Example

$$p(A|B) = \frac{p(A \cap B)}{p(B)}$$

Calculate the probability of obtaining 8 as the sum of two rolls of a die, given that the first roll was a 3.

IMPORTANT: there is only 1 die that is thrown twice.

A = {The sum of two rolls is 8} $\Rightarrow$ $\overset{1/36}{(2,6)}, \boxed{\overset{1/36}{(3,5)}}, \overset{1/36}{(4,4)}, \overset{1/36}{(5,3)}, \overset{1/36}{(6,2)}$

B = {The first roll gives a 3} $\Rightarrow$ $p(B) = 1/6$

And what is $p(A)$?

$$p(A|B) = \frac{p(A \cap B)}{p(B)} = \frac{1/36}{1/6} = \frac{1}{6}$$

$$p(A) = \frac{n_A}{N} = \frac{5}{36}$$

Example

In a deck of 52 cards where two cards are being drawn, then let's consider the events be

A: Drawing a red card on the first draw, and

B: Drawing a red card on the second draw.

The conditional probability of drawing a red card on the second draw (B) given that we drew a red card on the first draw (A) is $p(B|A)$.

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After drawing a red card on the first draw, there are 25 red cards and 51 cards remaining in the deck. So, $p(B|A)$ = 25/51 ≈ 0.49 (approximately 49%).

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Applying the conditional probability formula:

$$p(truck|red) = \frac{p(truck \cap red)}{p(red)} = \frac{2/40}{18/40} = \frac{1}{9} \approx 0.11$$

Example (two-way table = table containing information of two categorical variables)

A survey asked full-time and part-time staff in a company how often they had visited the shared library in the last month. The results are shown below.

	One or fewer times	2-3 times	4 times or more	Total
Full-time	12	25	7	44
Part-time	2	5	5	12
Total	14	30	12	56

What is the probability the staff visited the library four or more times, given that the staff is full-time?

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$$p(A|B) = \frac{p(A \cap B)}{p(B)}$$

$$p(4\ or\ more|full\ time) = \frac{7}{44} \approx 0.159$$

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If the staff visited the library four or more times, what is the probability the staff is part-time?

$$p(A|B) = \frac{p(A \cap B)}{p(B)}$$

$$p(part\ time|4\ or\ more) = \frac{5}{12} \approx 0.42$$

$$p(A|B) = \frac{p(A \cap B)}{p(B)} \qquad\qquad p(B|A) = \frac{p(B \cap A)}{p(A)}$$

But given the commutative property of the intersection of two sets, $A \cap B = B \cap A$:

$$\boxed{p(A|B) = \frac{p(B|A)p(A)}{p(B)}}$$

Bayes' Theorem!

$p(A) \neq 0 \qquad p(B) \neq 0$

H = hypothesis
E = evidence

Likelihood: how probable is the evidence given that the hypothesis is true?

Prior: how probable is our hypothesis before any evidence is presented? Note that this is also a marginal probability!

$$\boxed{p(H|E) = \frac{p(E|H)p(H)}{p(E)}}$$

Posterior: how probable is our hypothesis given the observed evidence?

Marginal probability of the evidence (considering all possible hypotheses)

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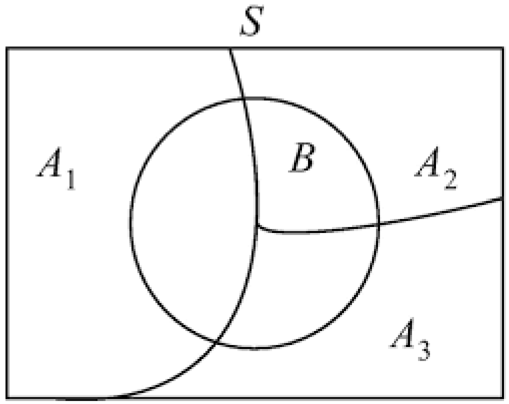

This can be generalized for a partition $\{A_i\}$

$$\boxed{p(A_i|B) = \frac{p(B|A_i)p(A_i)}{\sum_j p(B|A_j)p(A_j)}}$$

Law of total probability!