
QUESTION 1

(a) If $p > 0$ and $q > 0$ with $p + q = 1$ show that $|p + qe^{i\alpha}| < 1$ for any $0 < \alpha < \pi/2$. [6 marks]

(b) State without proof the theorem that gives the converse of the Cauchy-Riemann equations. [4 marks]

$\frac{1}{z-b}$

(c) Given that $u(x, y) = 2y^3 - 6x^2y$ is harmonic, find a harmonic conjugate $v(x, y)$ for it. [6 Marks]

(d) Let $a \in \mathbb{C}$, $r > 0$ and n an integer. Evaluate $\int_{C(a,r)}(z - a)^n dz$. [10 marks]

TOTAL MARKS FOR QUESTION 1 = 26 MARKS

$\frac{1}{z-b}$

QUESTION 2

(a) State Cauchy's Integral Theorem Revisited. [3 Marks]

(b) State and prove Cauchy's Integral formula. [10 Marks]

(c) Evaluate the contour integral

$$\int_{C(0,1)} \frac{\exp(z)}{z^2 - z + 2} dz.$$

[6 marks]

(d) Evaluate the contour integral

$$\int_{C(0,2)} \frac{\exp(z)}{z^2 - 2z - 3} dz.$$

[5 marks]

TOTAL MARKS FOR QUESTION 2 = 24 MARKS

QUESTION 1

(a) State without proof the theorem that gives the converse of the Cauchy-Riemann equations.

[4 marks]

(b)(i) Let $f(x+iy) = u(x,y) + iv(x,y)$ where $u(x,y) = \sqrt{|xy|}$ and $v(x,y) = 0$. Show that the Cauchy-Riemann equations are satisfied at $z = 0$ but that f is not complex differentiable at $z = 0$.

[14 marks]

(b)(ii) Explain why the converse of the Cauchy-Riemann equations does not apply in this case. **[4 marks]**

(d) Show that $u(x,y) = x^3 - 3xy^2$ is harmonic and find a harmonic conjugate for it.

[10 Marks]

TOTAL MARKS FOR QUESTION 1 = 32 MARKS

QUESTION 2

(a) Evaluate $\int_\gamma |z|^3 dz$, where $\gamma = \{e^{it} : t \in [0, \frac{\pi}{4}]\}$. **[6 Marks]**

(b) State and prove the theorem giving the upper estimate for the contour integral $|\int_\gamma f|$.

[7 marks]

(c) Use Cauchy's Integral Theorem to evaluate

$$\int_{C(0,1)} \frac{\sin(z)}{z^2 - 5z + 6} dz.$$

[5 marks]

(d) Let $a \in \mathbb{C}$, $r > 0$ and n an integer. Evaluate $\int_{C(a,r)} (z-a)^n dz$. **[10 marks]**

TOTAL MARKS FOR QUESTION 2 = 28 MARKS

QUESTION 1

(a) Compute $(1-i)^6/(1+i)^{10}$ in real-imaginary form. **[6 marks]**

(b) State without proof the theorem that gives the converse of the Cauchy-Riemann equations.

[4 marks]

(c) At which points $z \in \mathbb{C}$ is the function $f(z) = 3|z|^2 - i$ complex differentiable? **[6 marks]**

(d) Let $a \in \mathbb{C}$, $r > 0$ and n an integer. Evaluate $\int_{C(a,r)}(z-a)^n dz$. **[10 marks]**

TOTAL MARKS FOR QUESTION 1 = 26 MARKS

QUESTION 2

(a) State Cauchy's Integral Theorem Revisited. **[3 Marks]**

(b) State and prove Cauchy's Integral formula. **[10 Marks]**

(c) Evaluate the contour integral

$$\int_{C(0,1)} \frac{\sin(z)}{z^2 - z + 2} dz.$$

[6 marks]

(d) Evaluate the contour integral

$$\int_{C(0,3/2)} \frac{\exp(z)}{z^2 - z - 2} dz.$$

[5 marks]

TOTAL MARKS FOR QUESTION 2 = 24 MARKS

$\frac{(1-i)^6}{(1+i)^{10}}$

$= \frac{(1-i)^6(1-i)^{10}}{(1+i)^{10}(1-i)^{10}}$ $(\sqrt{2})^{10}$ $(1-i)^{16}$ $16|z|^2$

$= \frac{(1-i)^{16}}{2^{10}}$ $= (\sqrt{2})^{16}$ $(\sqrt{2})^2(2^{10})$

1028 $e^{i\frac{\pi}{4}\cdot 16}$

QUESTION 1

(a) State without proof the theorem that gives the converse of the Cauchy-Riemann equations. **[4 marks]**

(b) At which points $z = x + iy \in \mathbb{C}$ is the function $f(x + iy) = x^2y + i(xy + 3x)$ complex differentiable? **[14 marks]**

(c) Show that $u(x, y) = x^3 - 3xy^2$ is harmonic and find a harmonic conjugate for it. **[10 Marks]**

TOTAL MARKS FOR QUESTION 1 = 28 MARKS

QUESTION 2 V $f = u + iv$

(a) Evaluate $\int_\gamma |z|^3 dz$, where $\gamma = \{e^{it} : t \in [0, \frac{\pi}{4}]\}$. **[6 Marks]**

(b) State and prove the theorem giving the upper estimate for the contour integral $|\int_\gamma f|$.
Thm 2.14
[7 marks]

(c) Use Cauchy's Integral Theorem to evaluate

$$\int_{C(0,1)} \frac{\sin(z)}{z^2 - 5z + 6} dz.$$

[5 marks]

(d) State Cauchy's Integral Theorem Revisited. Thm 2.27. **[4 Marks]**

(e) Let $a \in \mathbb{C}$, $r > 0$ and n an integer. Evaluate $\int_{C(a,r)} (z - a)^n dz$. **[10 marks]**
Thm 2.20

TOTAL MARKS FOR QUESTION 2 = 32 MARKS

$(z-3)(z-2)$

QUESTION 1

(a) Compute $(1-i)^6/(1+i)^{10}$ in real-imaginary form. **[8 marks]**

(b) Express in polar form $(\sqrt{3}+i)^{1/2}$. **[6 marks]**

(c) State without proof the theorem that gives the converse of the Cauchy-Riemann equations. **[4 marks]**

(d) At which points $z \in \mathbb{C}$ is the function $f(z) = 3|z|^2 - i$ complex differentiable? **[6 marks]**

TOTAL MARKS FOR QUESTION 1 = 24 MARKS

QUESTION 2

(a) Evaluate $\int_\gamma |z|^3 dz$, where $\gamma = \{e^{it} : t \in [0, \frac{\pi}{4}]\}$. **[6 Marks]**

(b) Let $a \in \mathbb{C}$, $r > 0$ and n an integer. Evaluate $\int_{C(a,r)} (z-a)^n dz$. **[10 marks]**

(c) Use Cauchy's Integral Theorem to evaluate

$$\int_{C(0,1)} \frac{\exp(z)}{z^2 - z + 2} dz.$$

[6 marks]

(d) State Cauchy's Integral Theorem Revisited. **[4 Marks]**

TOTAL MARKS FOR QUESTION 2 = 26 MARKS

COMPLEX ANALYSIS Page 1 of 1 pages SEPTEMBER 2011

QUESTION 1

(a) Prove that for $z, w \in \mathbb{C}$, $|z + w| \le |z| + |w|$. Tut question

(b) If $p > 0$ and $q > 0$ with $p + q = 1$ show that $|p + qe^{i\alpha}| < 1$ for any $0 < \alpha < \pi/2$. [7 marks]

[7 marks]

(c) State without proof the theorem that gives the converse of the Cauchy-Riemann equations. → thrm. 1.14. see notes

[4 marks]

(d) At which points $z = x + iy \in \mathbb{C}$ is the function $f(x + iy) = x^2y + i(xy + 3x)$ complex differentiable? → similar to Q4 tut 1. – 2013 2010.

[14 marks]

TOTAL MARKS FOR QUESTION 1 = 32 MARKS

QUESTION 2

(a) Evaluate $\int_\gamma |z|^3 dz$, where $\gamma = \{e^{it} : t \in [0, \frac{\pi}{4}]\}$.) tut2 Q6 done

[6 Marks]

(b) State and prove the theorem giving the upper estimate for the contour integral $|\int_\gamma f|$.
↳ thrm. 2.14

[7 marks]

(c) Use Cauchy's Integral Theorem to evaluate
↳ by thrm. 2.25

$$\int_{C(0,1)} \frac{\sin(z)}{z^2 - 5z + 6} dz.$$

Done.

[5 marks]

(d) Show that $u(x, y) = x^3 - 3xy^2$ is harmonic and find a harmonic conjugate for it. Done

[10 Marks]

TOTAL MARKS FOR QUESTION 2 = 28 MARKS

COMPLEX ANALYSIS

QUESTION 1

→ tut 1 Q4 Similar

(a) Compute $(1-i)^{1/2}$ by using complex logarithms.

[5 marks]

(b) If $y \neq 0$ show $\cos(x+iy)$ is real iff x is an integer multiple of π.

↳ tut.1 Q12 a). Learn.

[9 marks]

(c) State without proof the theorem that gives the converse of the Cauchy-Riemann equations. ↳ theorem 1.14. See notes.

[4 marks]

(d) At which points $z = x+iy \in \mathbb{C}$ is the function $f(x+iy) = x^2y + i(xy+3x)$ complex differentiable?

↳ similar to tut 1 Q14

[14 marks]

TOTAL MARKS FOR QUESTION 1 = 32 MARKS

QUESTION 2

→ See tut 2 Q7

(a) Let $f(x+iy) = xy$. Compute the integral of f along the semi-circle $\{e^{it} : t \in [0,\pi]\}$ traversed anticlockwise.

[9 Marks]

(b) Let $a \in \mathbb{C}$, $r > 0$ and n an integer. Evaluate $\int_{C(a,r)} (z-a)^n dz$.

[10 marks]

(c) Use Cauchy's Integral Theorem to evaluate

$$\int_{C(0,1)} \frac{\sin(z)}{z^2 - 5z + 6} dz.$$

[5 marks]

(d) State Cauchy's Integral Theorem Revisited.

[4 Marks]

↳ theorem 2.27. Notes

TOTAL MARKS FOR QUESTION 2 = 28 MARKS

$\cos(x+y) = \cos x \cos y - \sin x \sin y$

COMPLEX ANALYSIS Page 1 of 1 pages SEPTEMBER 2009

QUESTION 1

(a) Compute $(1-i)^6/(1+i)^{10}$ in real-imaginary form. ⇒ Similar to Q3 tut 1.

(b) Express in polar form $(\sqrt{3}+i)^{1/2}$. [8 marks]

[6 marks]

(c) State without proof the theorem that gives the converse of the Cauchy-Riemann equations. ↳ See theorem ~~1~~.14 notes.

[4 marks]

(d) At which points $z \in \mathbb{C}$ is the function $f(z) = 3|z|^2 - i$ complex differentiable?

↳ similar Q14 Tut 1.

[6 marks]

TOTAL MARKS FOR QUESTION 1 = 24 MARKS

QUESTION 2

Review (a) Evaluate $\int_\gamma |z|^3 dz$, where $\gamma = \{e^{it} : t \in [0, \frac{\pi}{4}]\}$. ⇒ Similar to Q6 Tut2.

[6 Marks]

Review (b) Let $a \in \mathbb{C}$, $r > 0$ and n an integer. Evaluate $\int_{C(a,r)} (z-a)^n dz$. [10 marks]

⇒ See theorem 2.20 notes.

(c) Use Cauchy's Integral Theorem to evaluate

$$\int_{C(0,1)} \frac{\exp(z)}{z^2 - z + 2} dz.$$

↳ by C's ∫ theorem, f(z) is analytic on C and C(0,1) is closed path so $\int_C f(z)\,dz$ is 0.

[6 marks]

(d) State Cauchy's Integral Theorem Revisited. [4 Marks]

↳ theorem 2.27 See notes

TOTAL MARKS FOR QUESTION 2 = 26 MARKS

→ Similar to Q14 Tut2. ()