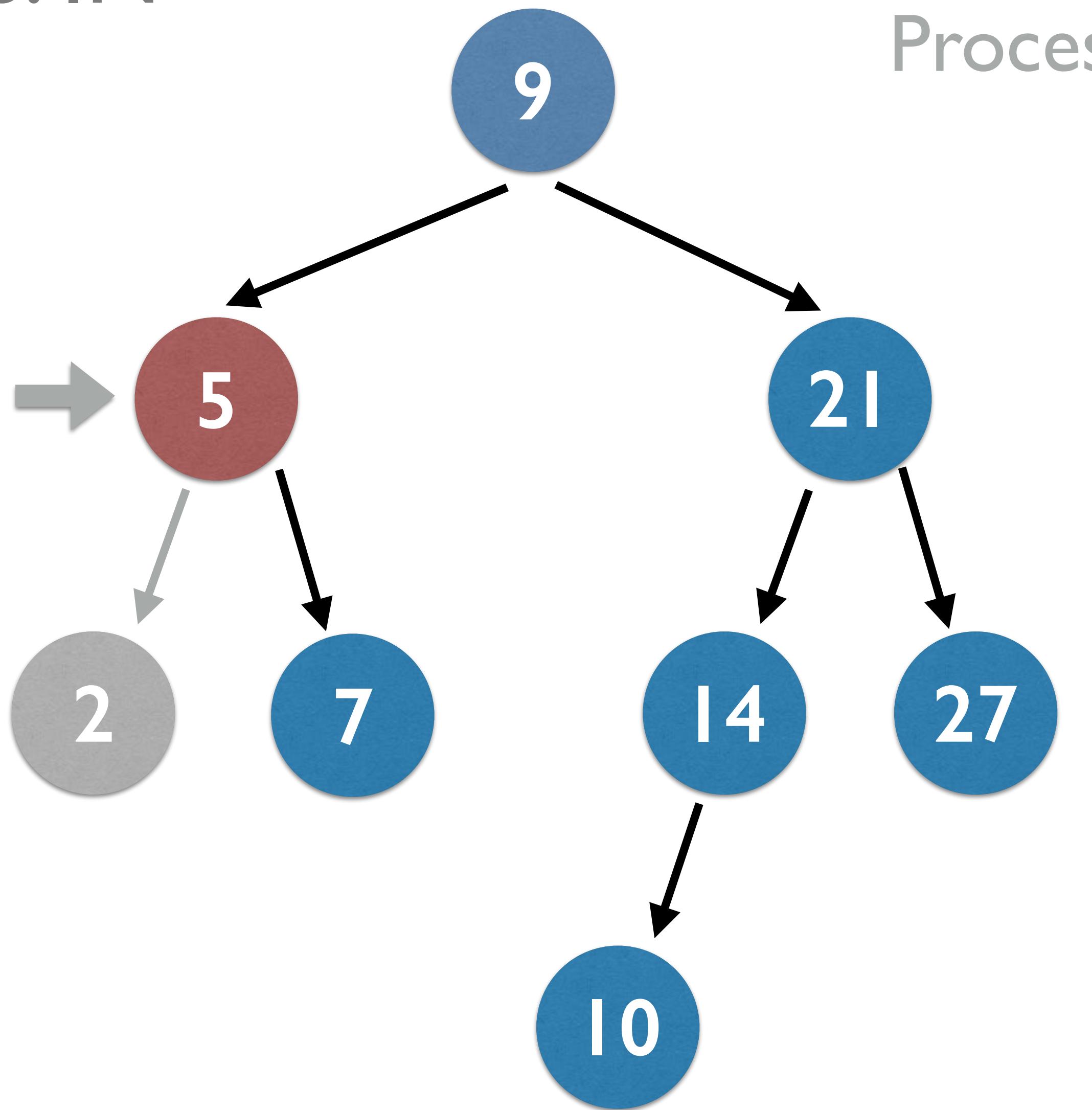
## DFS: IN



Process left · **Process root** · Process right

2

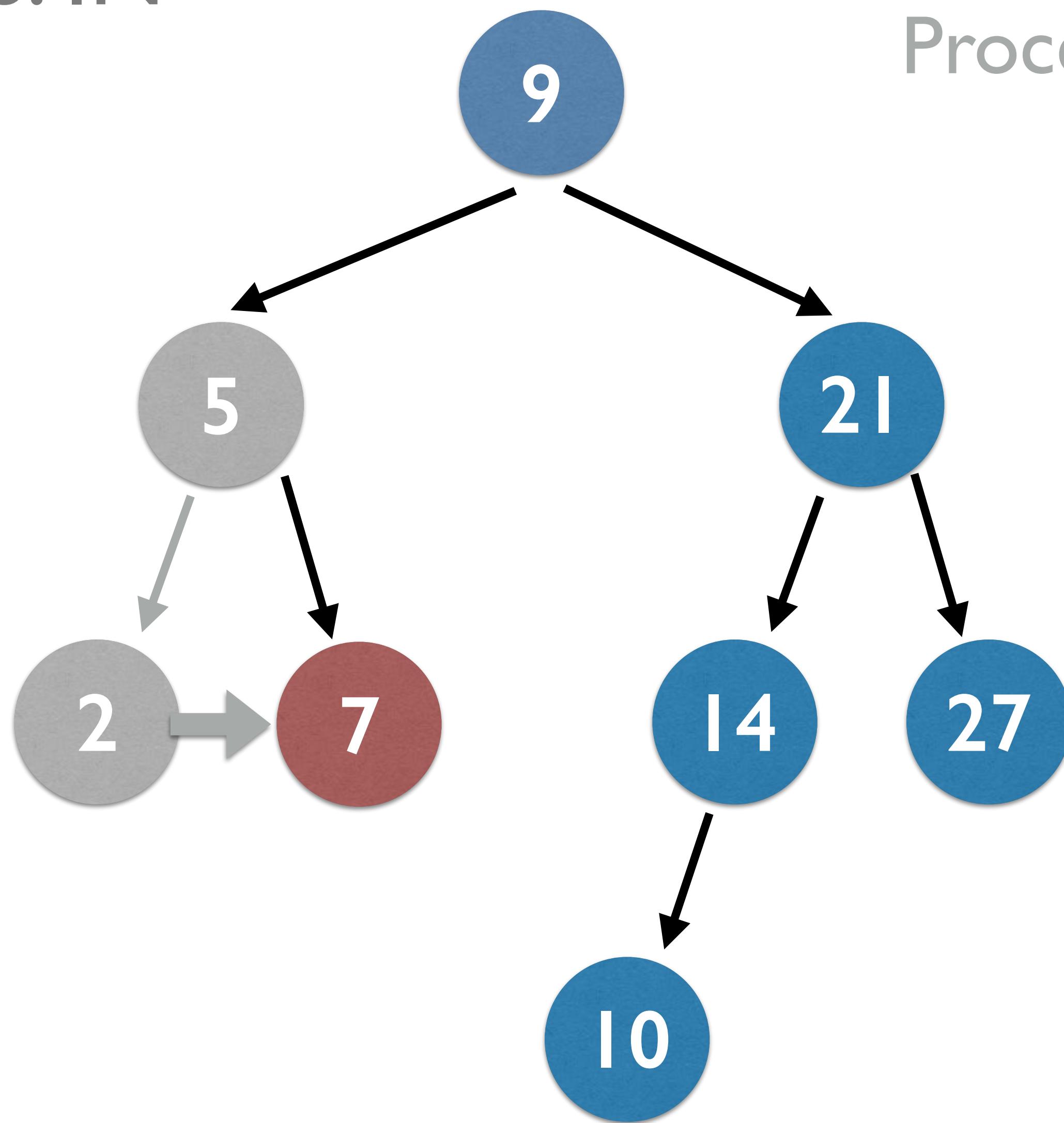
## DFS: IN



Process left · Process root · Process right

2, 5

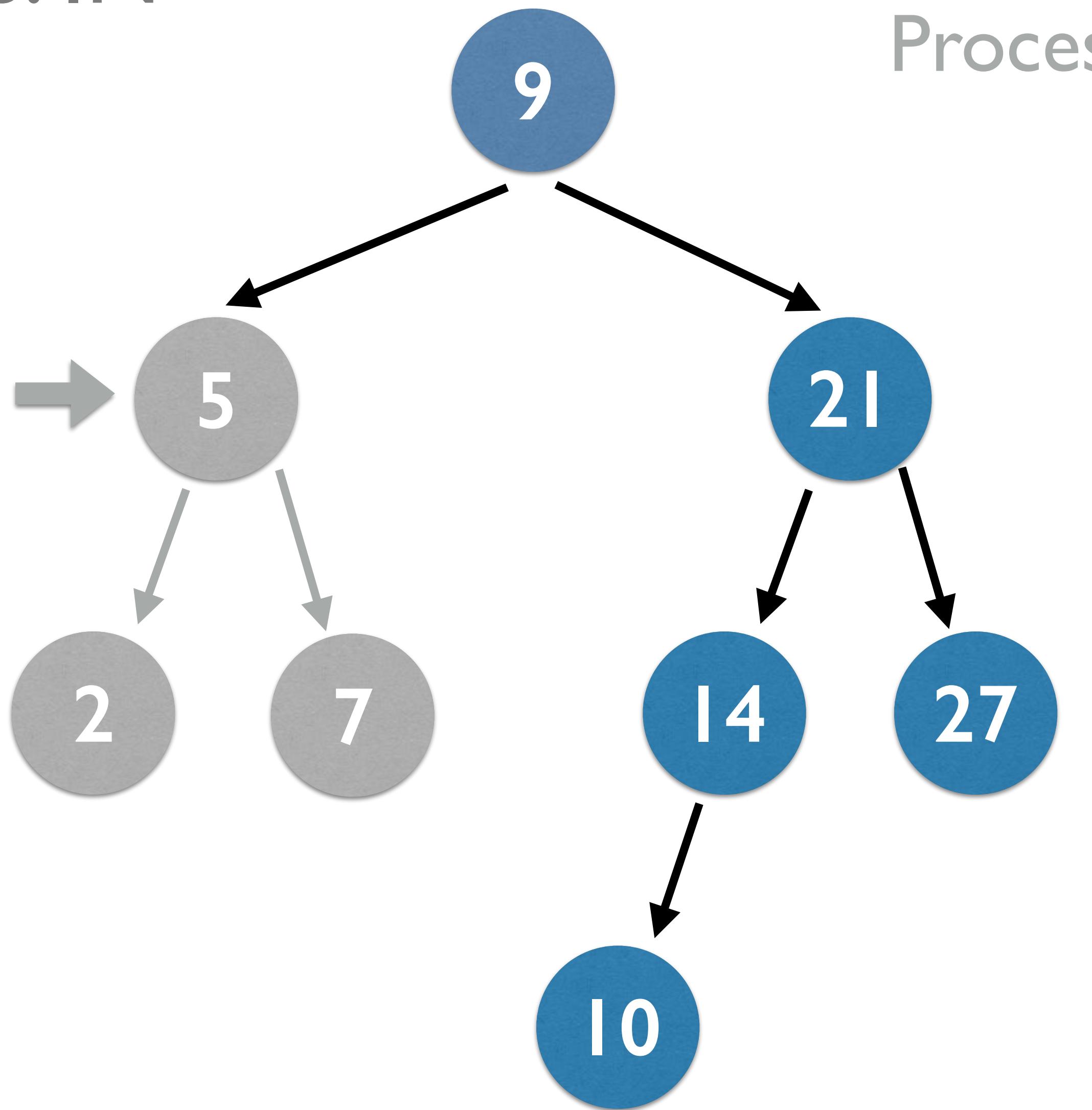
## DFS: IN



Process left · **Process root** · Process right

2, 5, 7

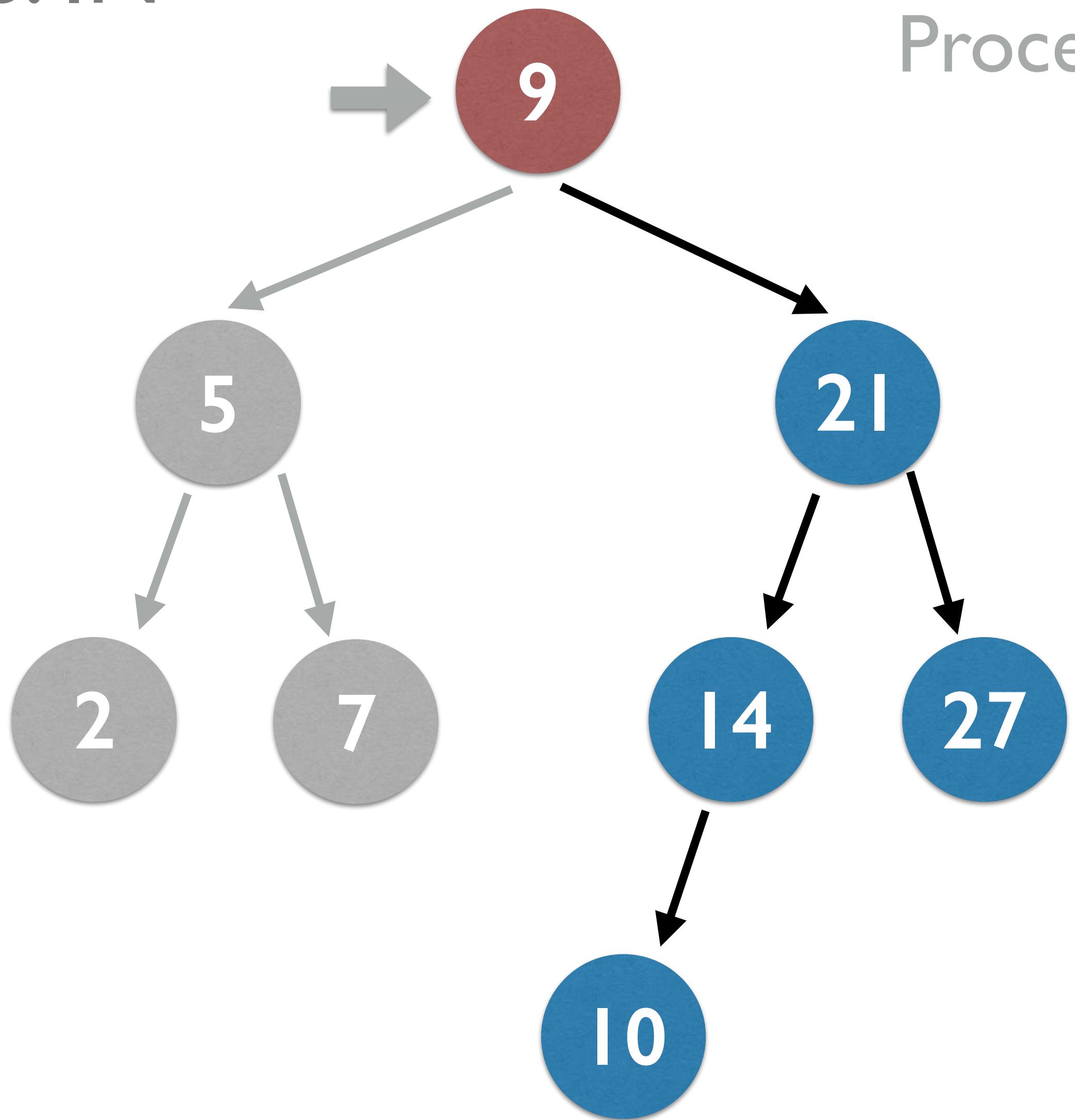
## DFS: IN



Process left · Process root · Process right

2, 5, 7

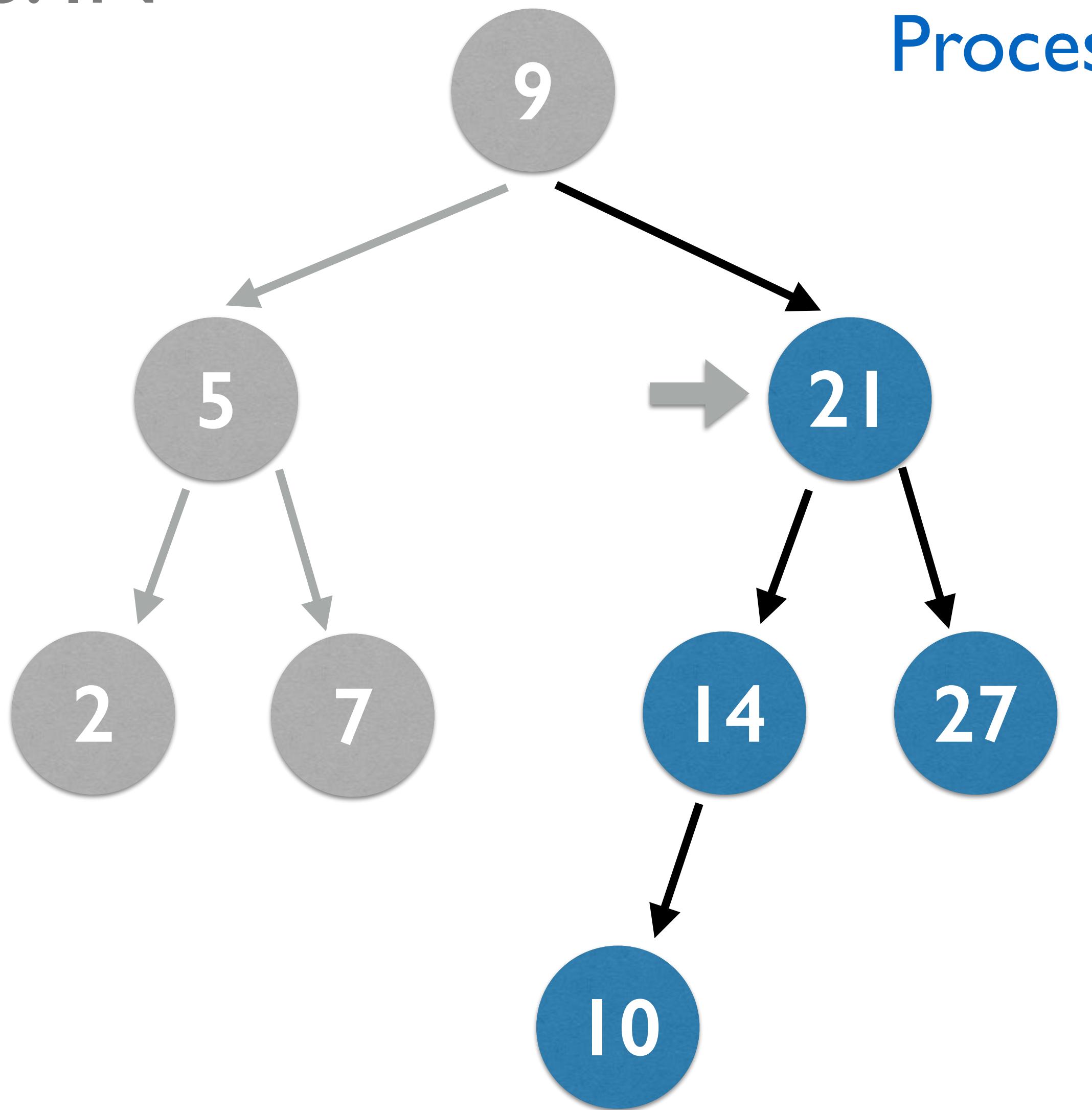
## DFS: IN



Process left · Process root · Process right

2, 5, 7, 9

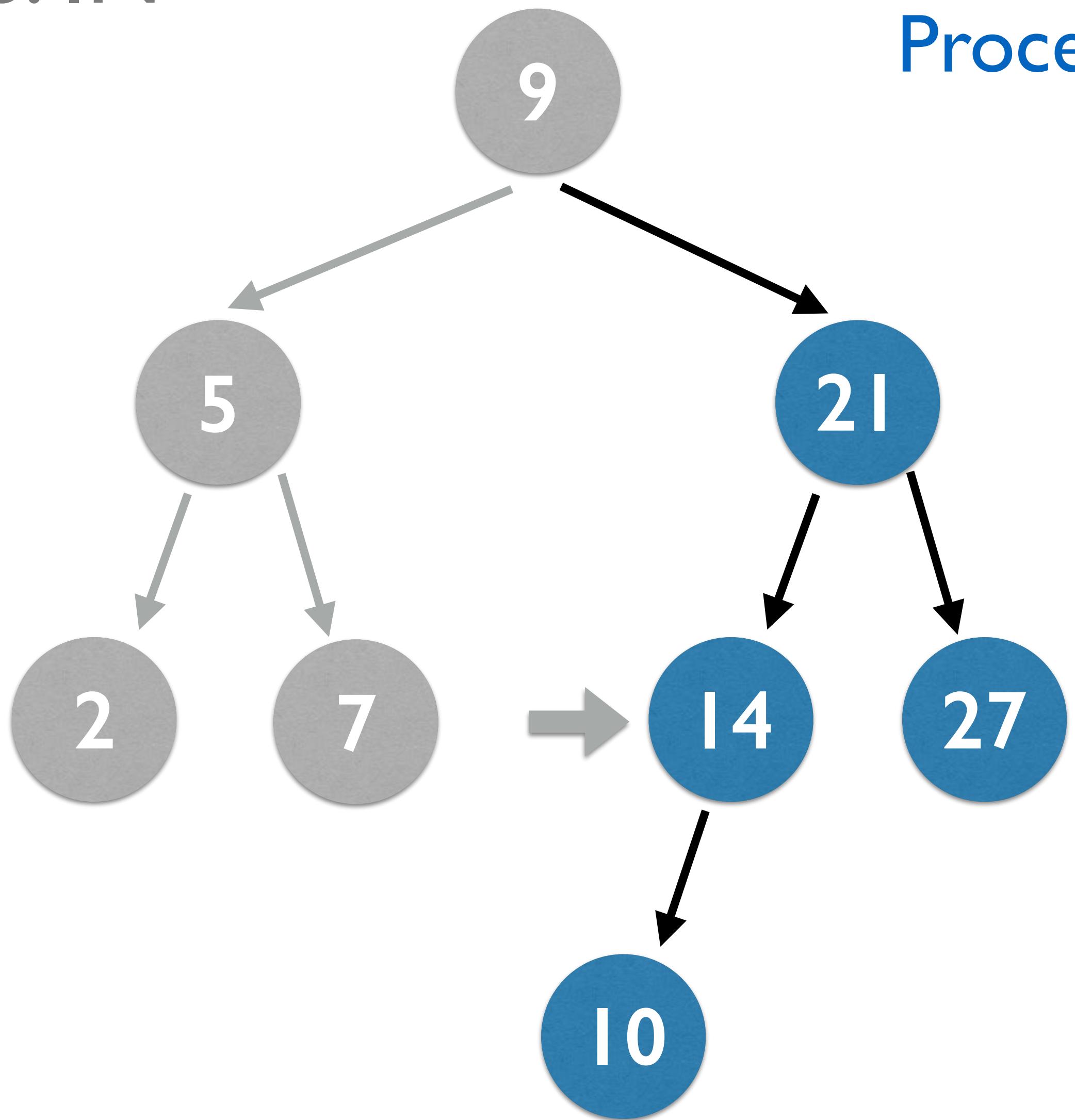
## DFS: IN



Process left · Process root · Process right

2, 5, 7, 9

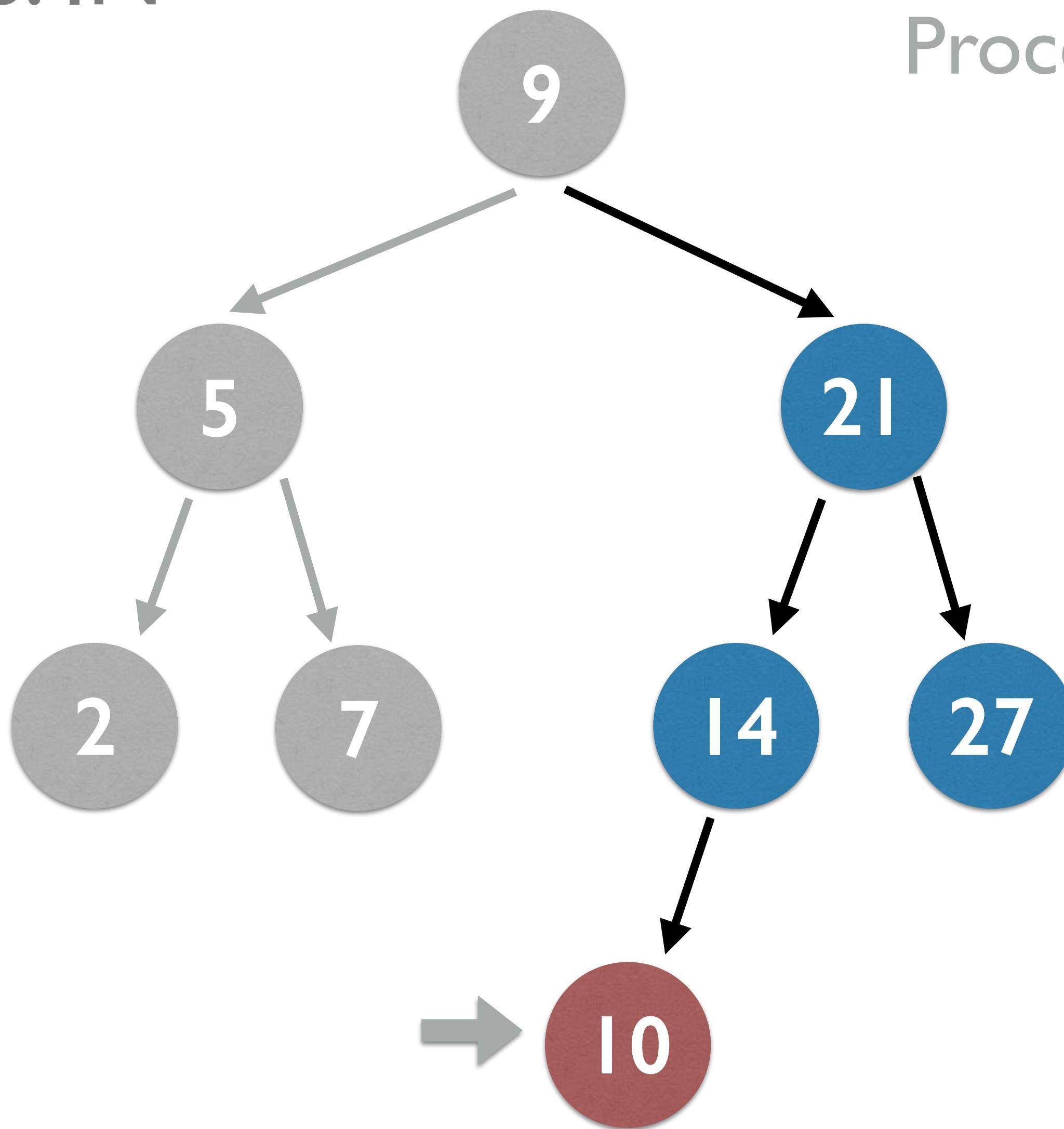
## DFS: IN



Process left · Process root · Process right

2, 5, 7, 9

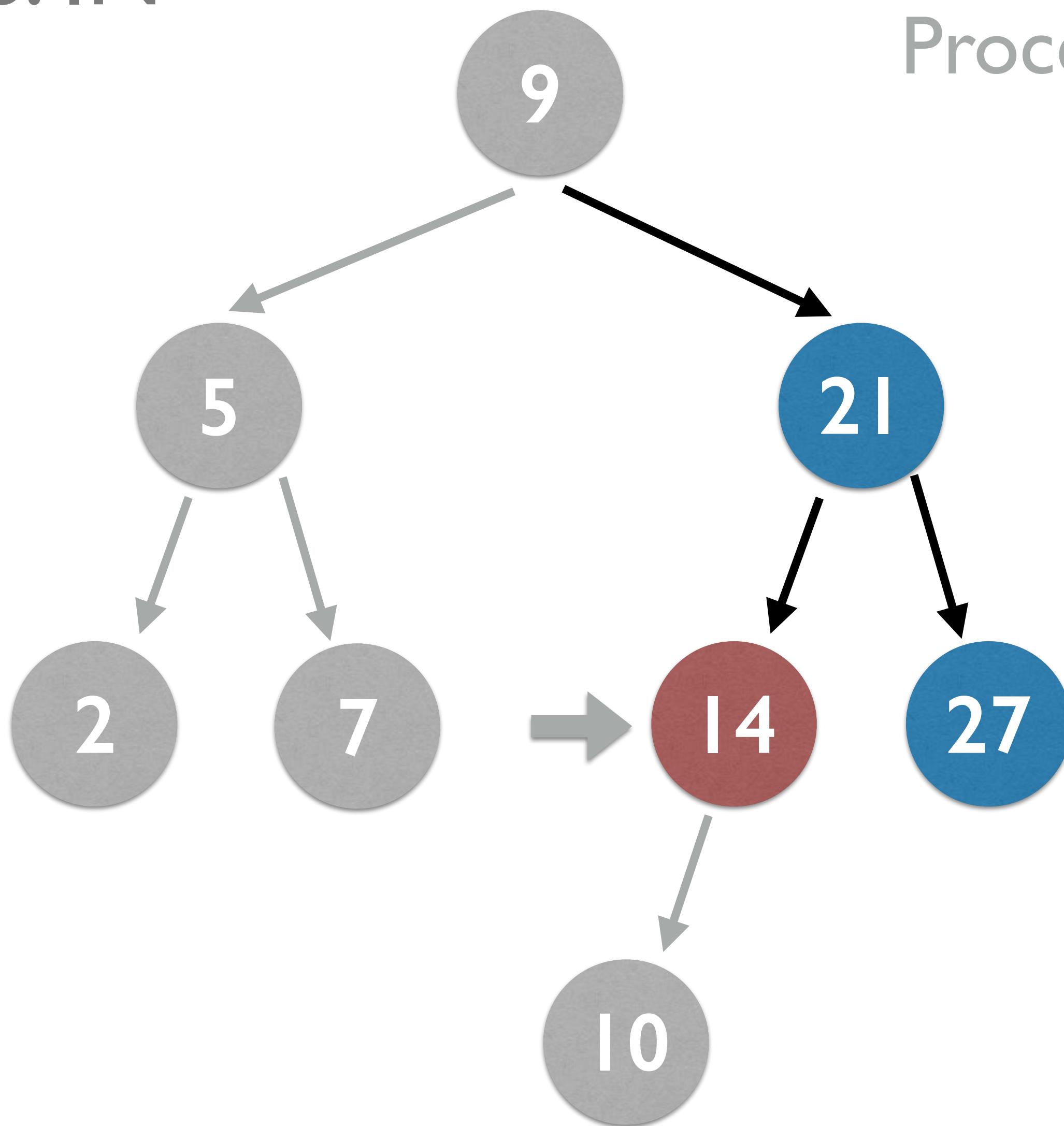
## DFS: IN



Process left · **Process root** · Process right

2, 5, 7, 9, 10

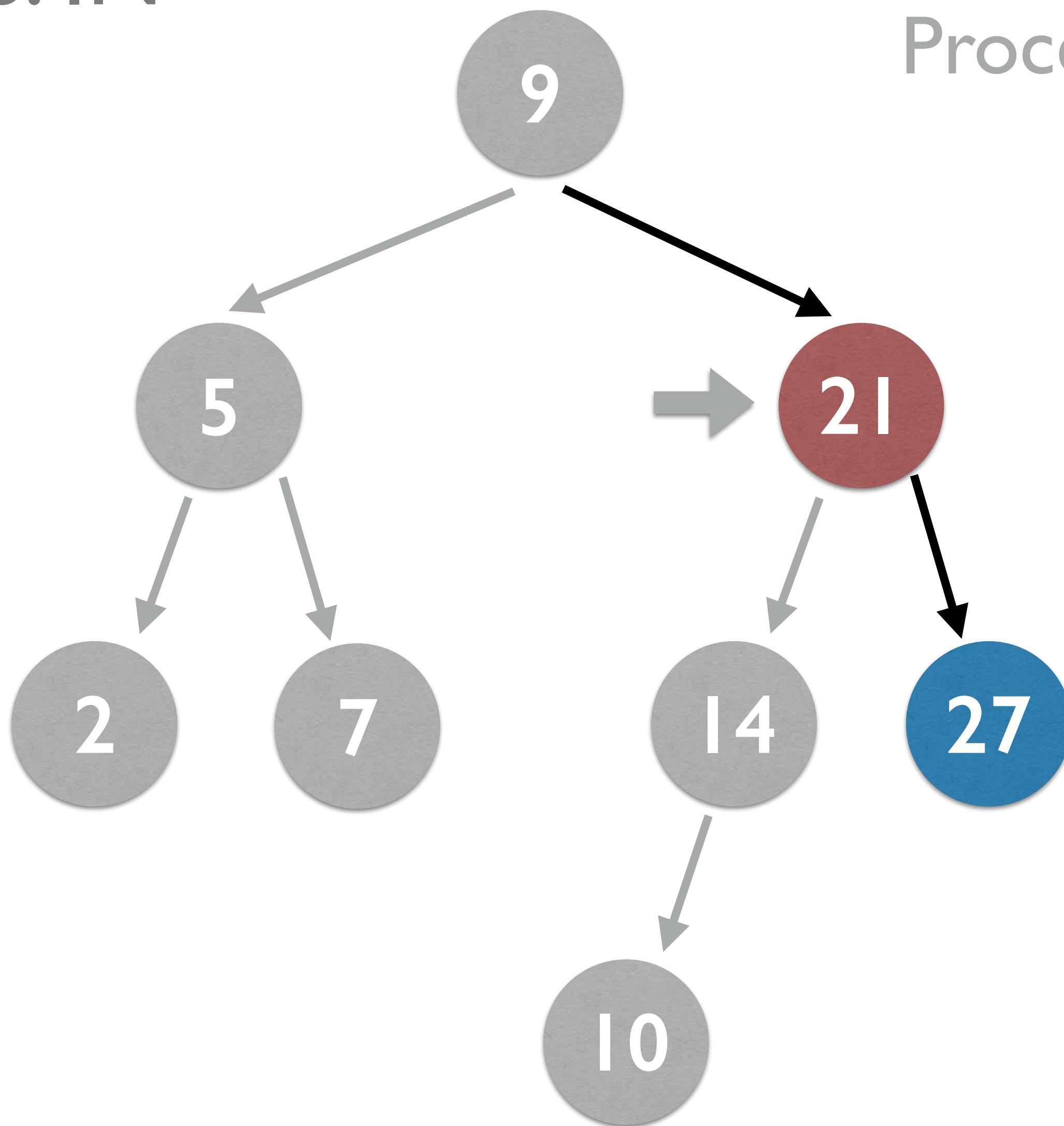
## DFS: IN



Process left · **Process root** · Process right

2, 5, 7, 9, 10, 14

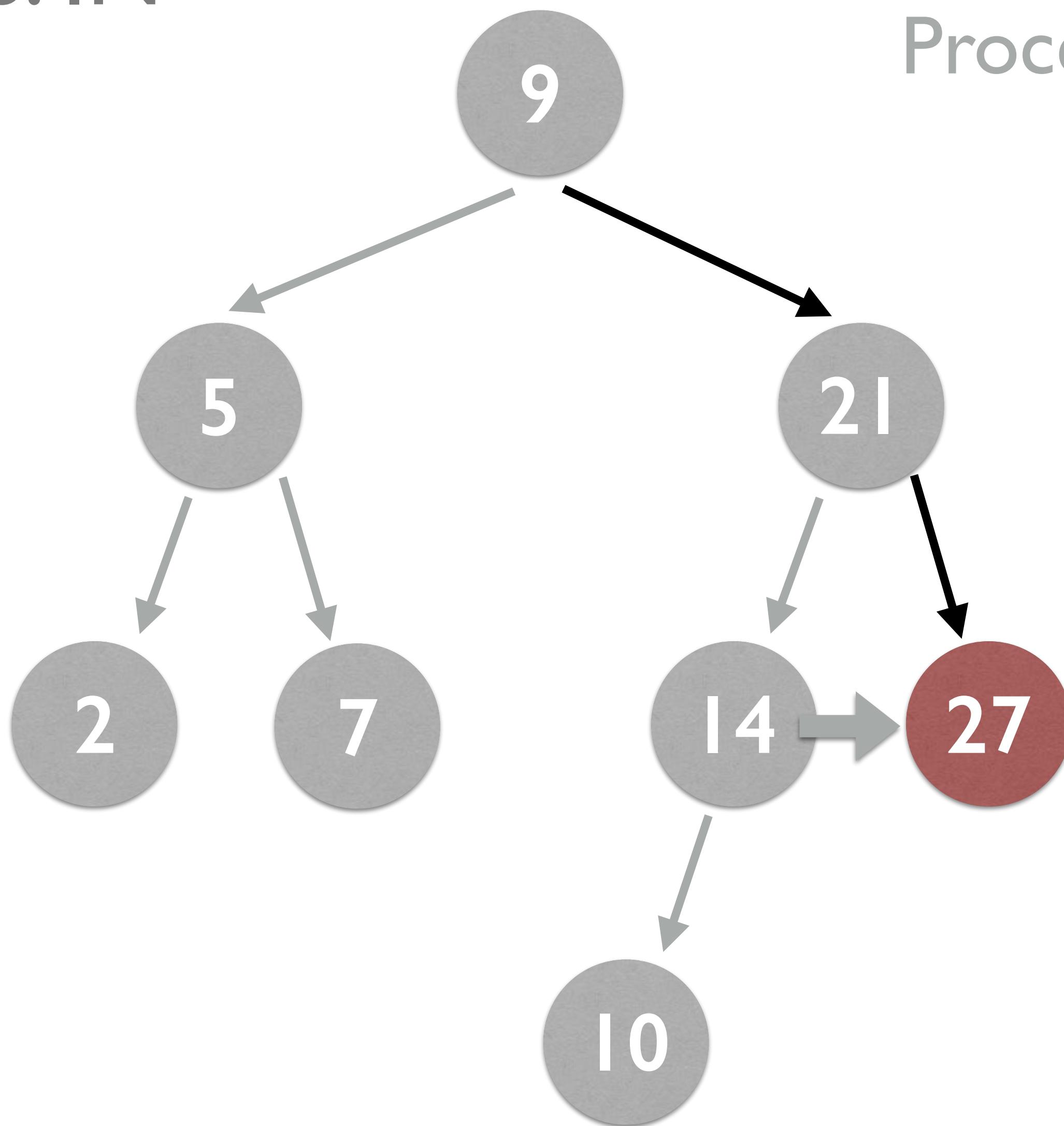
## DFS: IN



Process left · Process root · Process right

2, 5, 7, 9, 10, 14, 21

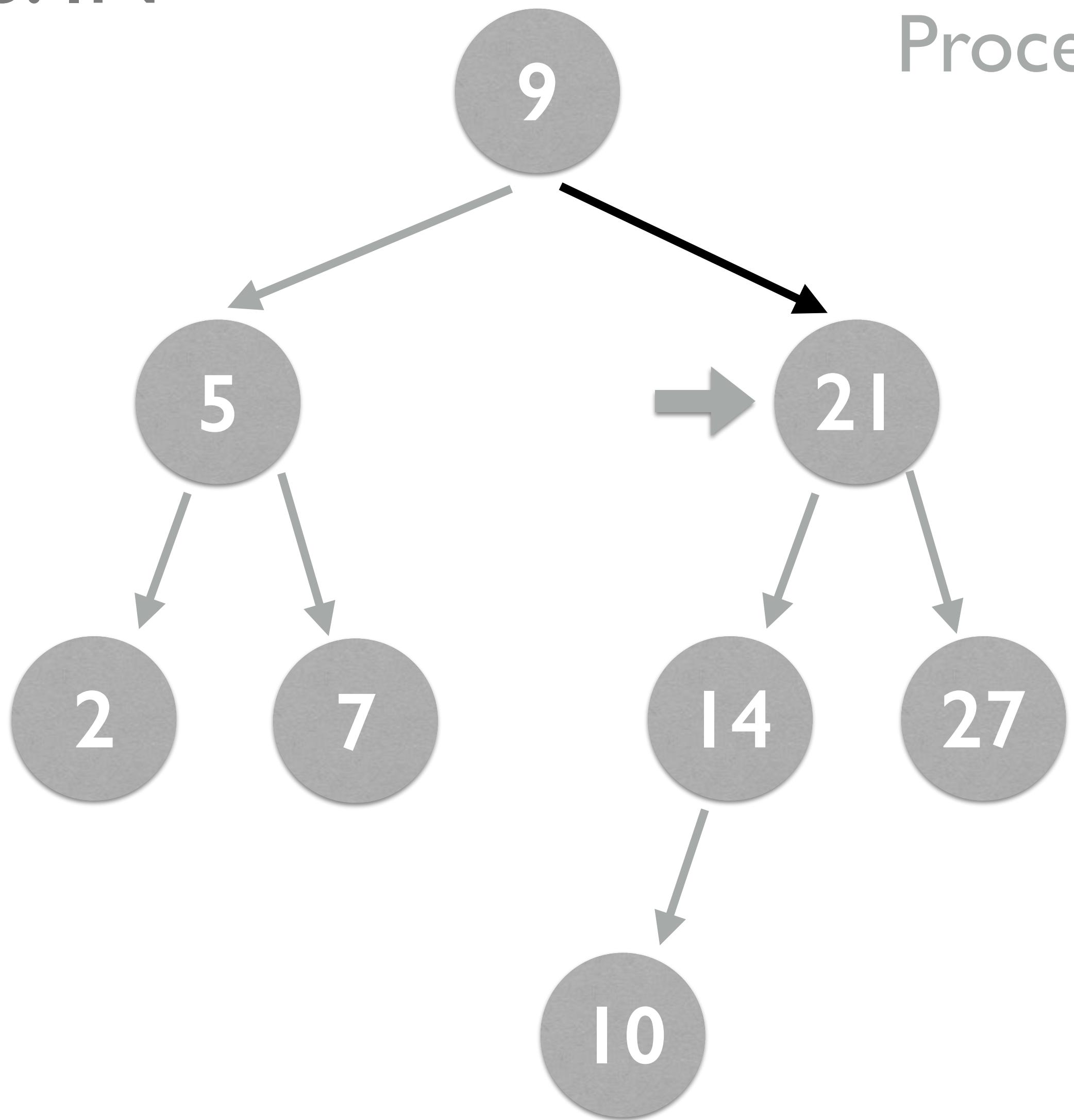
## DFS: IN



Process left · **Process root** · Process right

2, 5, 7, 9, 10, 14, 21, 27

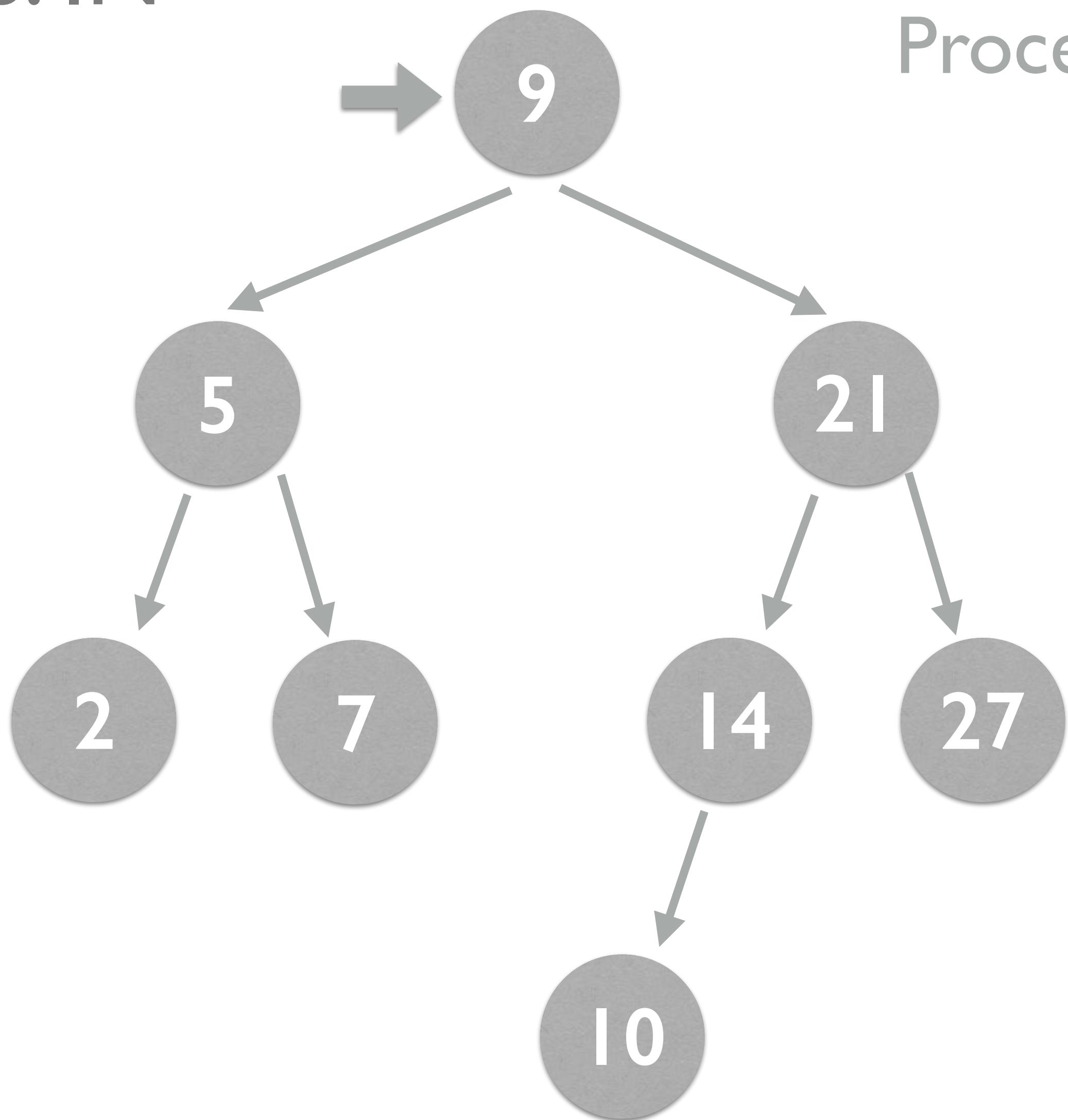
## DFS: IN



Process left · Process root · Process right

2, 5, 7, 9, 10, 14, 21, 27

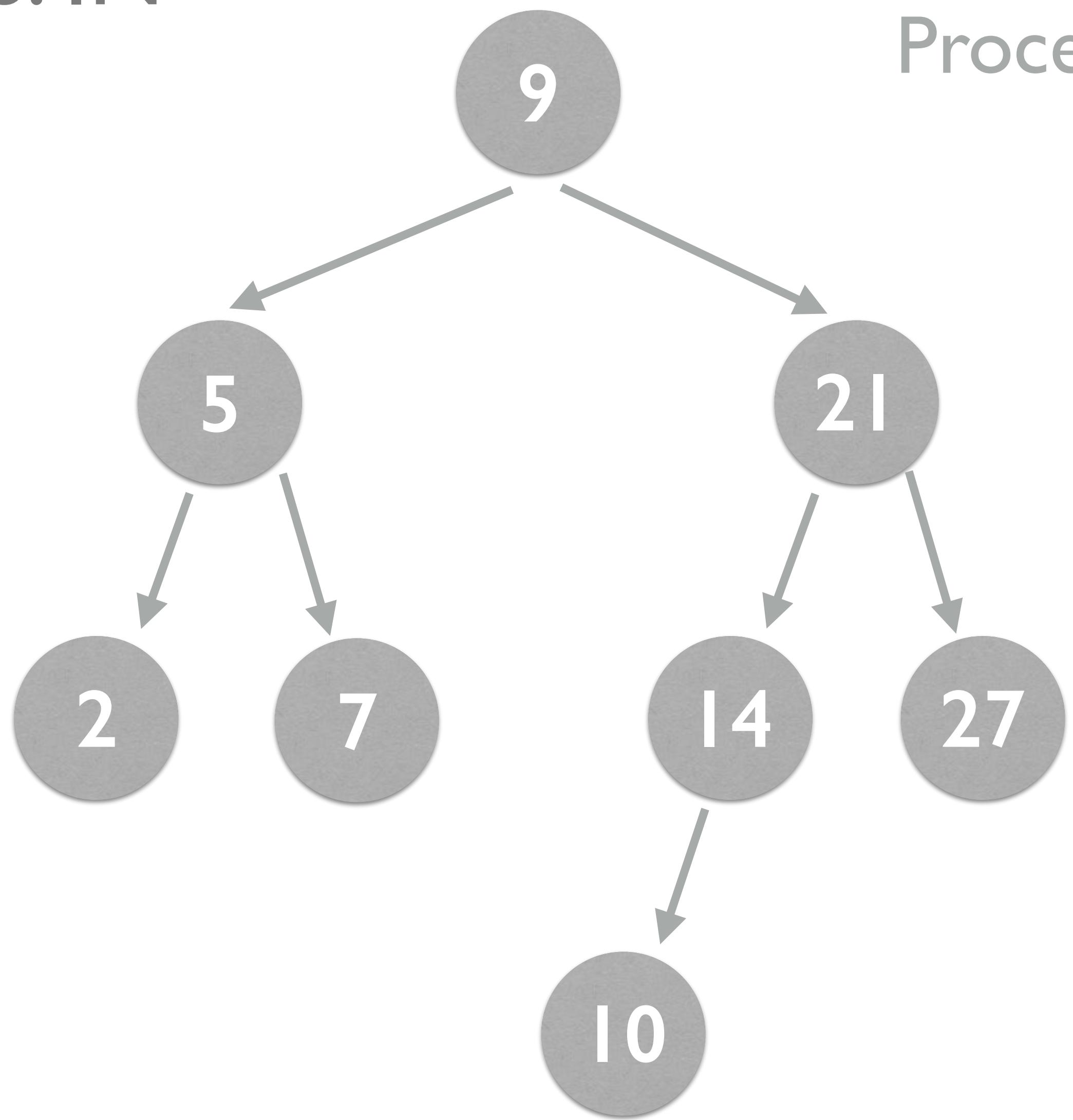
## DFS: IN



Process left · Process root · Process right

2, 5, 7, 9, 10, 14, 21, 27

## DFS: IN



Process left · Process root · Process right

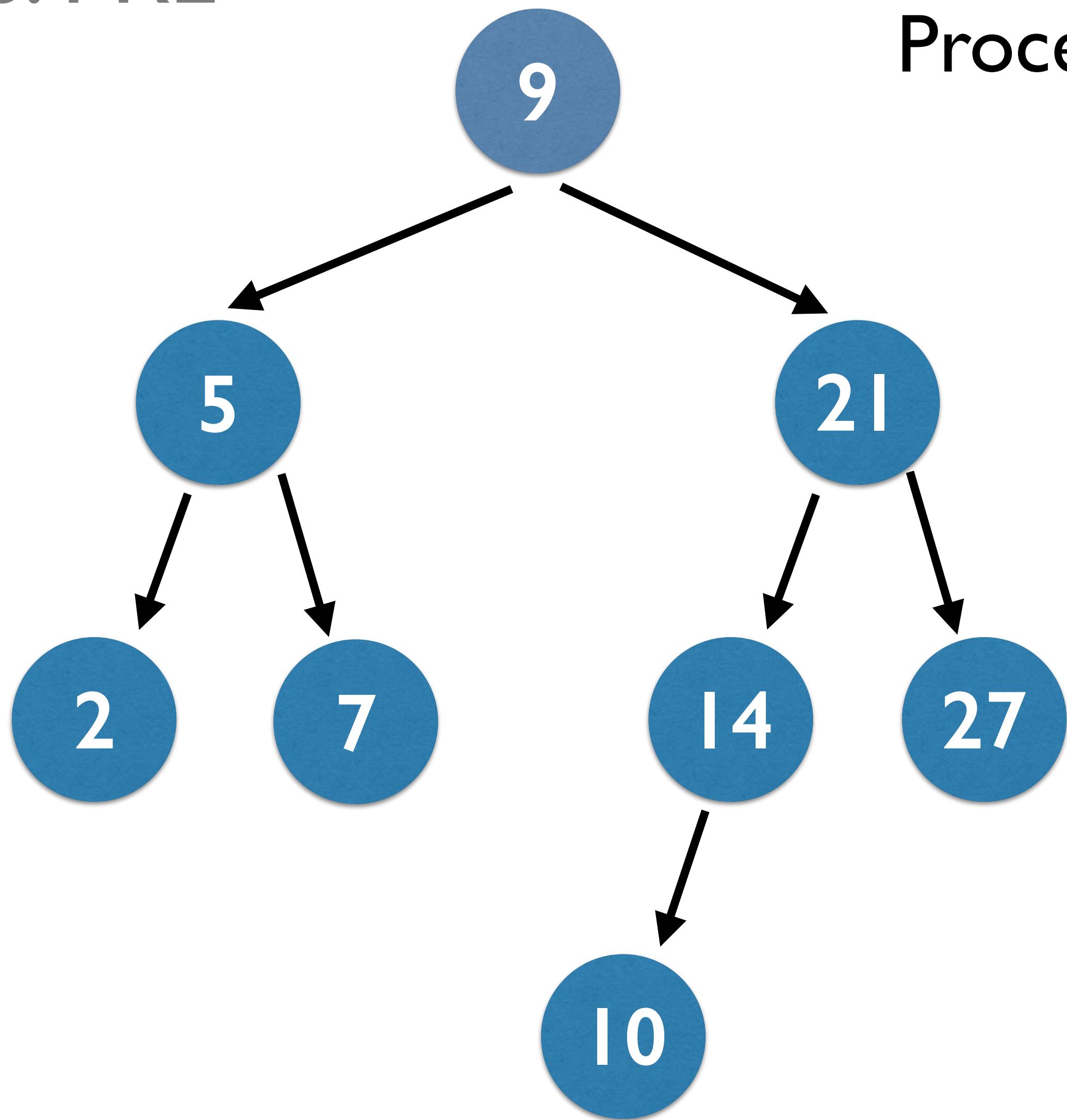
2, 5, 7, 9, 10, 14, 21, 27

- In-order because respects ordering of nodes — nodes are processed smallest to largest value (leftmost to rightmost).
- The most generally useful DFS strategy for BSTs.

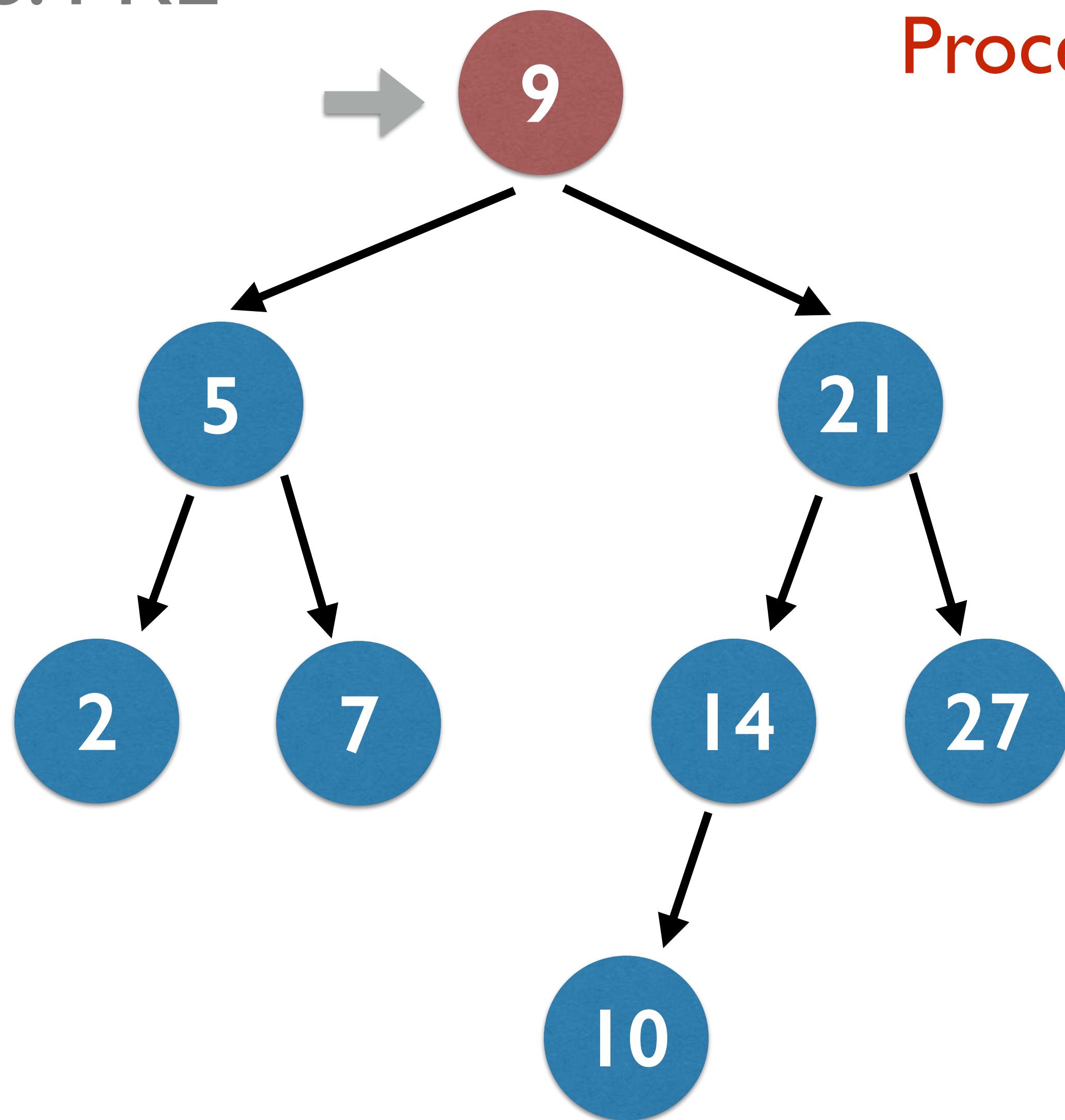
# Depth First: Pre-Order

## DFS: PRE

Process root · Process left · Process right



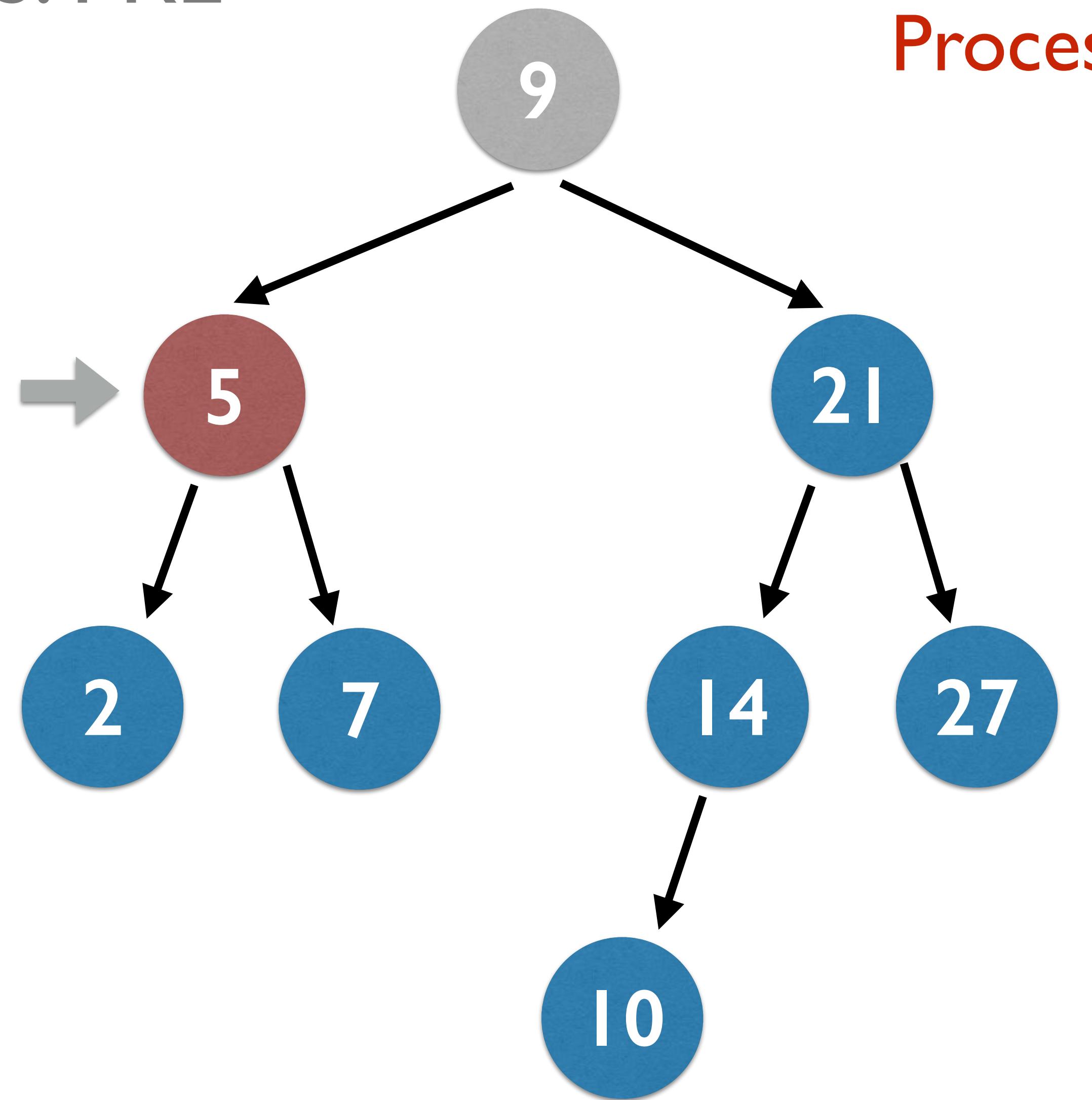
## DFS: PRE



Process root • Process left • Process right

9

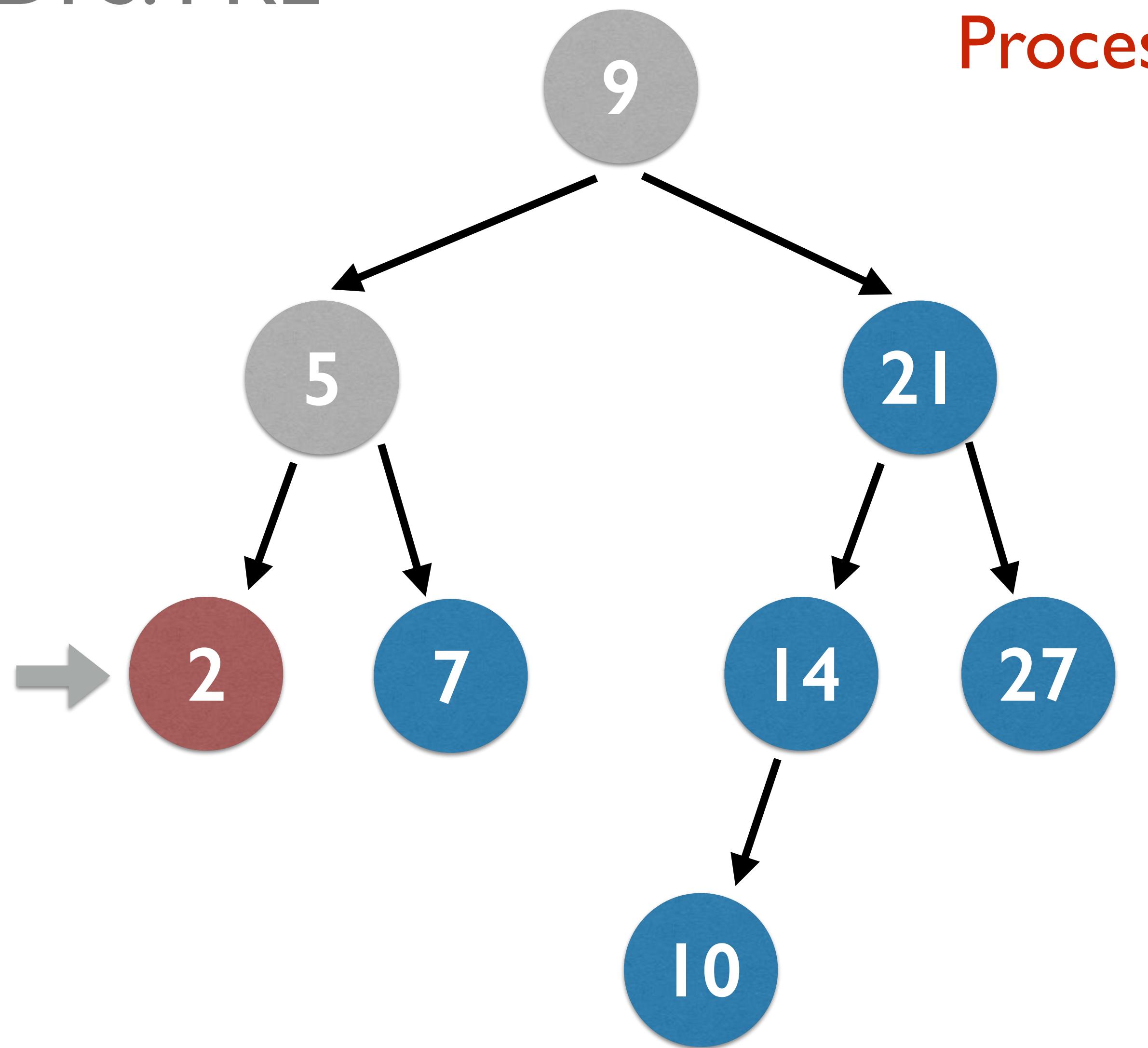
## DFS: PRE



Process root • Process left • Process right

9, 5

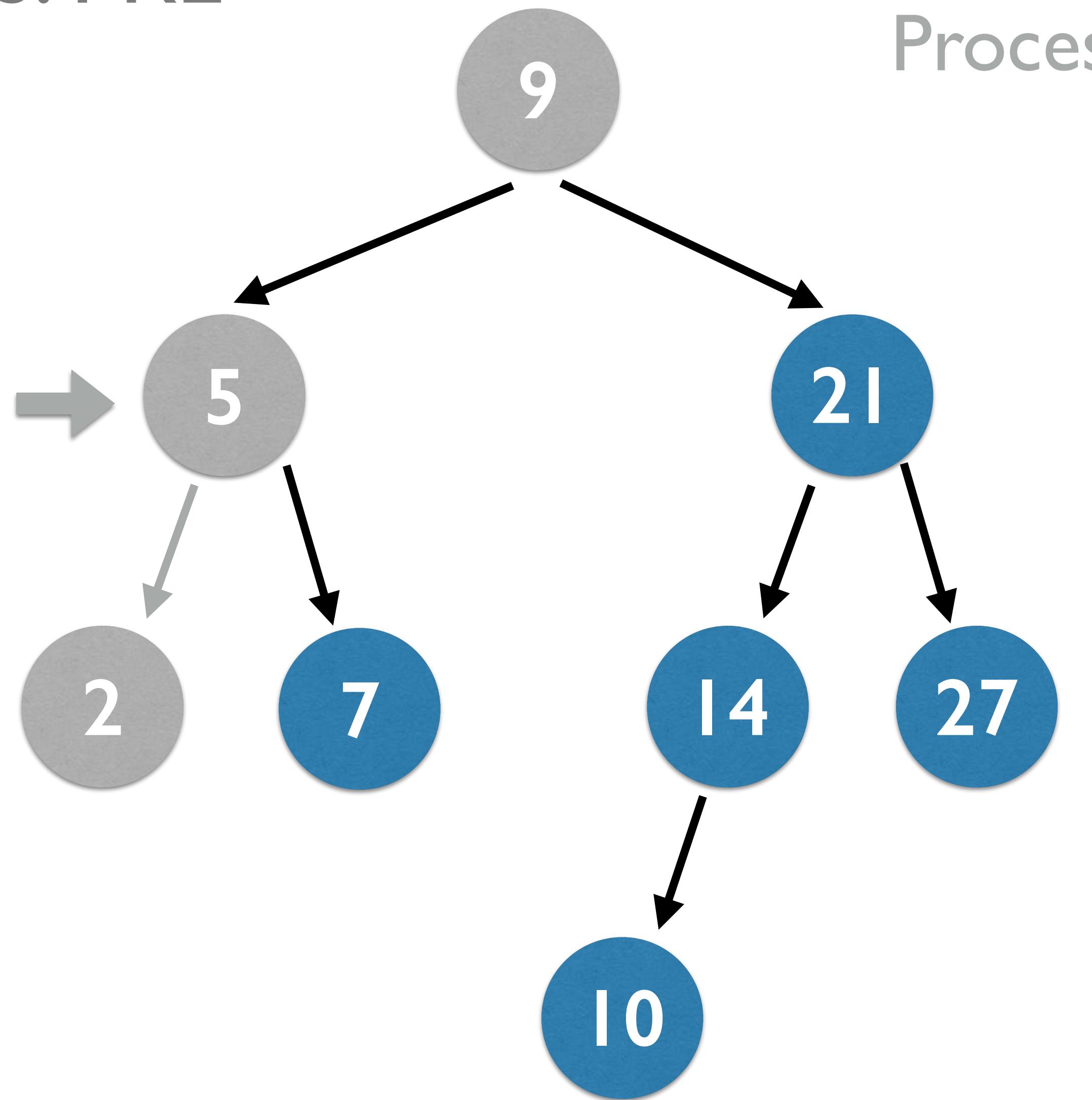
## DFS: PRE



**Process root** • Process left • Process right

9, 5, 2

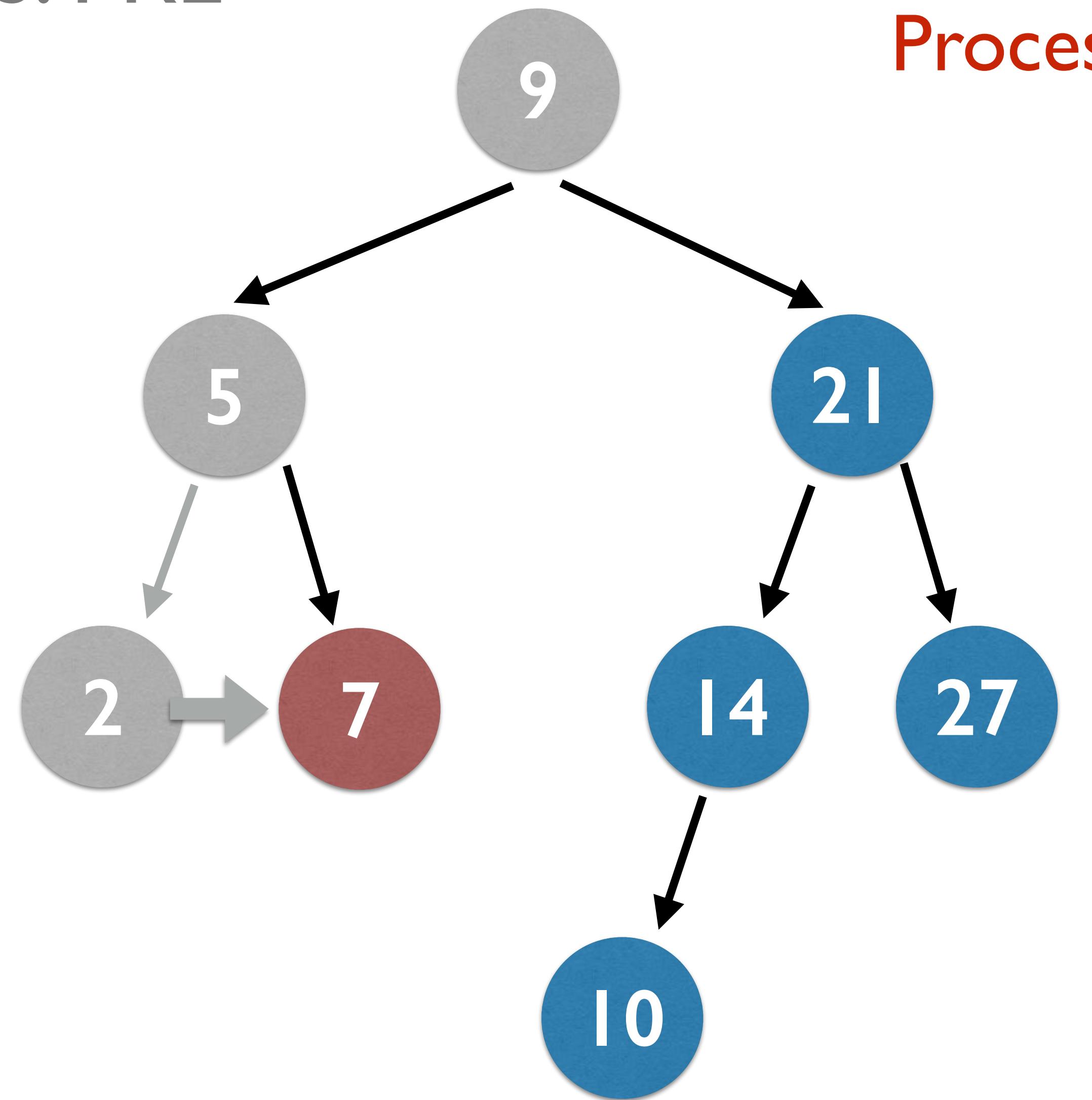
## DFS: PRE



Process root · Process left · **Process right**

9, 5, 2

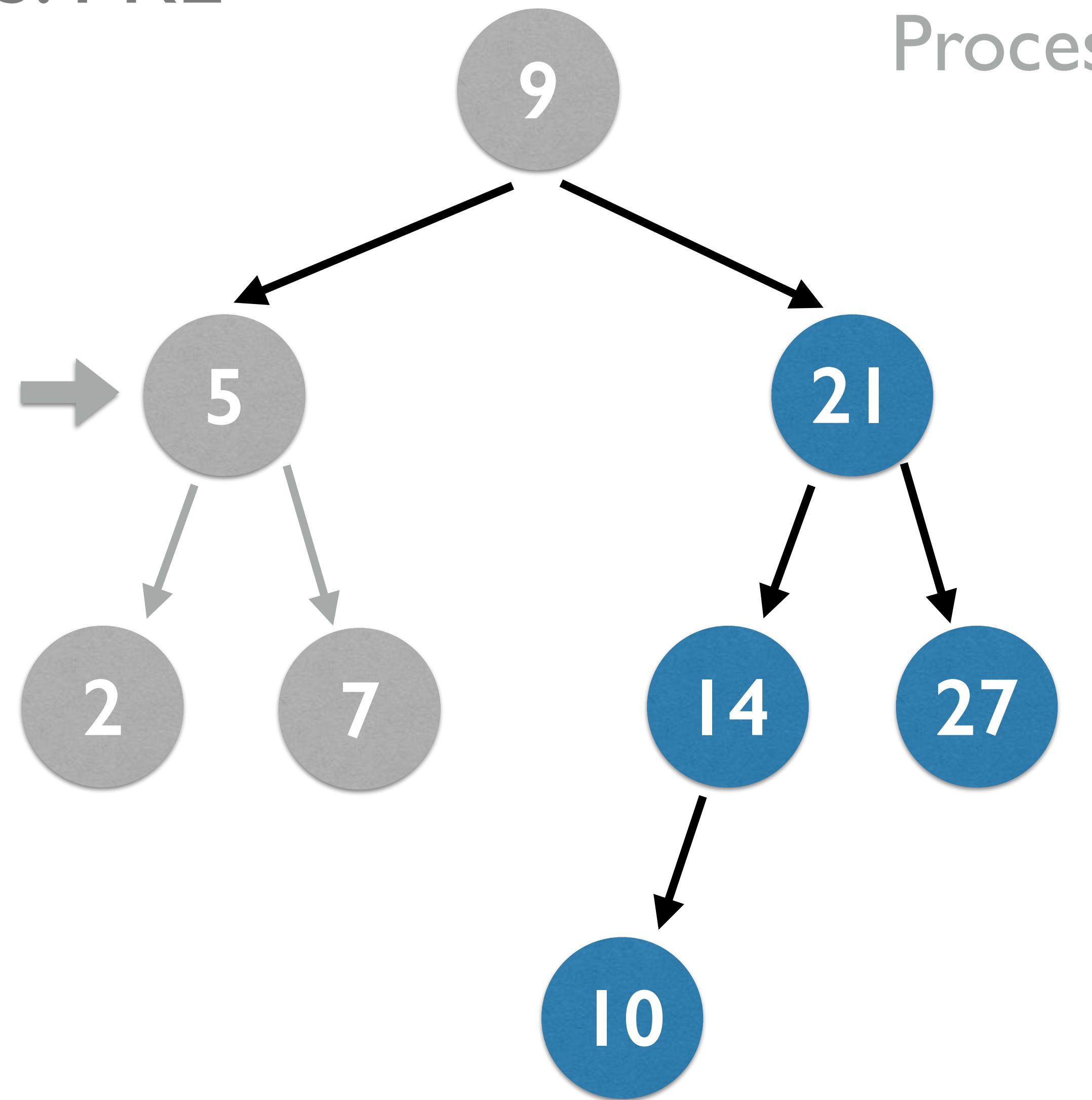
## DFS: PRE



**Process root** • Process left • Process right

9, 5, 2, 7

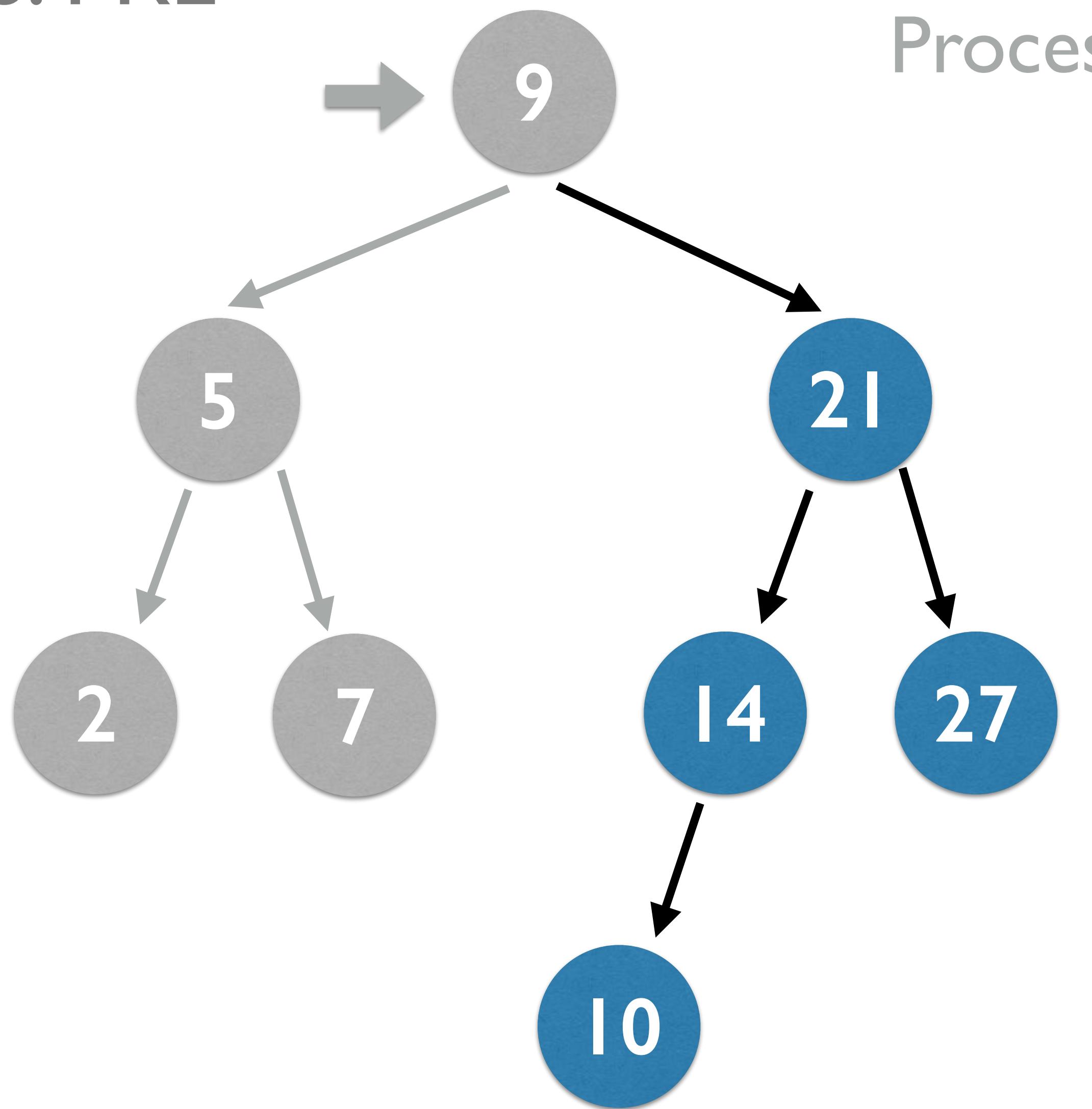
## DFS: PRE



Process root · Process left · Process right

9, 5, 2, 7

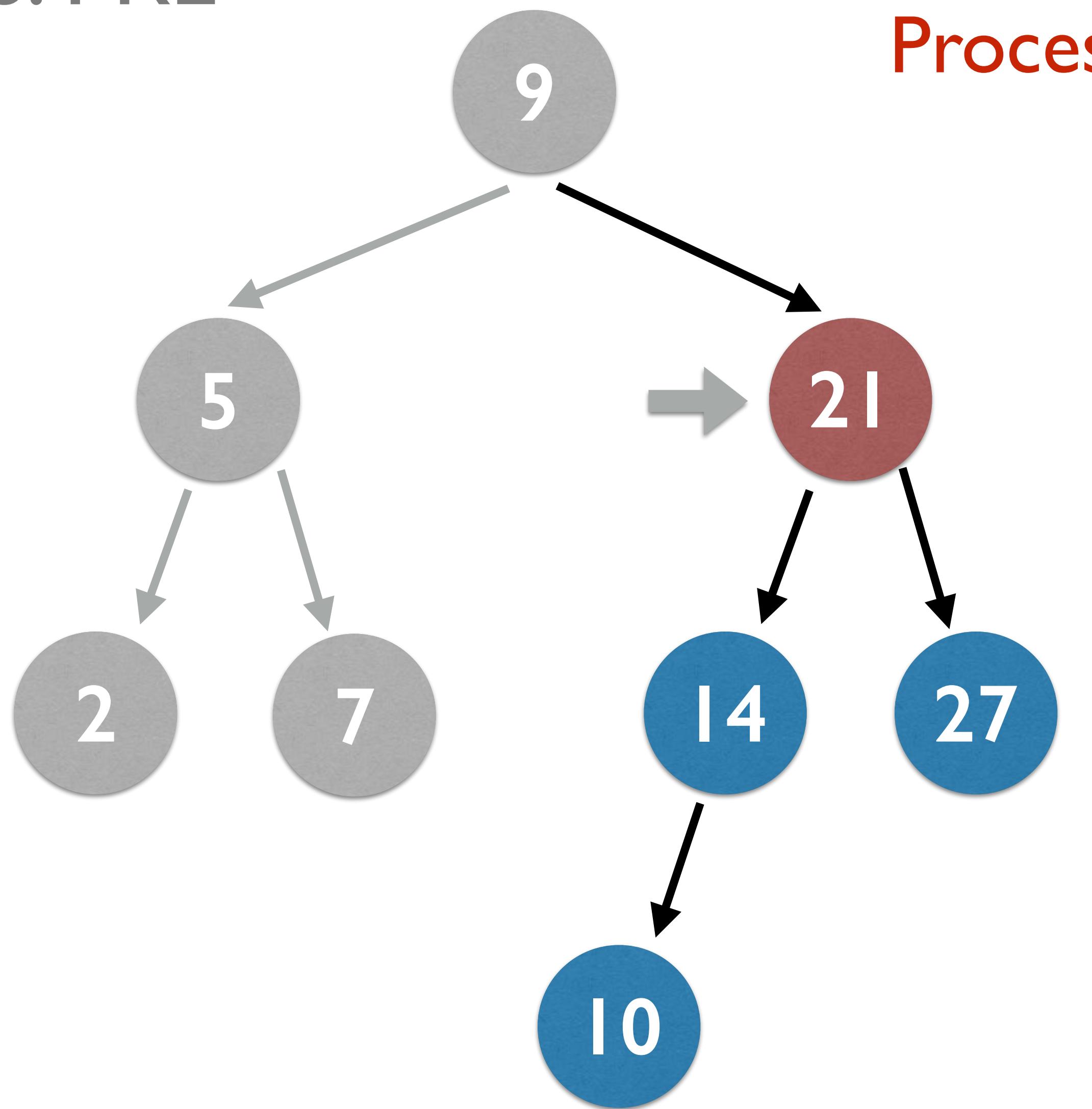
## DFS: PRE



Process root · Process left · **Process right**

9, 5, 2, 7

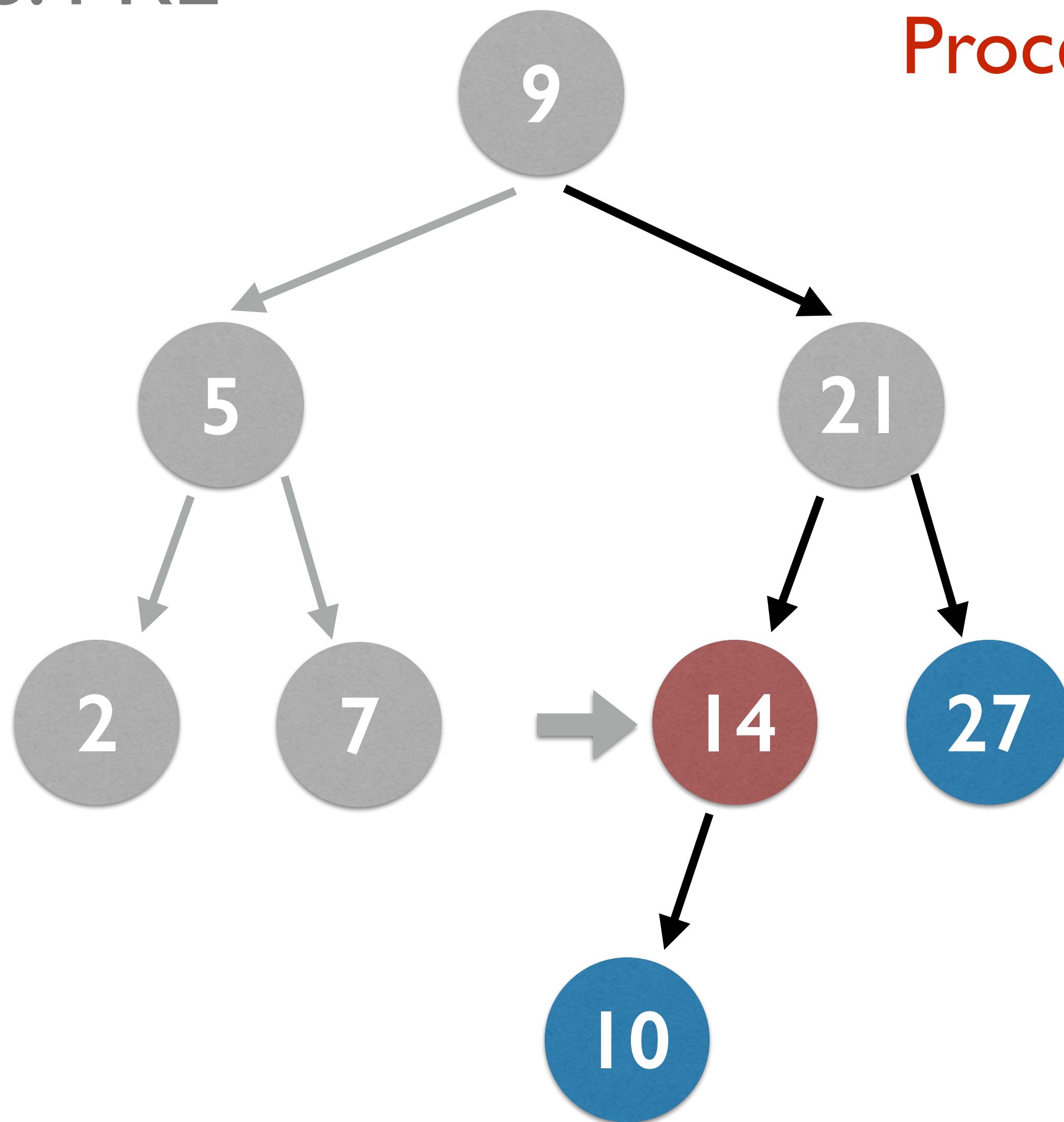
## DFS: PRE



Process root · Process left · Process right

9, 5, 2, 7, 21

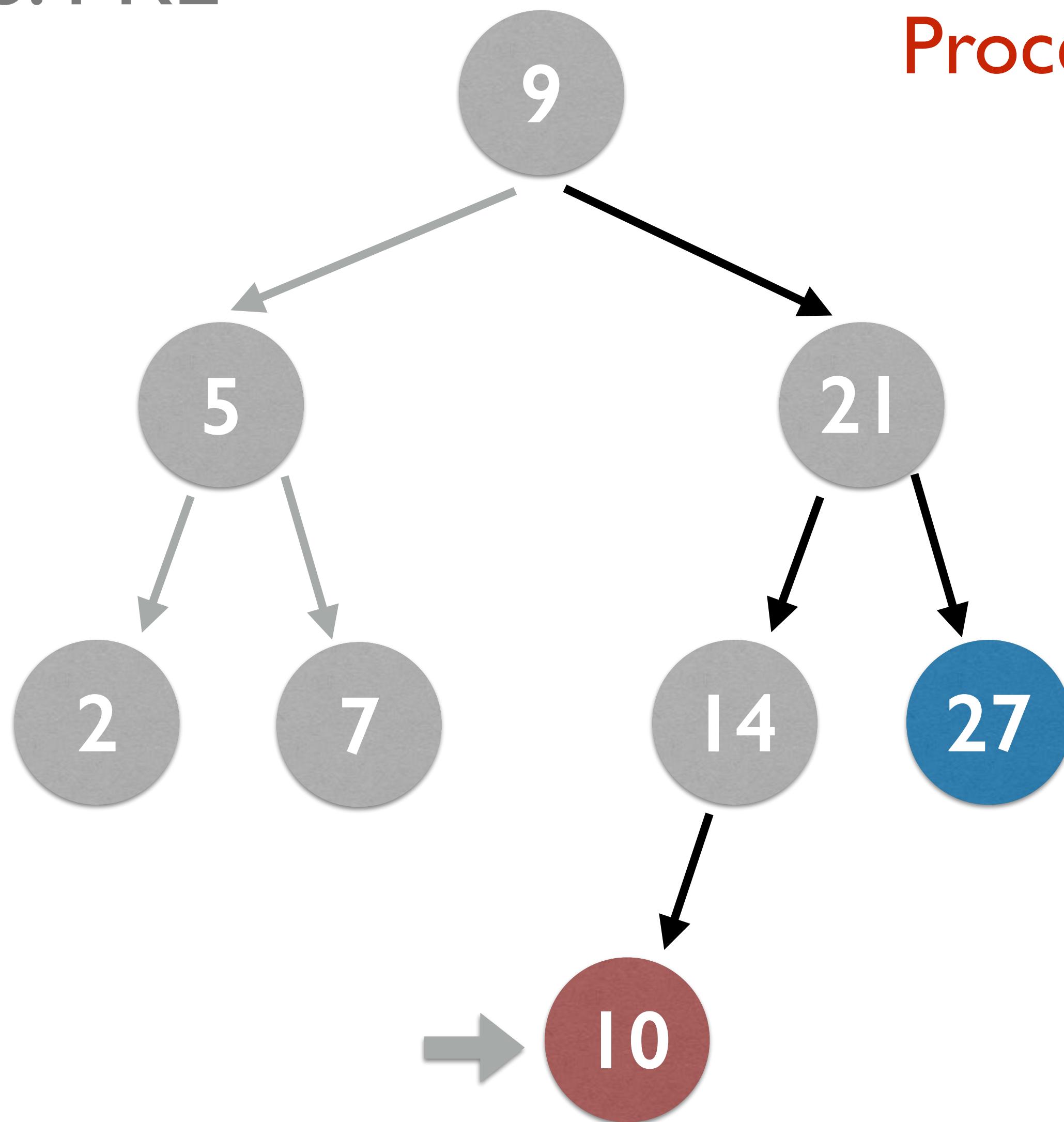
## DFS: PRE



Process root · Process left · Process right

9, 5, 2, 7, 21, 14

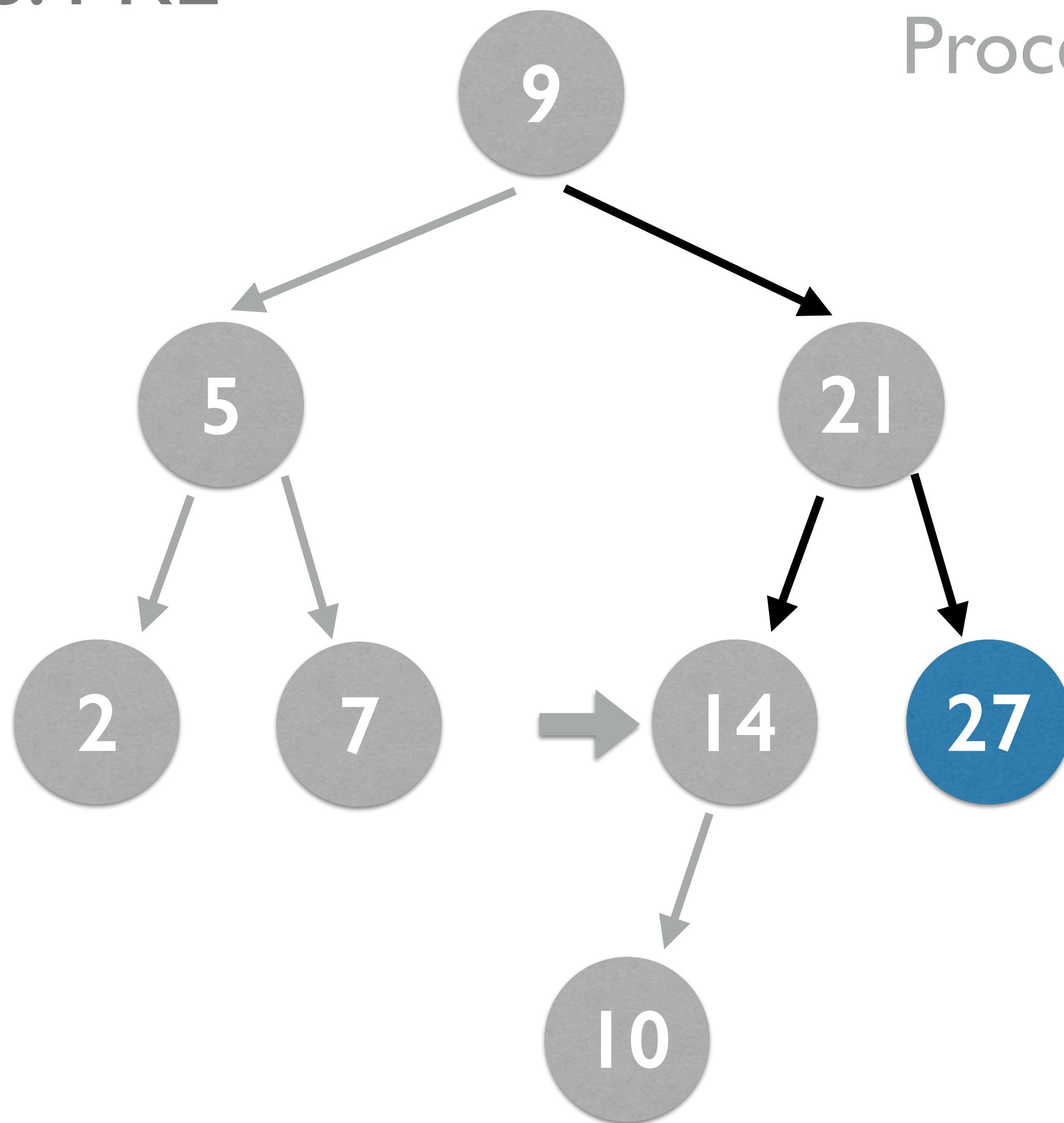
## DFS: PRE



**Process root** · Process left · Process right

9, 5, 2, 7, 21, 14, 10

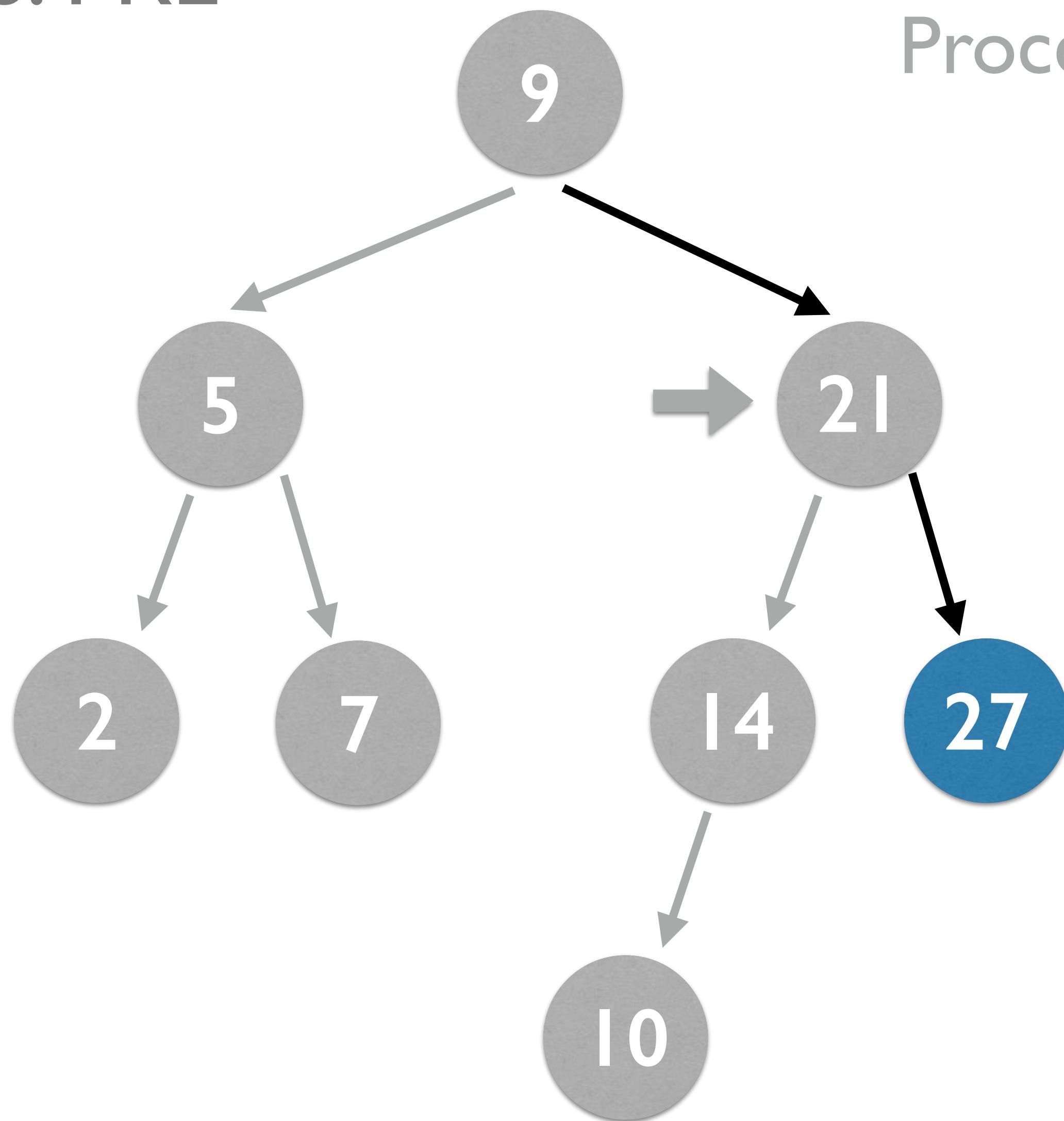
## DFS: PRE



Process root · Process left · Process right

9, 5, 2, 7, 21, 14, 10

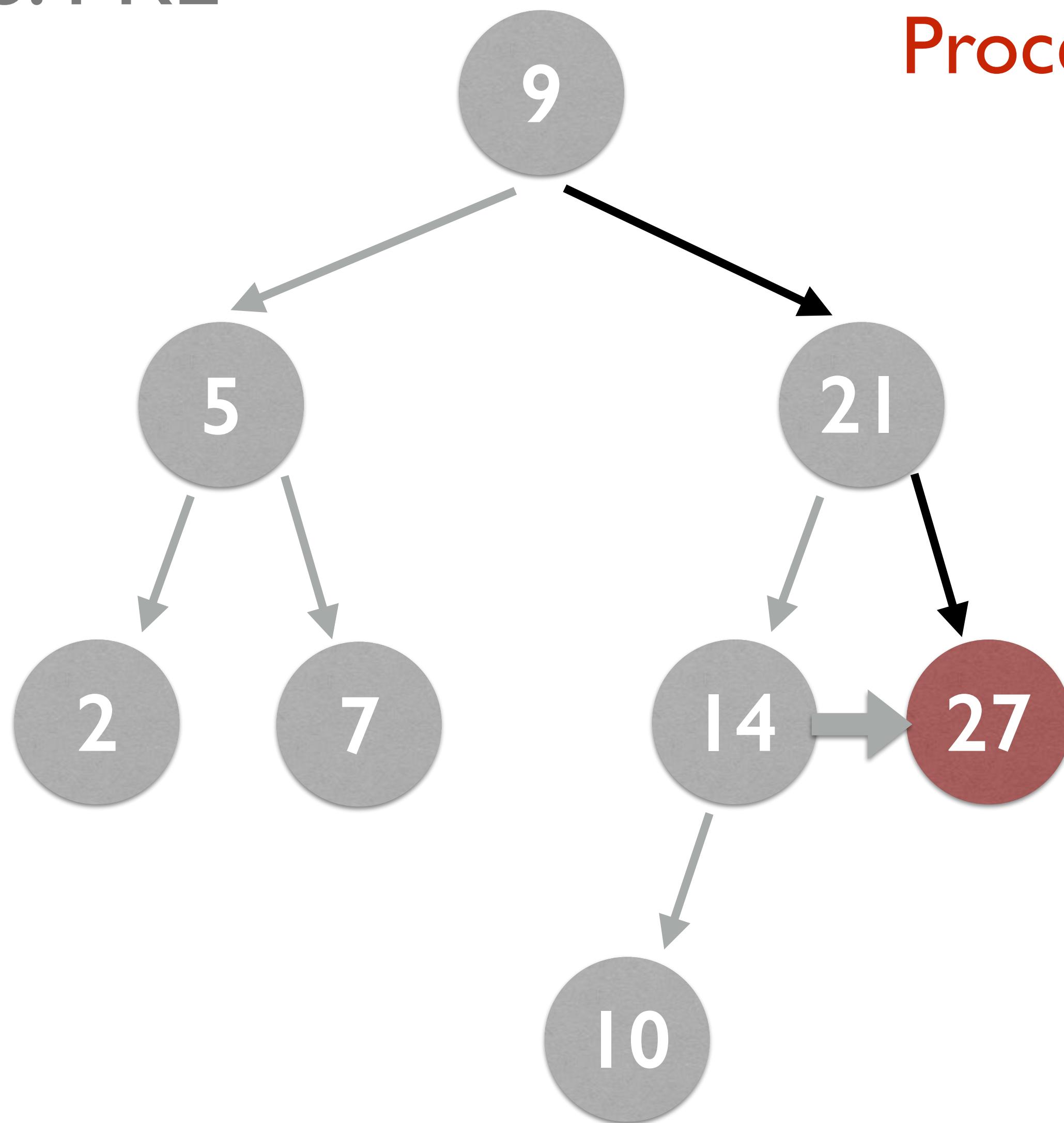
## DFS: PRE



Process root · Process left · **Process right**

9, 5, 2, 7, 21, 14, 10

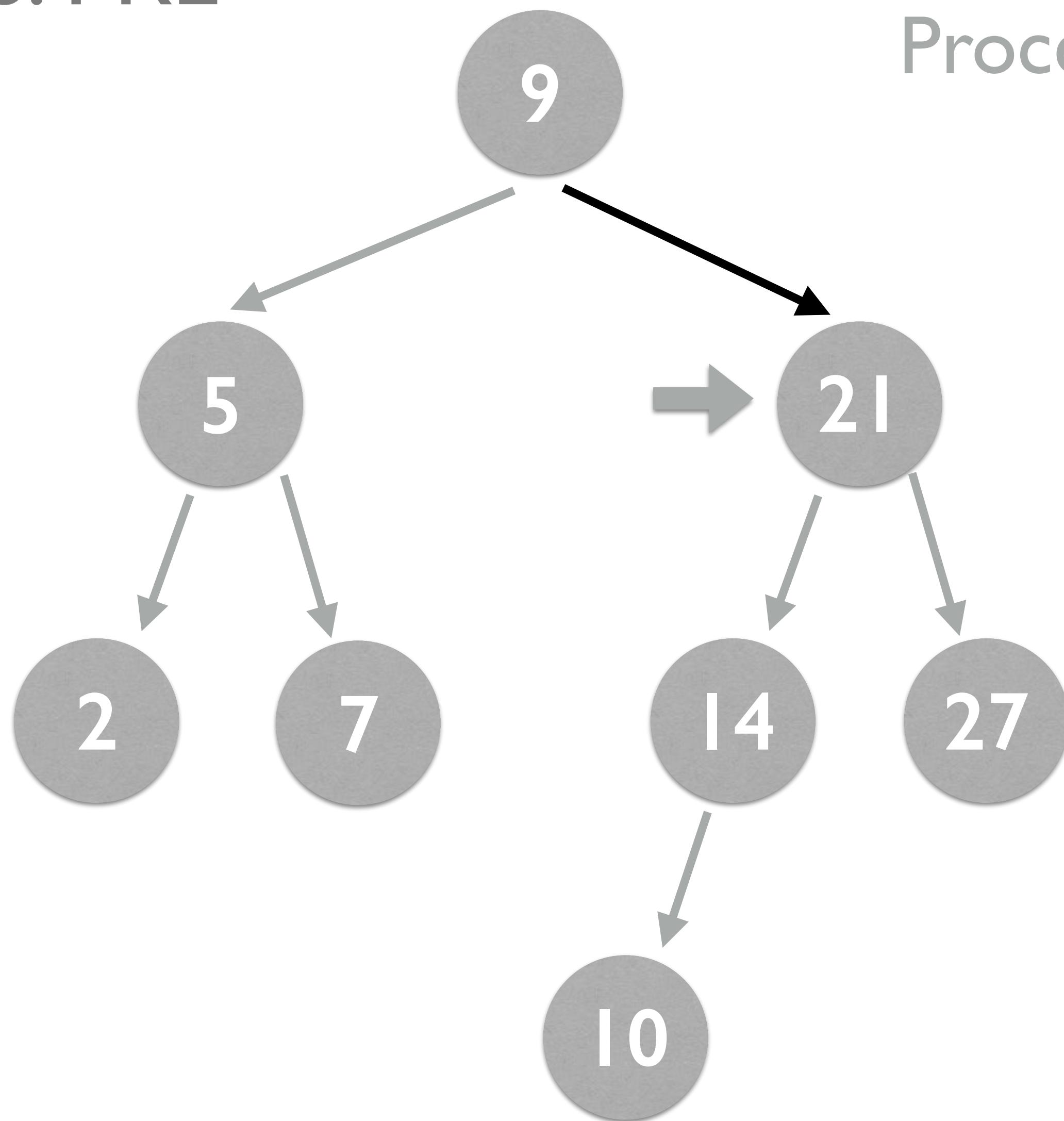
## DFS: PRE



**Process root** · Process left · Process right

9, 5, 2, 7, 21, 14, 10, 27

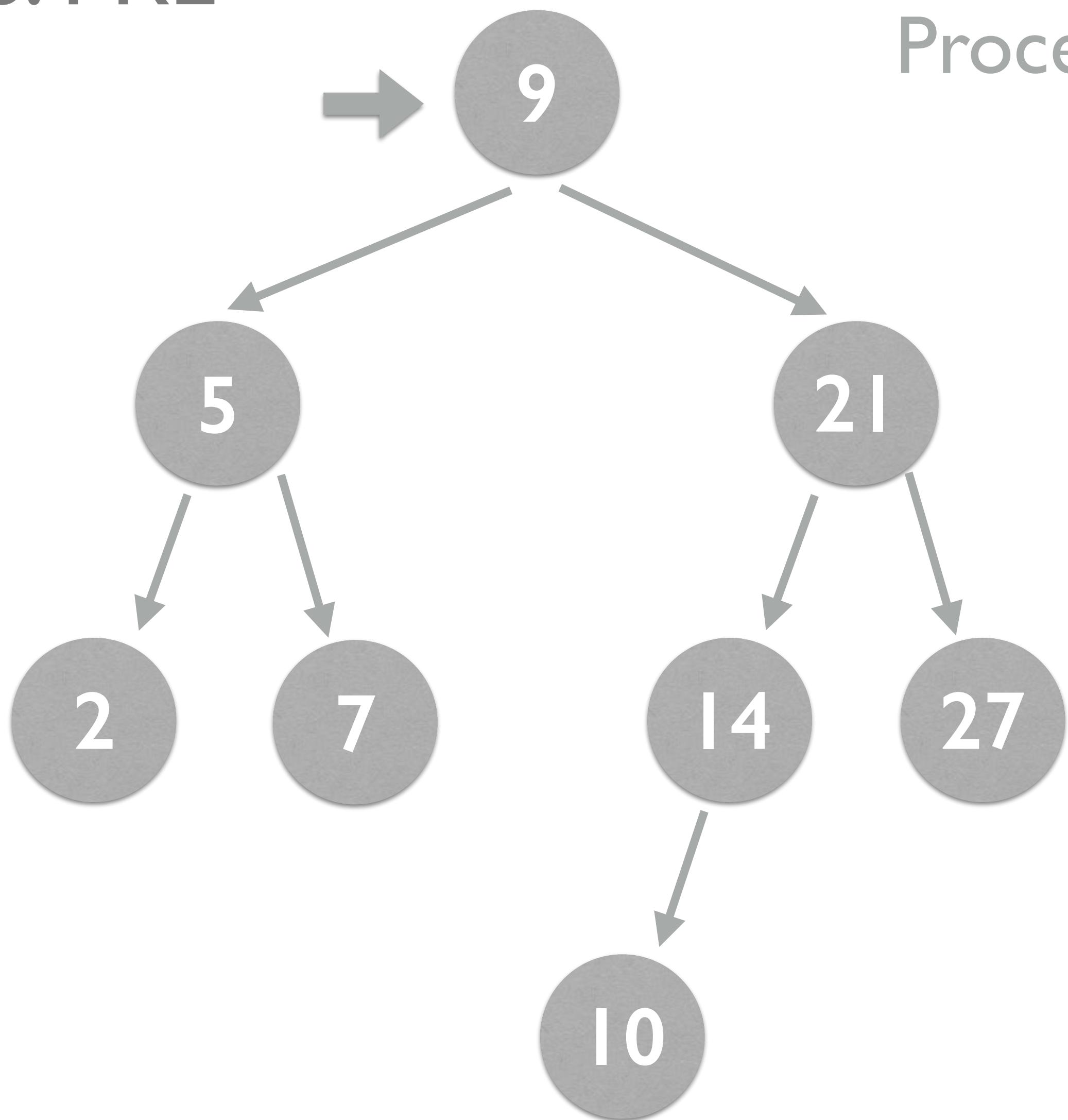
## DFS: PRE



Process root · Process left · Process right

9, 5, 2, 7, 21, 14, 10, 27

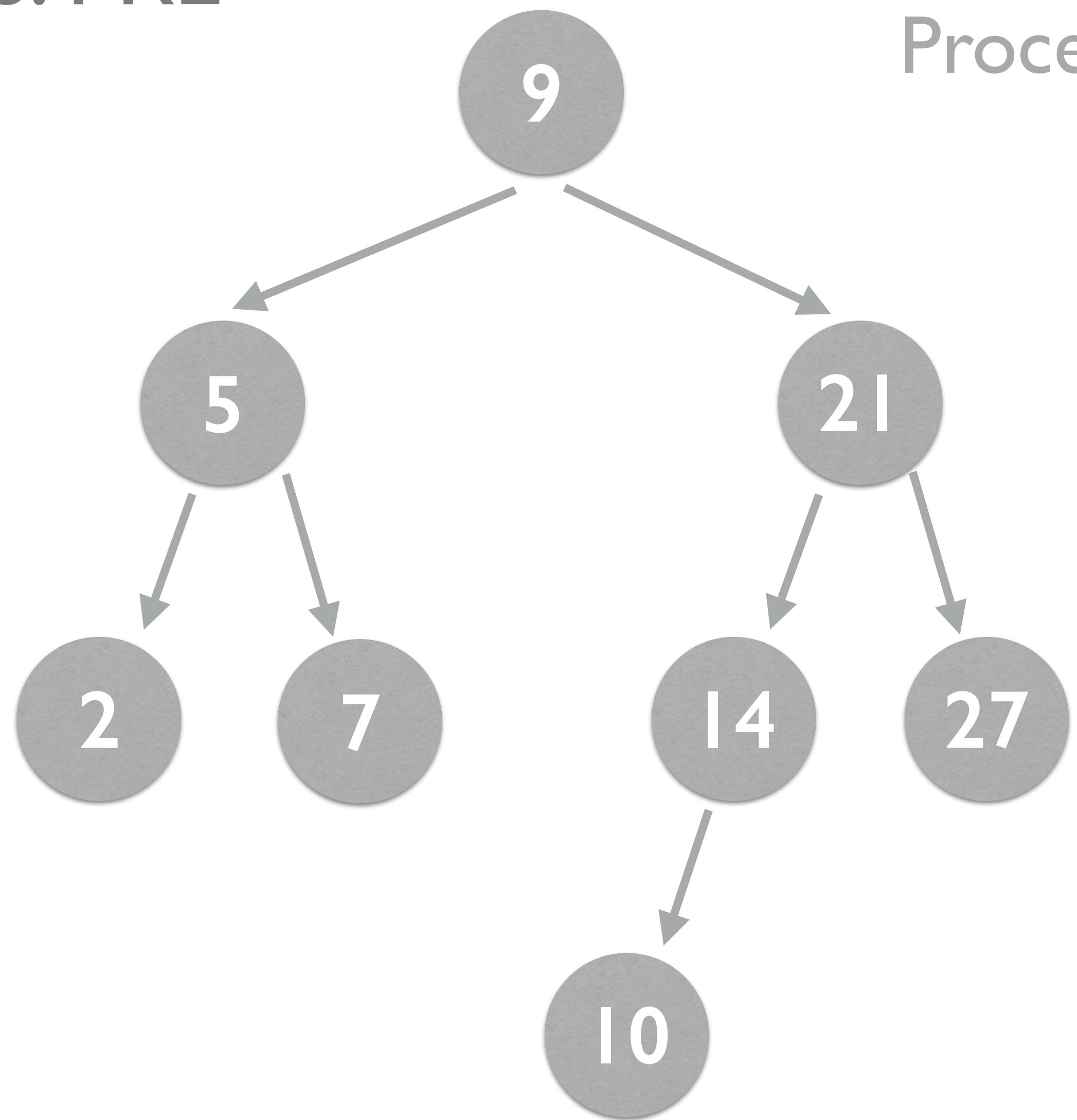
## DFS: PRE



Process root · Process left · Process right

9, 5, 2, 7, 21, 14, 10, 27

## DFS: PRE



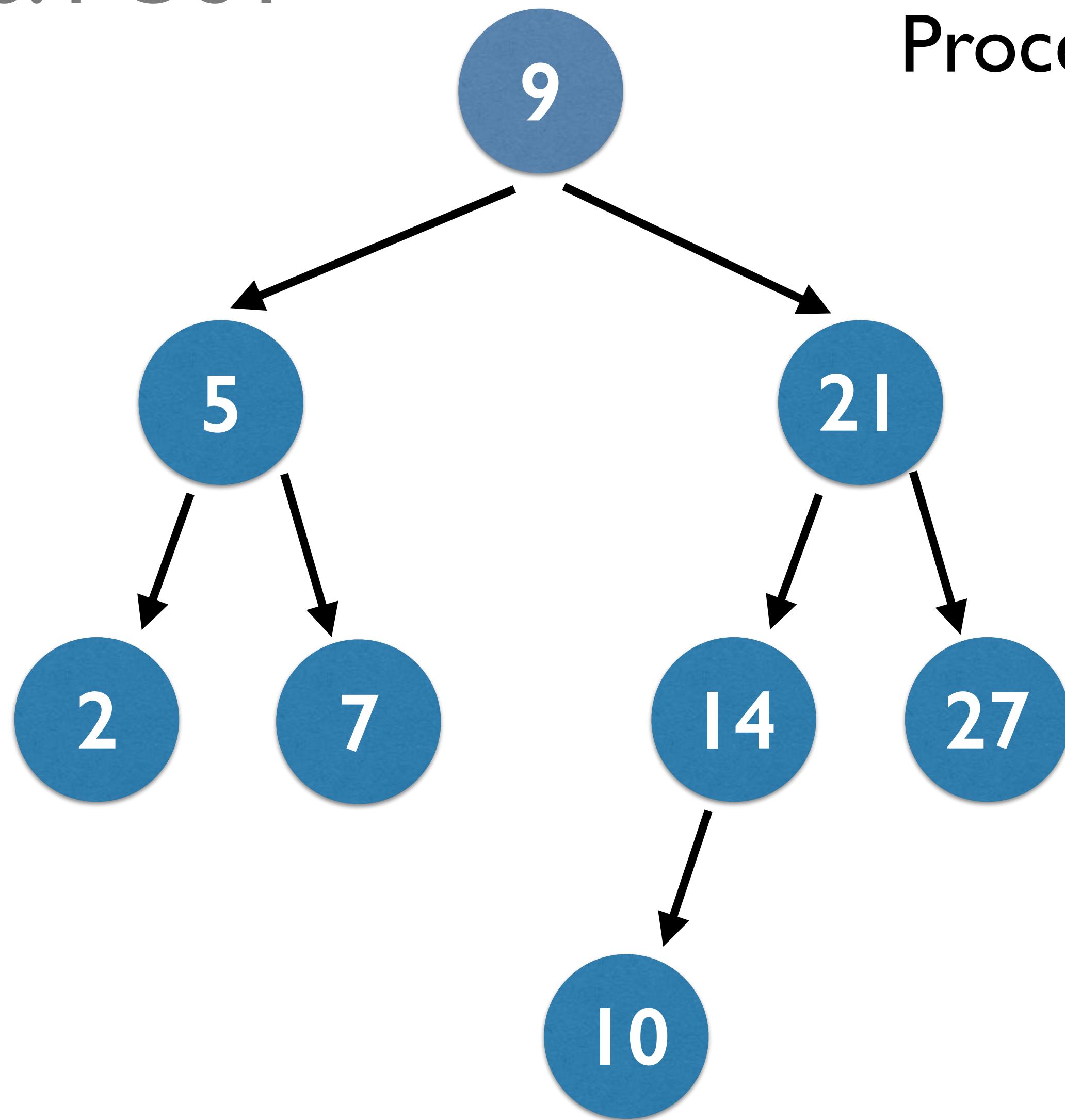
Process root · Process left · Process right

9, 5, 2, 7, 21, 14, 10, 27

- Output seems random, but actually this has one notable use case. If you create a BST by inserting these values in this order, you get a copy of the original tree. However, the same is true for breadth-first.

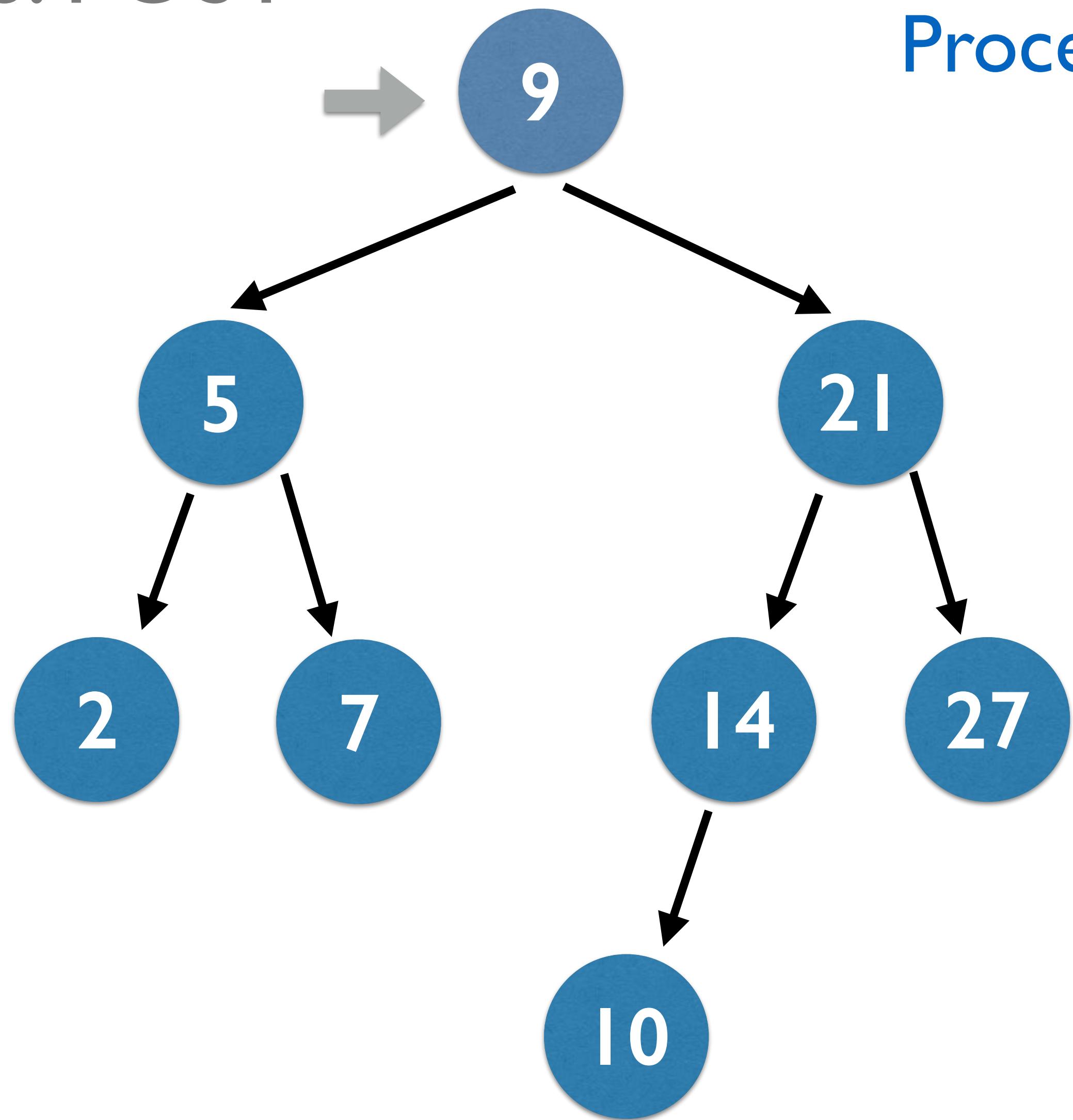
# Depth First: Post-Order

## DFS: POST



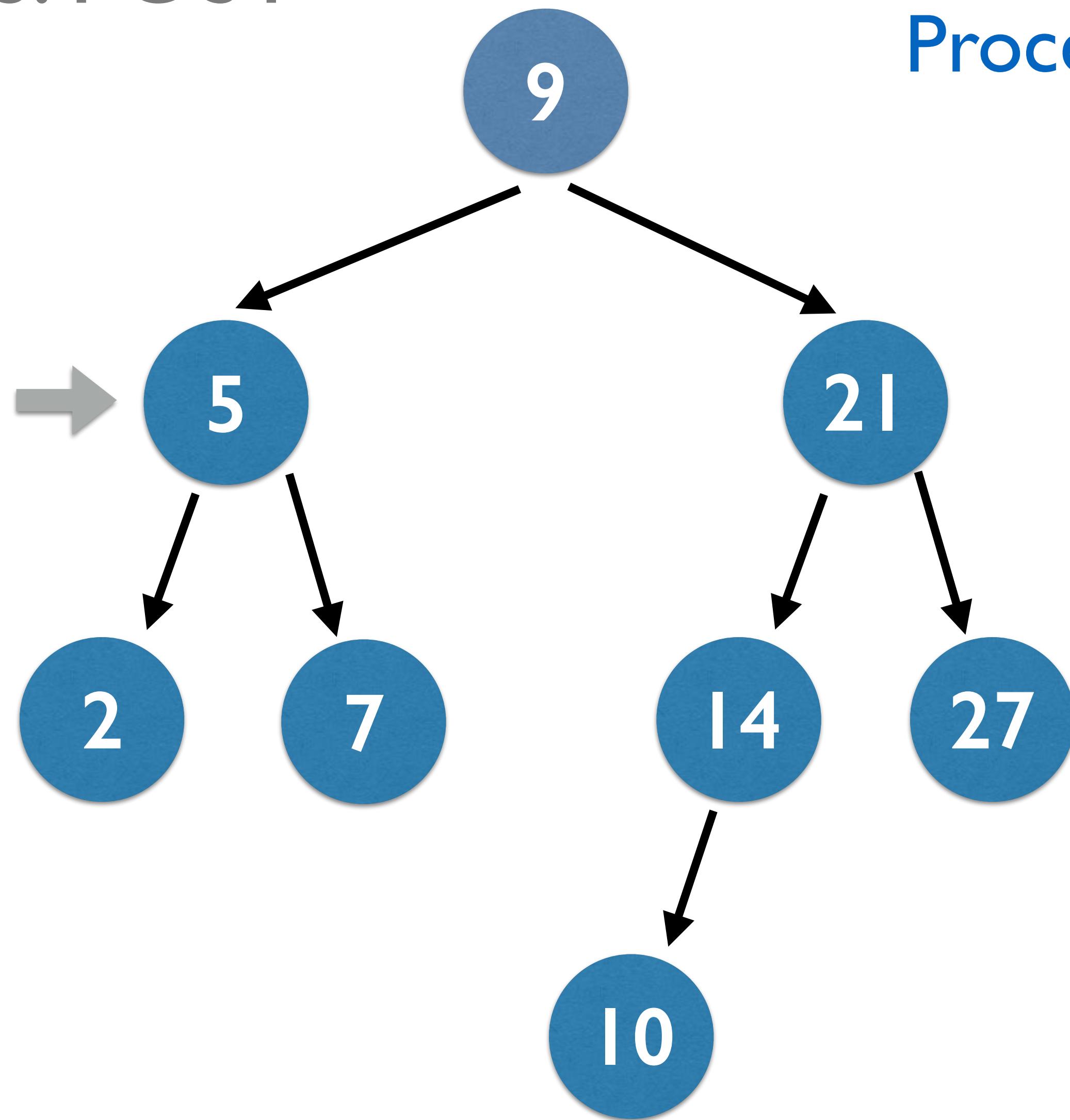
Process left · Process right · Process root

## DFS: POST



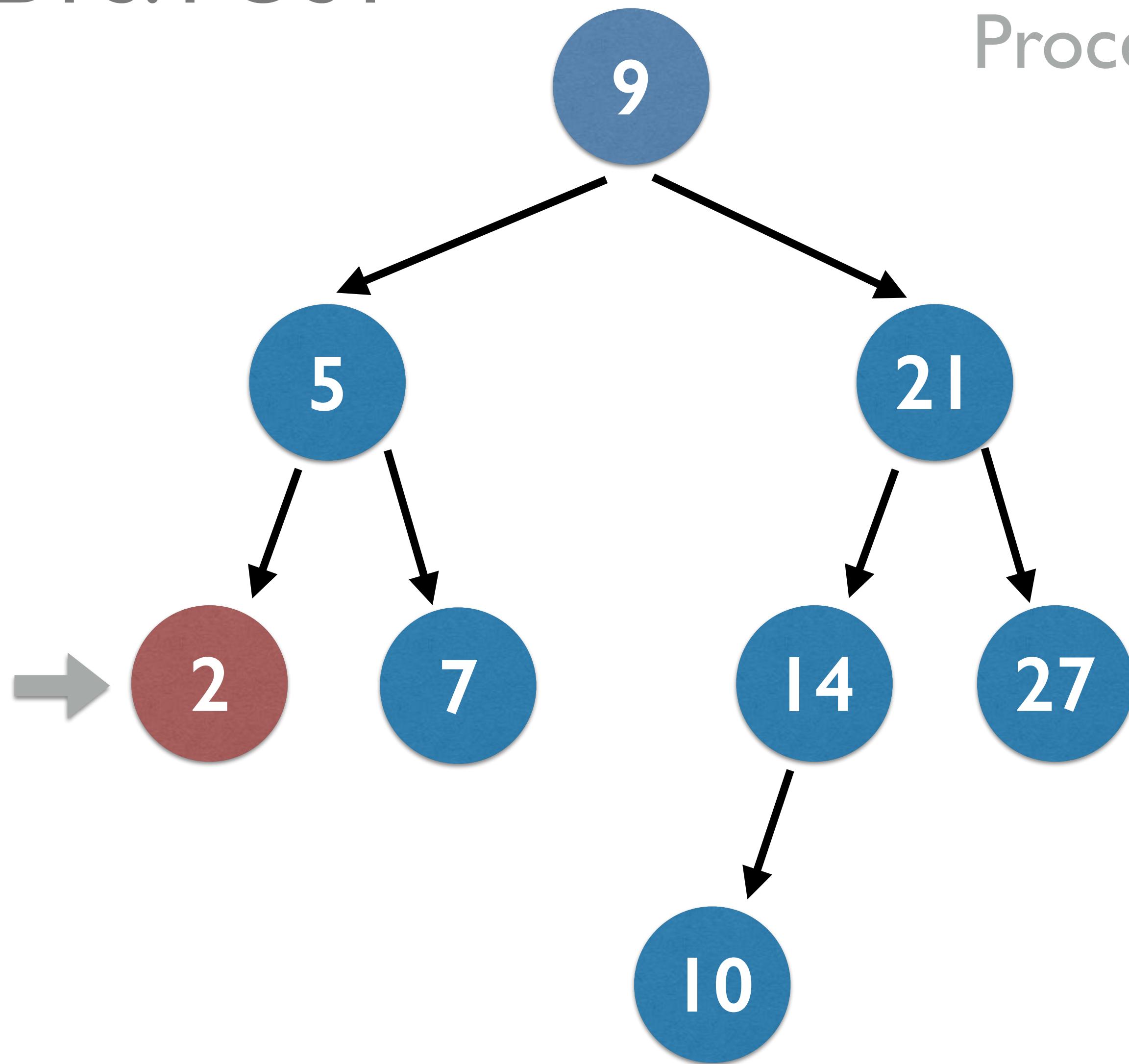
Process left · Process right · Process root

## DFS: POST



Process left · Process right · Process root

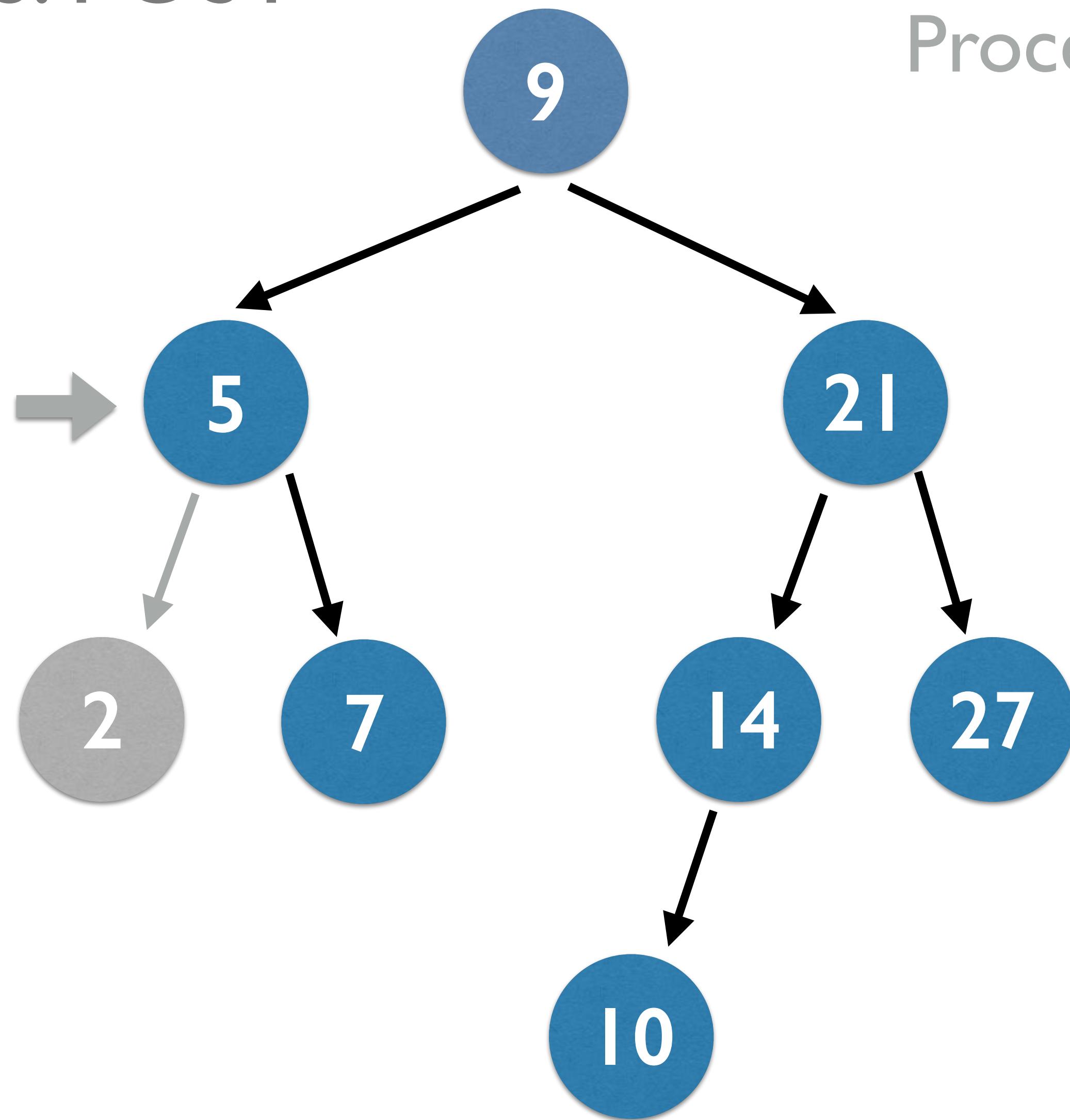
## DFS: POST



Process left · Process right · **Process root**

2

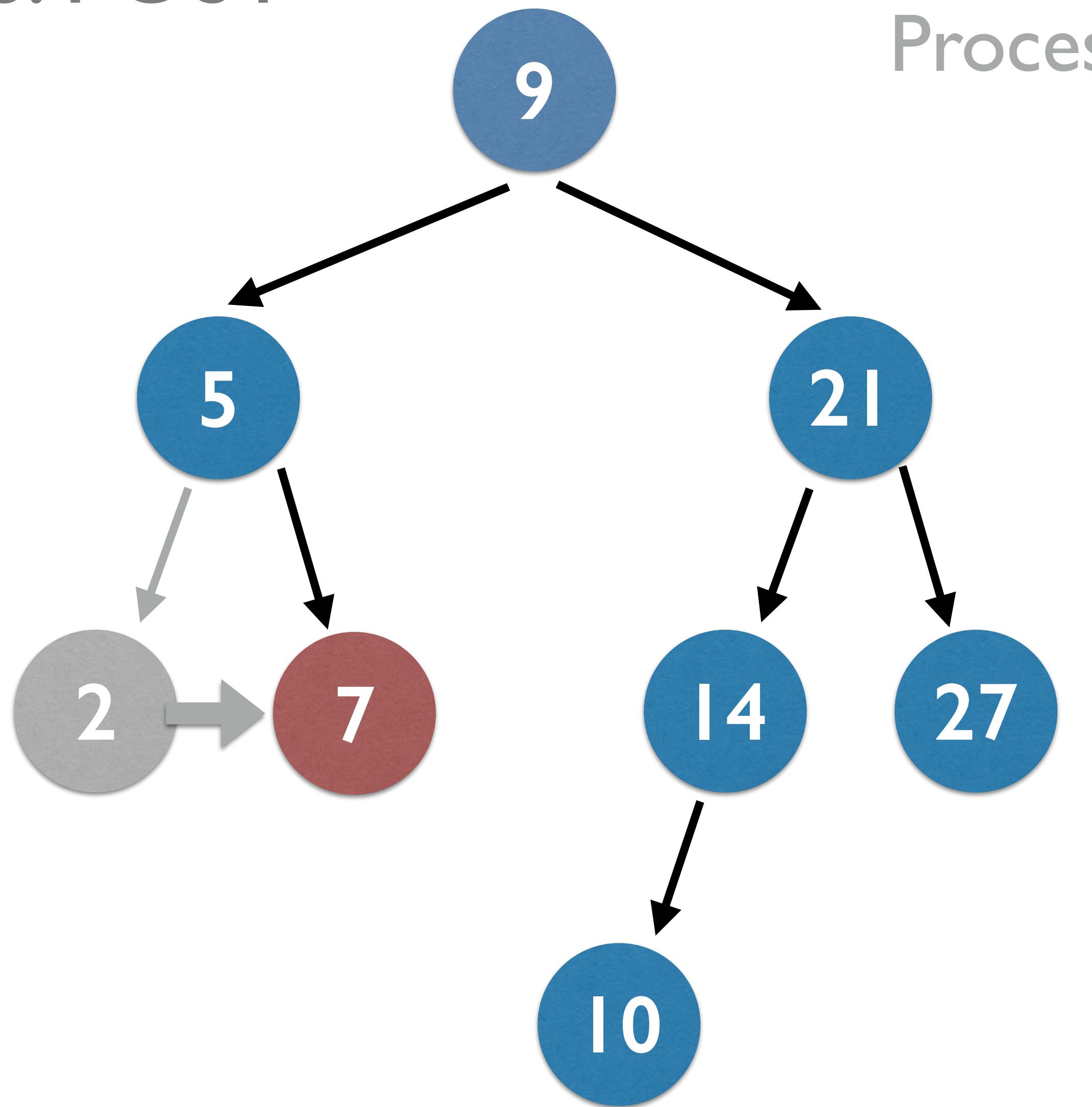
## DFS: POST



Process left · Process right · Process root

2

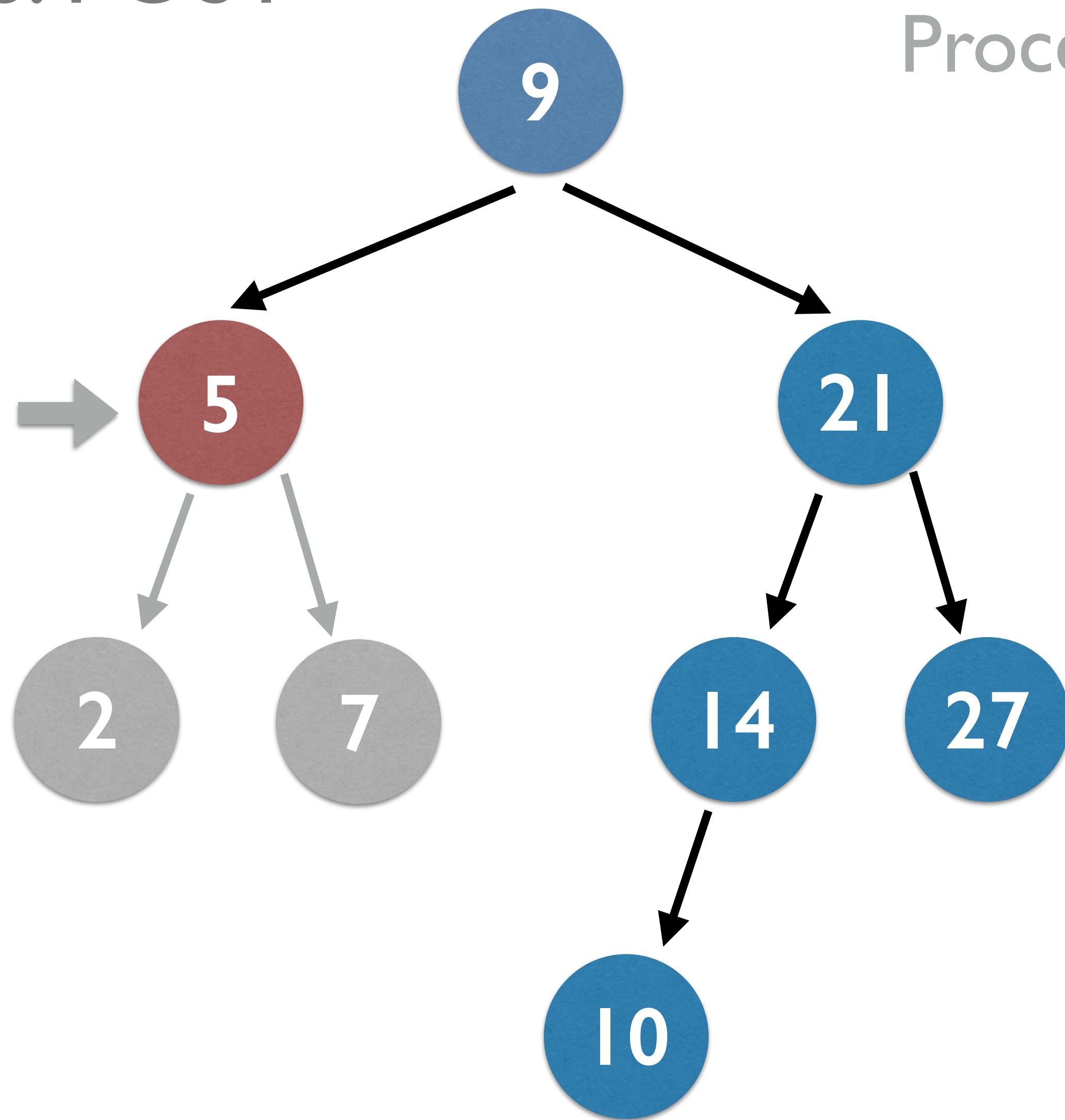
## DFS: POST



Process left · Process right · **Process root**

2, 7

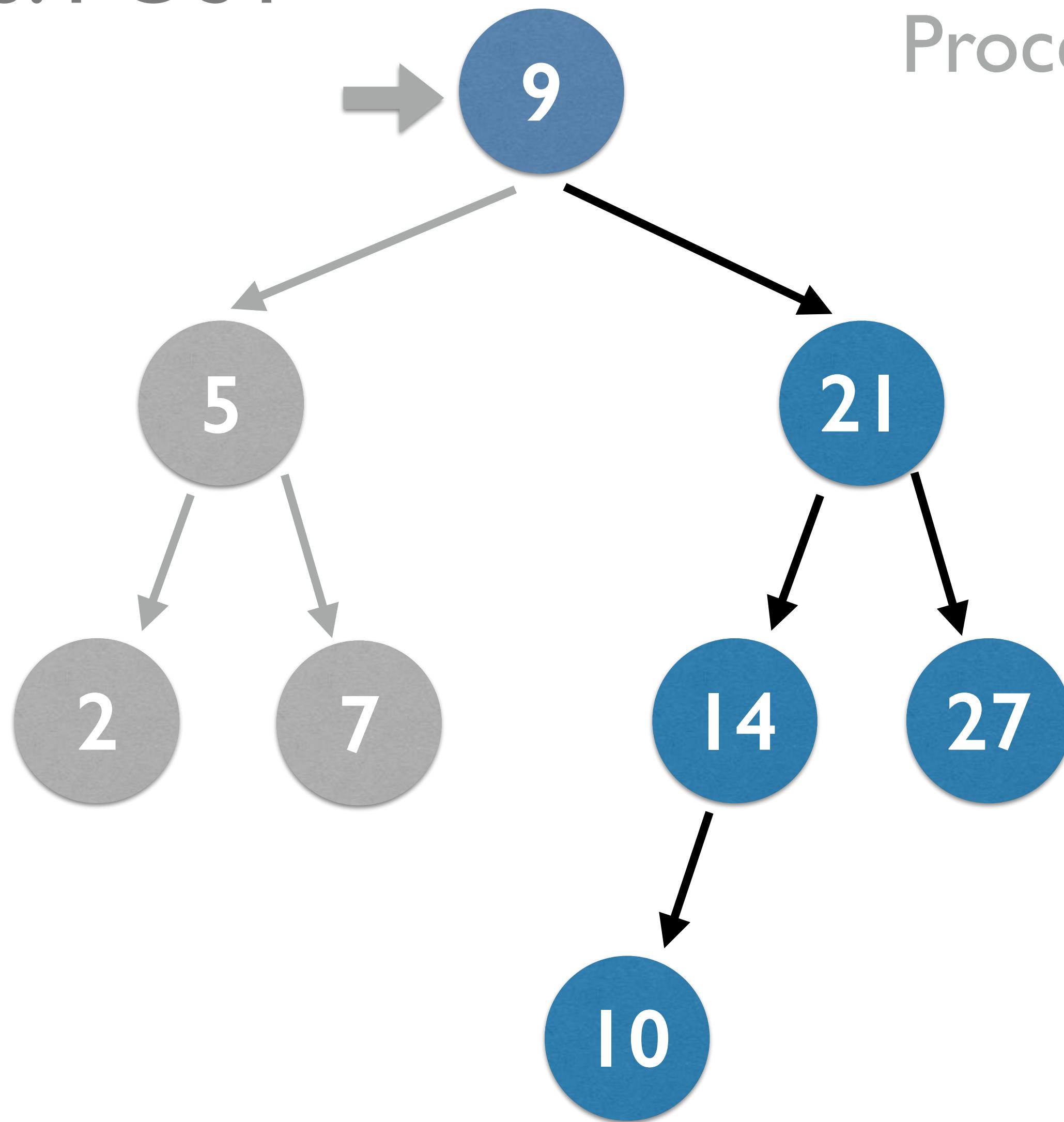
## DFS: POST



Process left · Process right · **Process root**

2, 7, 5

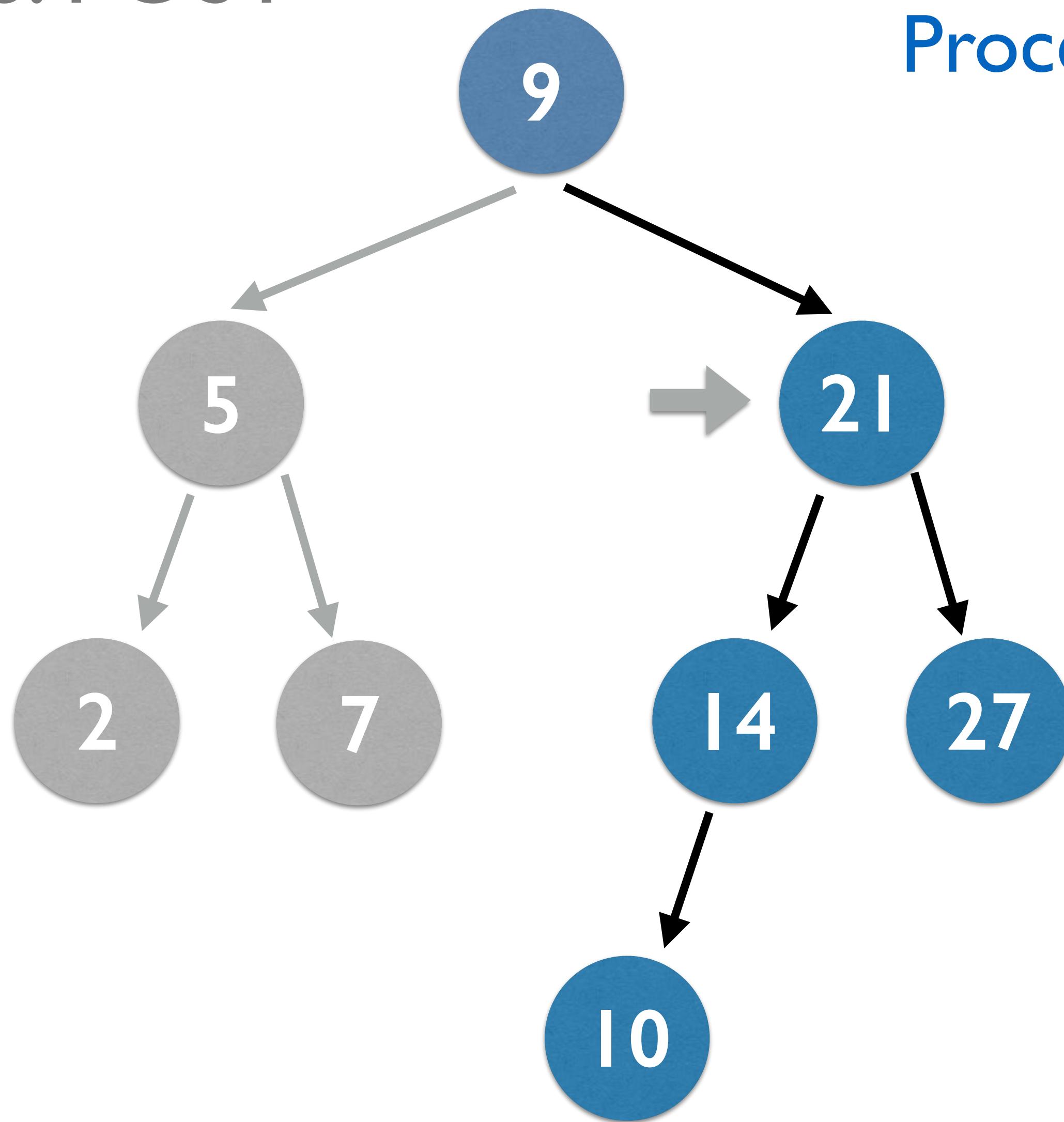
## DFS: POST



Process left • Process right • Process root

2, 7, 5

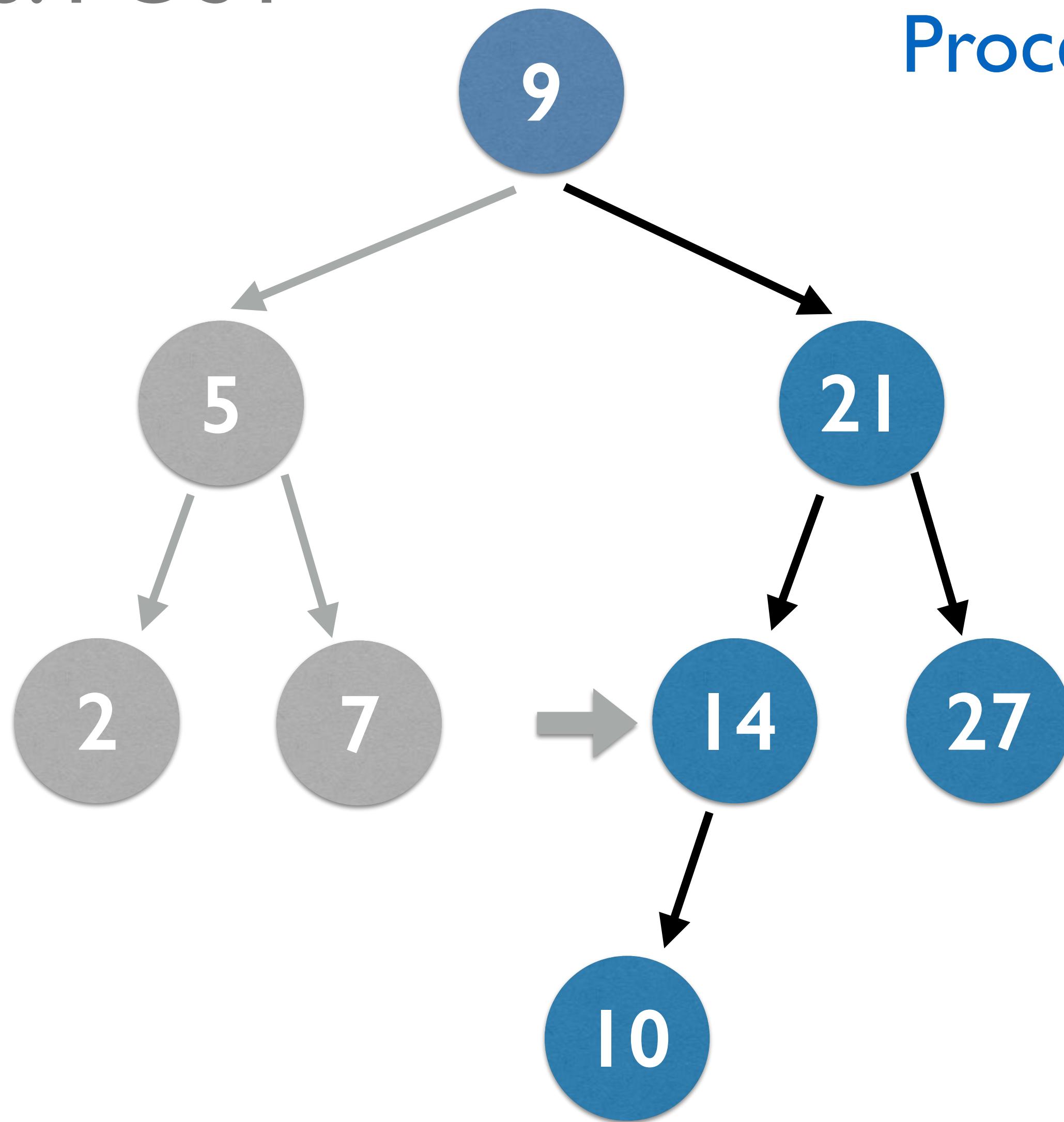
## DFS: POST



Process left · Process right · Process root

2, 7, 5

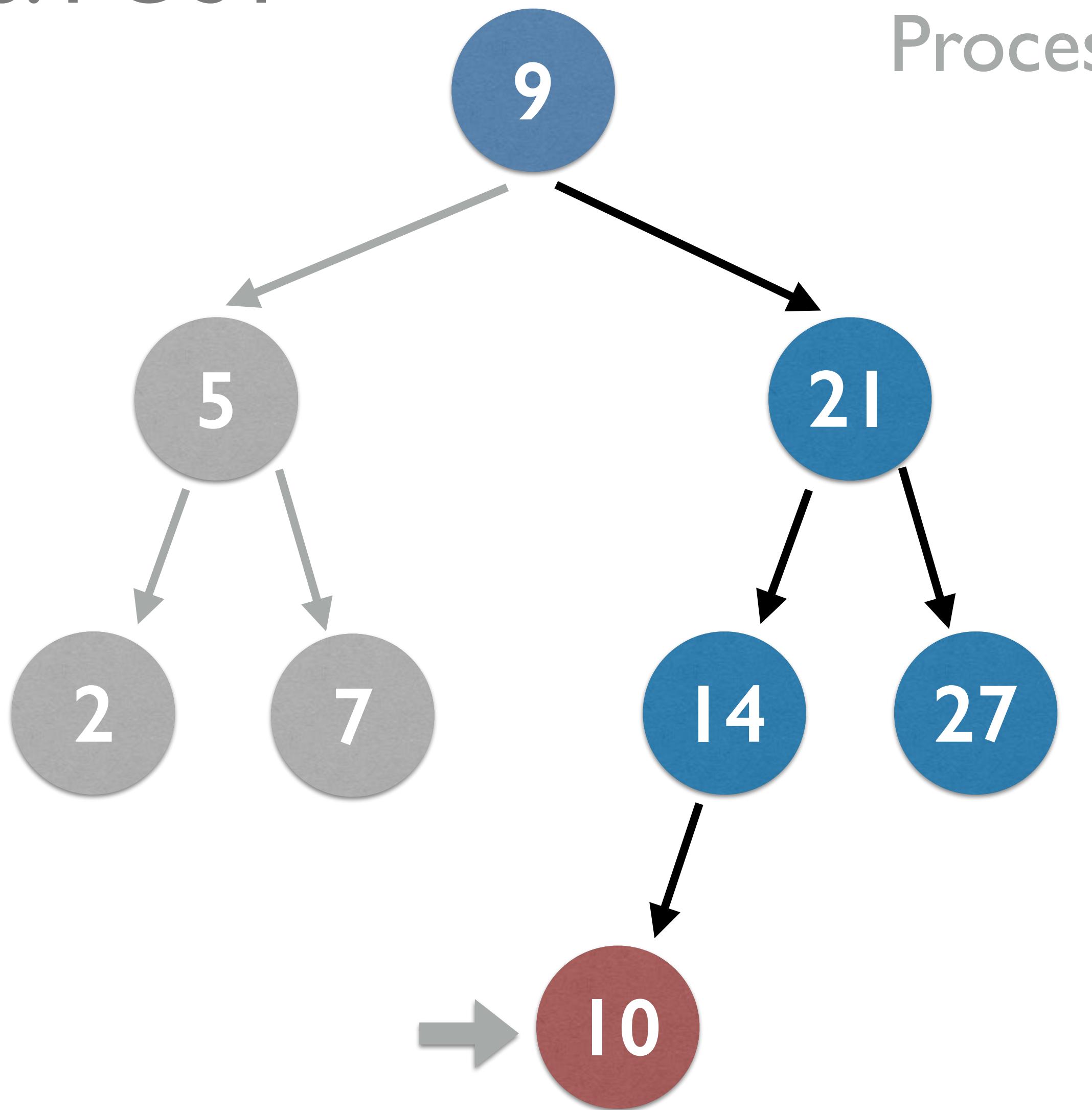
## DFS: POST



Process left · Process right · Process root

2, 7, 5

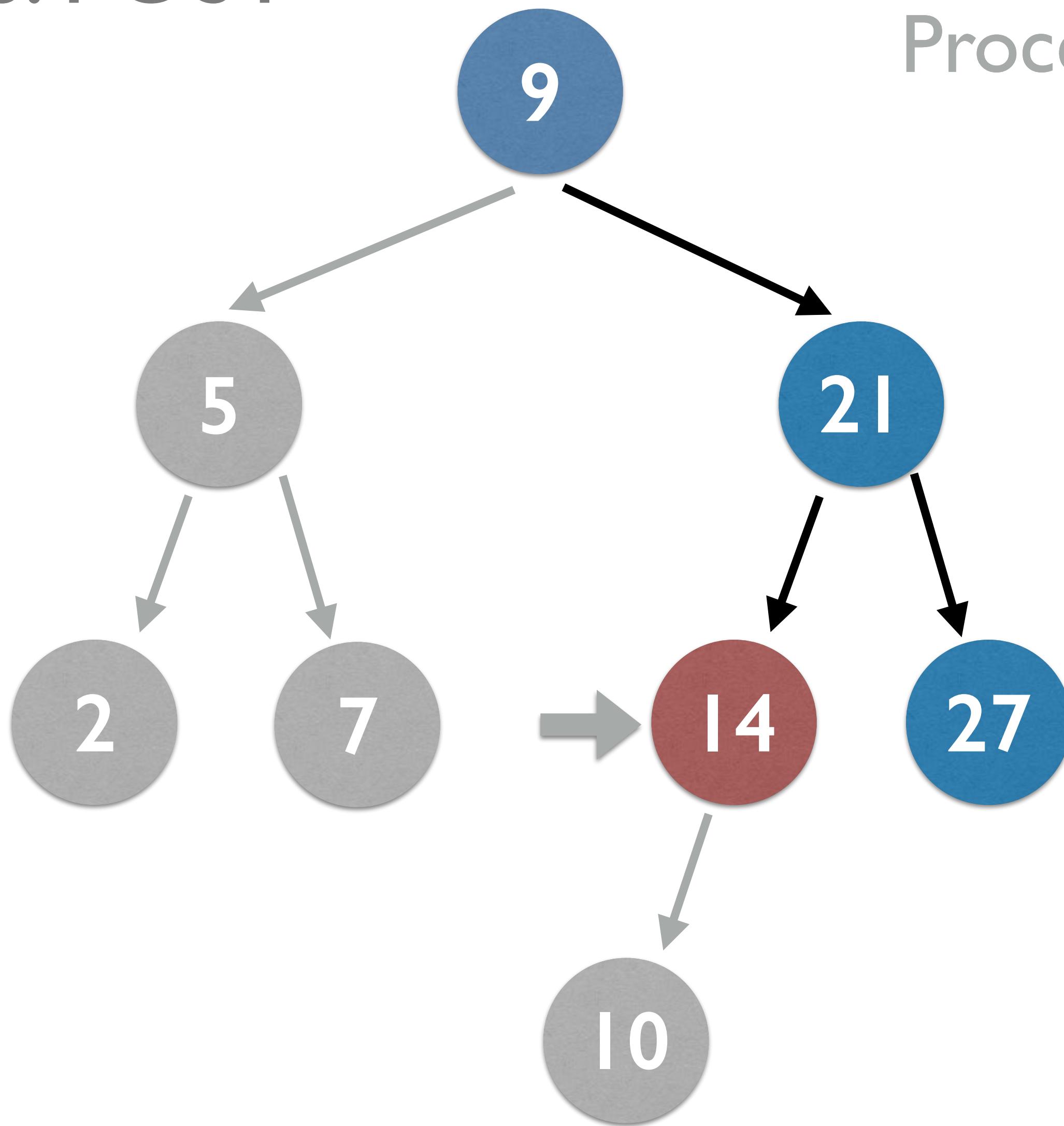
## DFS: POST



Process left · Process right · **Process root**

2, 7, 5, 10

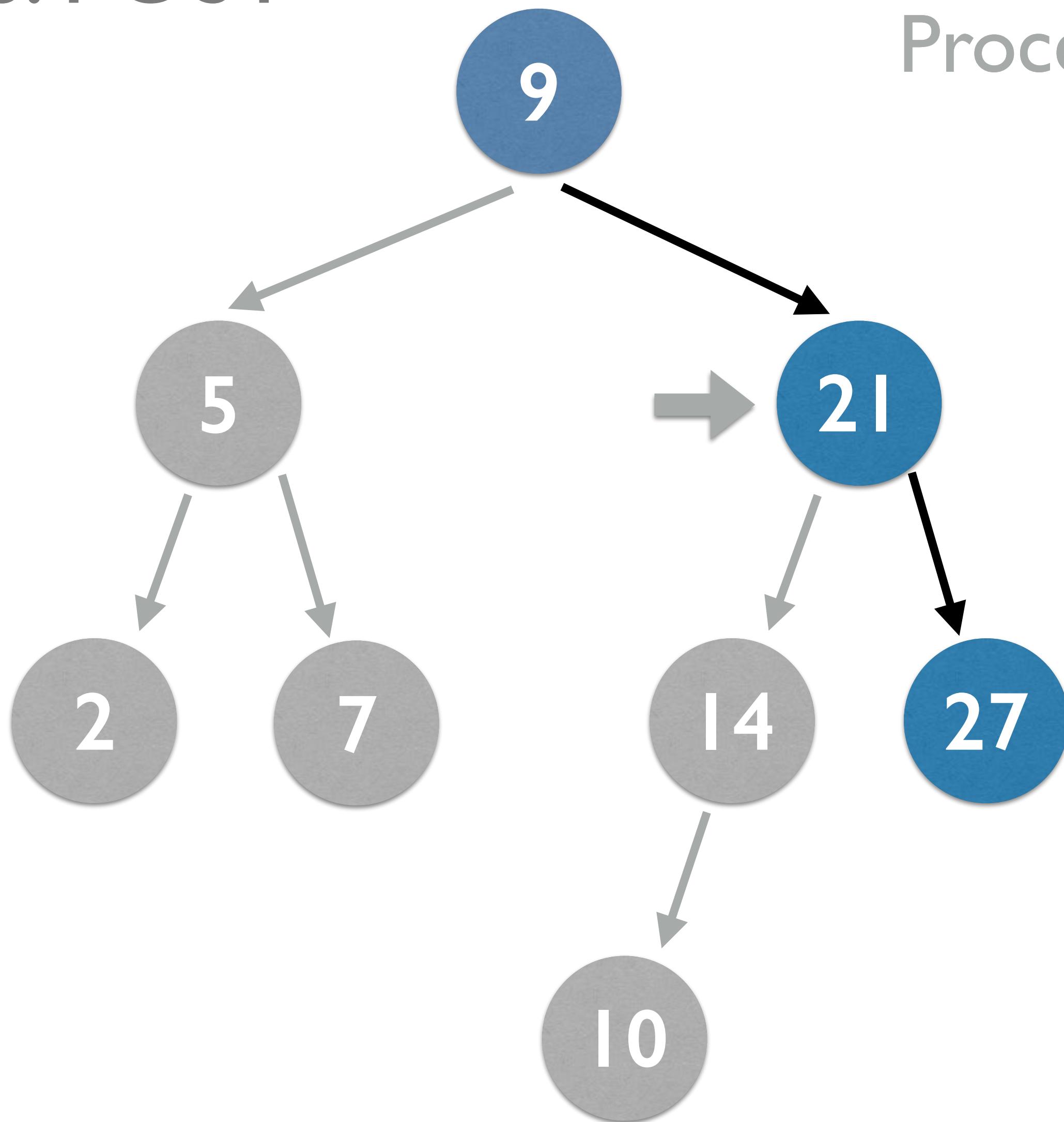
## DFS: POST



Process left · Process right · **Process root**

2, 7, 5, 10, 14

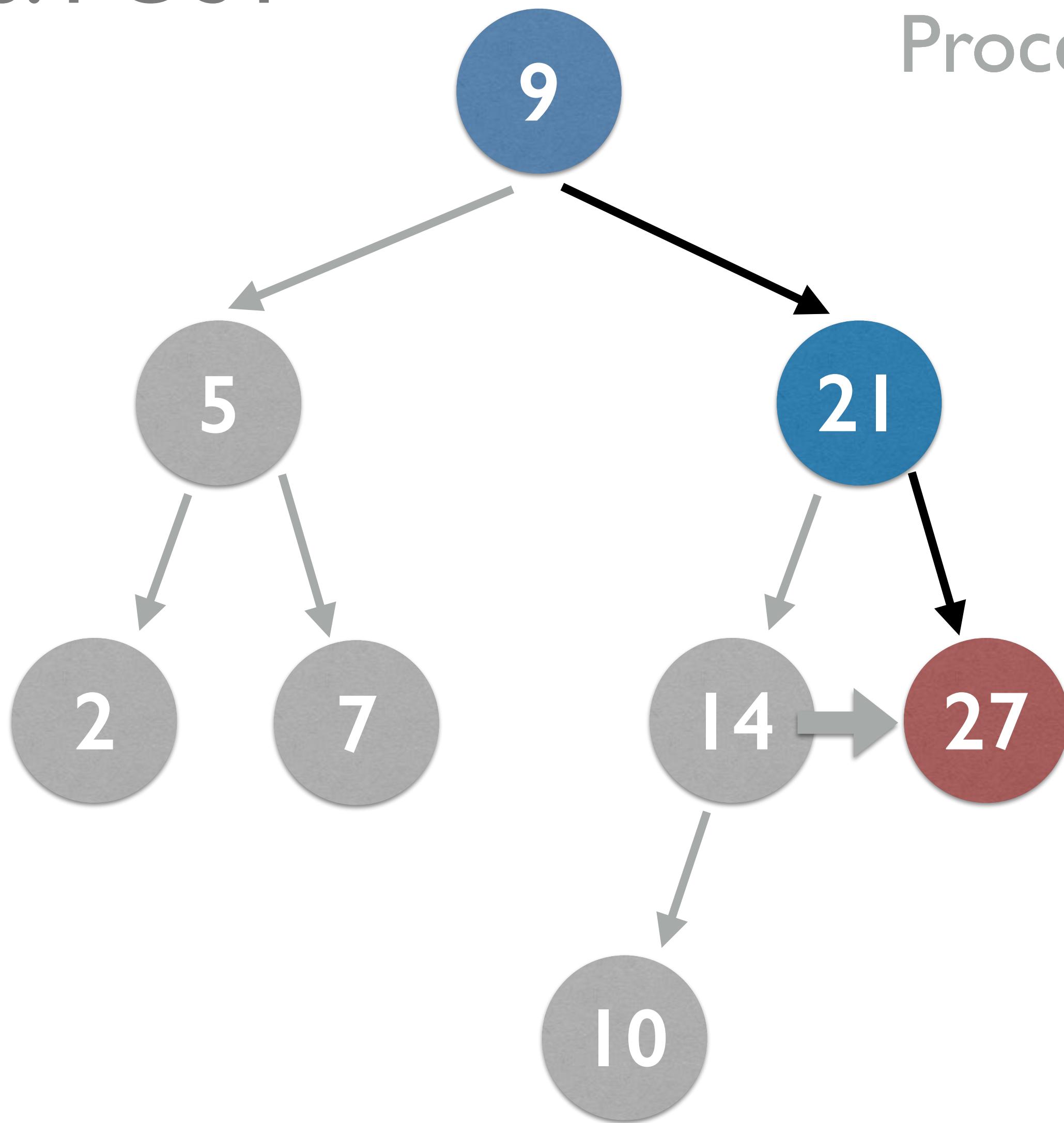
## DFS: POST



Process left · Process right · Process root

2, 7, 5, 10, 14

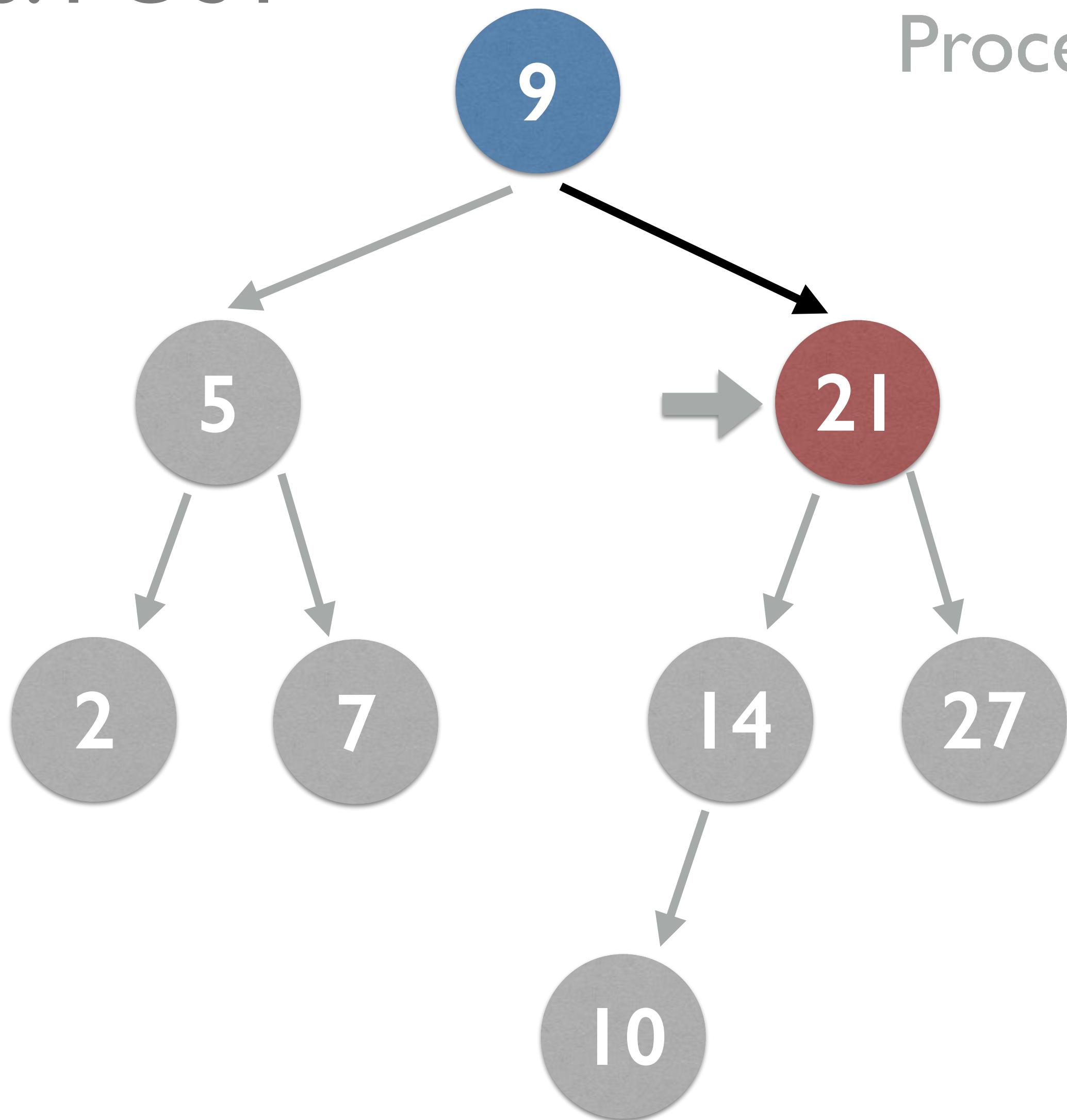
## DFS: POST



Process left · Process right · **Process root**

2, 7, 5, 10, 14, 27

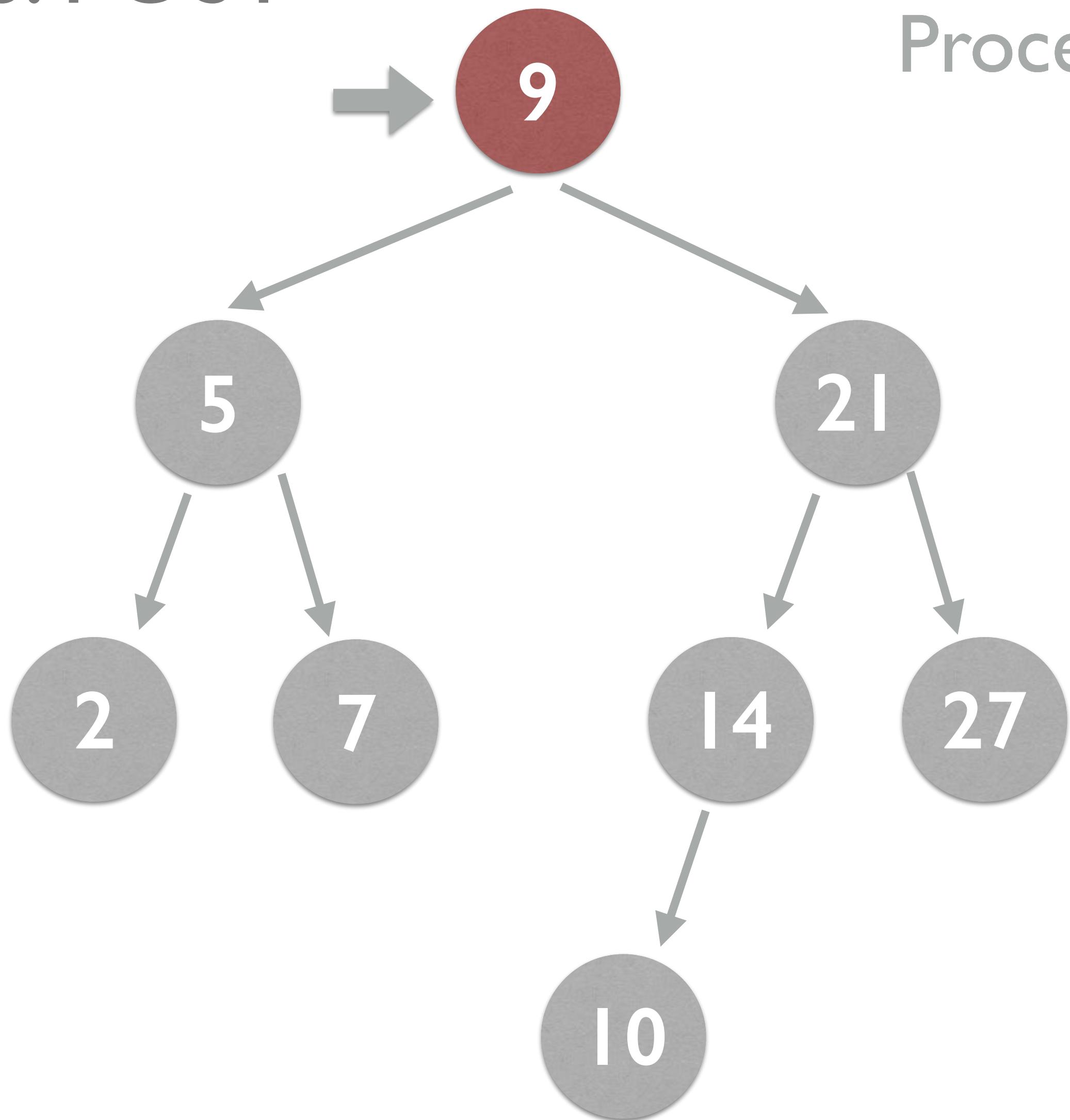
## DFS: POST



Process left · Process right · **Process root**

2, 7, 5, 10, 14, 27, 21

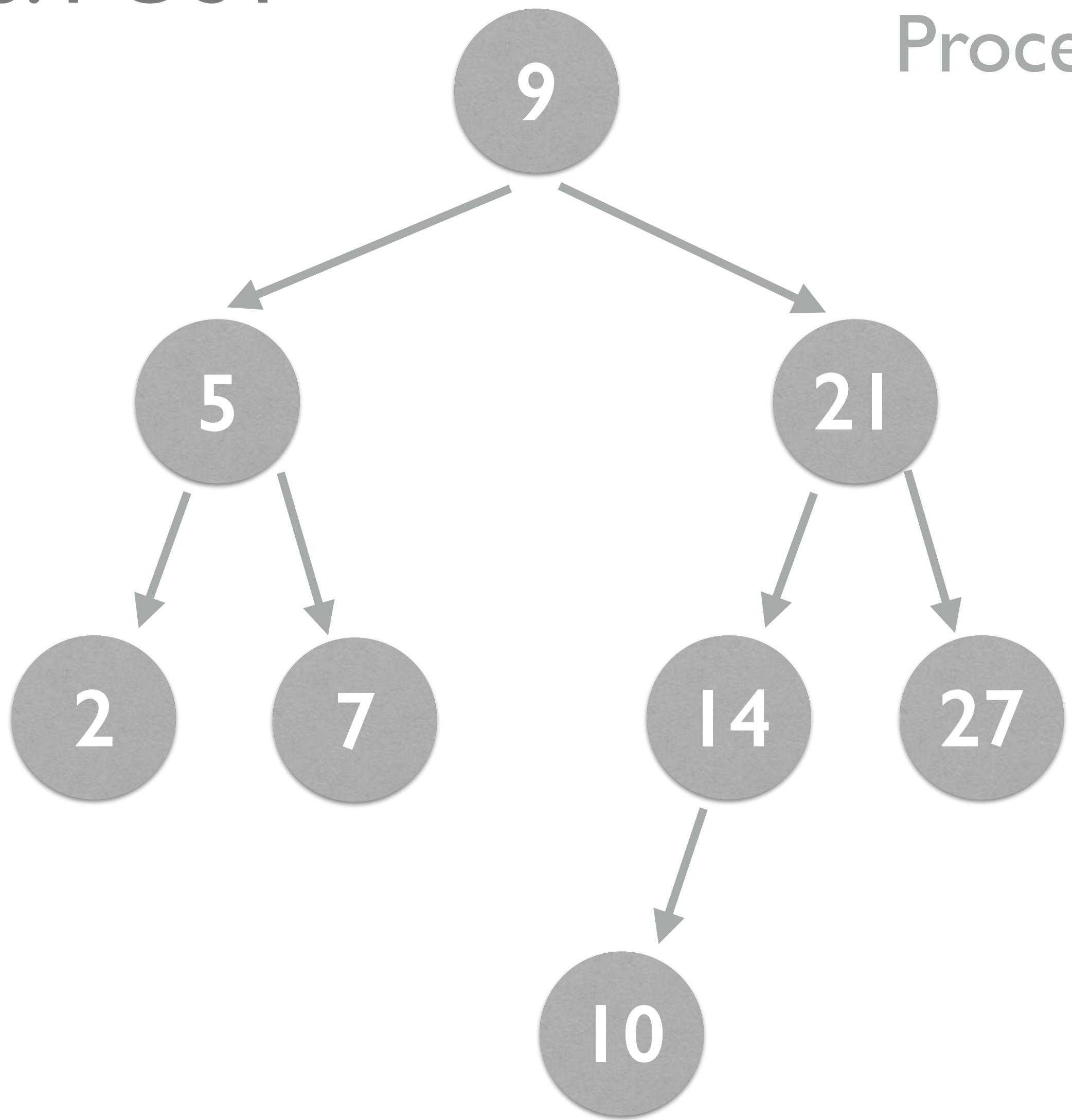
## DFS: POST



Process left · Process right · **Process root**

2, 7, 5, 10, 14, 27, 21, 9

## DFS: POST



Process left · Process right · Process root

2, 7, 5, 10, 14, 27, 21, 9

- ◎ Main use case is to safely delete a tree leaf by leaf, in lower-level languages (e.g. C) with no automatic garbage collection. Nodes are only processed once all their descendants have been processed.

# Recursive Problem Solving

# Recursive Problem Solving

- Any problem that can be solved iteratively can be solved recursively, and vice versa
- The catch: this doesn't mean it's equally easy to solve a particular problem one way or the other
- Binary Search Trees are a recursive data structure - it's much easier to write them if you employ recursive problem solving



# 1. Identify simplest possible input

```
function factorial (n) {  
}  
}
```

## 2. Solve just for the simplest input

```
function factorial (n) {  
    // We know that 1! and 0! are 1 - no calculation needed.  
    // Therefore, we can simply return 1 in those cases  
    if (n === 1 || n === 0) {  
        return 1  
    }  
}
```



### 3. Solve the problem for the second simplest possible input

```
function factorial (n) {  
  if (n === 1 || n === 0) {  
    return 1  
  } else {  
    // we know for sure that we must do two things:  
    // a. invoke the func again with the  
    //     base case (factorial(1) or factorial(0))  
    // b. get our input to factorial by "shrinking" the value of n  
    //     (for example, by subtracting 1)  
  
    return 2 * factorial(n - 1)  
  }  
}
```



## 4. Generalize in terms of input

```
function factorial (n) {  
  if (n === 1 || n === 0) {  
    return 1  
  } else {  
    return n * factorial(n - 1)  
  }  
}
```

# WORKSHOP

