Question 1:Bayes' on test accuracy

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Question 1: Bayes' Theorem on Test Accuracy

We are given three companies that manufacture personal locator beacons (PLBs), along with the percentage of the PLBs each company produces and the defect rates for their products. The data is as follows:

- Altitude Manufacturing Company: Produces 81% of PLBs with a defect rate of 4%.
- Bright Company: Produces 15% of PLBs with a defect rate of 6%.
- Camping Company: Produces 4% of PLBs with a defect rate of 9.5%.

We are asked to solve two parts:

1. (a) What is the probability that a randomly selected PLB was made by Altitude Manufacturing Company?
2. (b) If a randomly selected PLB is found to be defective, what is the probability that it was made by Altitude Manufacturing Company?

Part (a) - Probability of a PLB from Altitude Manufacturing

In this part, we are asked to find the probability that a randomly selected PLB was made by Altitude Manufacturing Company. This is a simple probability question. The probability that a randomly selected PLB comes from Altitude Manufacturing is the percentage of PLBs produced by that company.

The given data tells us that 81% of the PLBs are made by Altitude Manufacturing, so the probability is:

$$P(A) = 0.81$$

Thus, the probability is:

$$P(A) = 0.81 \quad \text{or} \quad 81\%$$

Part (b) - Probability That a Defective PLB is from Altitude Manufacturing

In this part, we are asked to find the probability that a defective PLB was made by Altitude Manufacturing Company. To solve this, we will use Bayes' Theorem.

Step 1: Bayes' Theorem

Bayes' Theorem is used to calculate the conditional probability of one event given that another event has occurred. The general form of Bayes' Theorem is:

$$P(A|D) = \frac{P(D|A) \cdot P(A)}{P(D)}$$

Where:

- $P(A|D)$ is the probability that a defective PLB (D) was made by Altitude Manufacturing (A).
- $P(D|A)$ is the probability that a PLB made by Altitude Manufacturing is defective. This is the defect rate for Altitude Manufacturing, which is 4% or 0.04.
- $P(A)$ is the probability that a PLB was made by Altitude Manufacturing, which we calculated in part (a) as 0.81.
- $P(D)$ is the total probability that a randomly selected PLB is defective, regardless of which company made it. We need to calculate this using the Law of Total Probability.

Step 2: Calculate $P(D)$ (Total Probability of Defect)

The total probability $P(D)$ of selecting a defective PLB can be found using the Law of Total Probability. Since the PLBs come from three different manufacturers, we must account for the defect rates of each company. The formula for $P(D)$ is:

$$P(D) = P(D|A) \cdot P(A) + P(D|B) \cdot P(B) + P(D|C) \cdot P(C)$$

Where:

- $P(D|A) = 0.04$ is the probability that a PLB from Altitude Manufacturing is defective.
- $P(A) = 0.81$ is the probability that a PLB comes from Altitude Manufacturing.
- $P(D|B) = 0.06$ is the probability that a PLB from Bright Company is defective.

- $P(B) = 0.15$ is the probability that a PLB comes from Bright Company.
- $P(D|C) = 0.095$ is the probability that a PLB from Camping Company is defective.
- $P(C) = 0.04$ is the probability that a PLB comes from Camping Company.

Substituting these values into the formula for $P(D)$:

$$P(D) = (0.04 \times 0.81) + (0.06 \times 0.15) + (0.095 \times 0.04)$$

Now, calculate each term:

$$P(D) = 0.0324 + 0.009 + 0.0038$$

Summing these values:

$$P(D) = 0.0452$$

Thus, the total probability that a randomly selected PLB is defective is:

$$P(D) = 0.0452$$

Step 3: Apply Bayes' Theorem

Now that we have all the necessary values, we can apply Bayes' Theorem to find $P(A|D)$, the probability that a defective PLB was made by Altitude Manufacturing:

$$P(A|D) = \frac{P(D|A) \cdot P(A)}{P(D)} = \frac{0.04 \times 0.81}{0.0452}$$

Performing the multiplication and division:

$$P(A|D) = \frac{0.0324}{0.0452} \approx 0.7168$$

Thus, the probability that a defective PLB was made by Altitude Manufacturing is approximately:

$$P(A|D) \approx 0.717 \quad \text{or} \quad 71.68\%$$

Conclusion

In conclusion, the probability that a randomly selected PLB was made by Altitude Manufacturing is 81%, and if the PLB is found to be defective, the probability that it was made by Altitude Manufacturing is approximately 71.68%.