1 Propositional Practice

Convert the following English sentences into propositional logic and the following propositions into English. State whether or not each statement is true with brief justification.

(a) There is a real number which is not rational.

A: $(\exists x \in \mathbb{R})(\mathrm{x} \notin \mathbb{Q})$ $\surd$

(b) All integers are natural numbers or are negative, but not both.

A: $(\forall x \in \mathbb{Z})\ (x \in \mathbb{N} \vee -x \in \mathbb{N})$ $\surd$

(c) If a natural number is divisible by 6, it is divisible by 2 or it is divisible by 3.

A: $(6 \mid x) \Rightarrow ((2 \mid x) \vee (3 \mid x))$ $\surd$

(d) $(\forall x \in \mathbb{Z})(x \in \mathbb{Q})$

A: All integers are rational number. $\surd$

(e) $(\forall x \in \mathbb{Z})(((2 \mid x) \vee (3 \mid x)) \Rightarrow (6 \mid x))$

A: Any integer can be divided by 2 or 3. It is divisible by 6. $\times$

(f) $(\forall x \in \mathbb{N})((x > 7) \Rightarrow ((\exists a, b \in \mathbb{N})(a + b = x)))$ $\surd$

2 Truth Tables

Determine whether the following equivalences hold, by writing out truth tables. Clearly state whether or not each pair is equivalent.

(a): $\mathbf{P} \wedge (\mathbf{Q} \vee \mathbf{P}) \equiv \mathbf{P} \wedge \mathbf{Q}$ *not equivalent*

(b): $(\mathbf{P} \vee \mathbf{Q}) \wedge \mathbf{R} \equiv (\mathbf{P} \wedge \mathbf{R}) \vee (\mathbf{Q} \wedge \mathbf{R})$ *equivalent*

(c): $(\mathbf{P} \wedge \mathbf{Q}) \vee \mathbf{R} \equiv (\mathbf{P} \vee \mathbf{R}) \wedge (\mathbf{Q} \vee \mathbf{R})$ *equivalent*

3 Logical Equivalence?

Decide whether each of the following logical equivalences is correct and justify your answer.

(a): $\forall x(P(x) \wedge Q(x)) \equiv \forall x P(x) \wedge \forall x Q(x)$ *correct*

(b): $\forall x(P(x) \vee Q(x)) \equiv \forall x P(x) \vee \forall x Q(x)$ *incorrect*

(c): $\exists(P(x) \vee Q(x)) \equiv \exists x P(x) \vee \exists x Q(x)$ *correct*

(d): $\exists x(P(x) \wedge Q(x)) \equiv \exists x P(x) \wedge \exists x Q(x)$ *incorrect*