

محاسبات زیر برای طول $l=1$ و $t=1$ در نظر گرفته شده است:

$$P_N = \sum_0^{\inf} \binom{N}{n_r} p^{n_r} q^{n_l}$$

$$\begin{cases} n_r - n_l = x \\ x = 2n_r - N \\ q + p = 1 \end{cases}$$

$$\langle x \rangle = 2 \langle n_r \rangle - N$$

$$\langle n_r \rangle = n_r P_N = \sum_0^{\inf} \binom{N}{n_r} n_r p^{n_r} q^{n_l}$$

$$\langle n_r \rangle = p \left(\frac{\partial}{\partial p} \right) P_N = p \left(\frac{\partial}{\partial p} \right) (p+q)^N = Np (p+q)^{N-1} = Np$$

$$\langle x \rangle = 2 \langle n_r \rangle - N = N(2p-1) = N(p-q) \Rightarrow \langle x \rangle^2 = N^2 (p-q)^2$$

$$\text{if } x = 2n_r - N \Rightarrow \sigma_x^2 = 4\sigma_{nr}^2$$

$$\sigma_{nr}^2 = \langle n_r^2 \rangle - \langle n_r \rangle^2$$

$$\langle n_r^2 \rangle = p \left(\frac{\partial}{\partial p} p \left(\frac{\partial}{\partial p} P_N \right) \right) = p \left(\frac{\partial}{\partial p} Np(p+q)^{N-1} \right) = N^2 p^2 + Npq$$

$$\sigma_{nr}^2 = N^2 p^2 + Npq - N^2 p^2 = Npq \Rightarrow \sigma_x^2 = 4pqN$$

متن بالا با استفاده از LATEX نوشته شده با این سورس کد:

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\begin{equation}
P_N=\sum_{0}^{\infty}\binom{N}{n_r}p^{n_r}q^{n_l}\\
\\
left\{\begin{matrix}\n
n_r-n_l=x
\\
x=2n_r-N \\
\\
q+p=1\\
\\
end{matrix}\right.\\
\\
\\
\\
x\geq 2n_r+N\\
n_r\geq n_rP_N=\sum_{0}^{\infty}\binom{N}{n_r}n_rp^{n_r}q^{n_l}>
n_r\geq p\left(\frac{\partial}{\partial p}\right)P_N=p\left(\frac{\partial}{\partial p}\right)\left(p+q\right)^N=Np\left(p+q\right)^{N-1}=Np>\\
\\
x\geq 2n_r-N=N(2p-1)=N(p-q)\displaystyle \Rightarrow x^2=N^2\left(p-q\right)^2\\
\\
if\ x=2n_r-N\Rightarrow \sigma_x^2=4\sigma_{n_r}^2\\
\sigma_{n_r}^2=<n_r^2>-<n_r>^2\\
\\
\\
n_r^2\geq p\left(\frac{\partial}{\partial p}\right)p\left(\frac{\partial}{\partial p}P_N\right)=p\left(\frac{\partial}{\partial p}Np(p+q)^{N-1}\right)=N^2p^2+Npq>
\\
\\
\sigma_{n_r}^2=N^2p^2+Npq-N^2p^2=Npq\Rightarrow \sigma_x^2=4pqN\end{equation}
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