

CS 372/469, Fall 2018  
Assignment 3, due 10/16, before class  
Total points: 100

1. (20 points) Use the master theorem to give bounds on the following recurrences:

- (a)  $T(n) = 2T(n/4) + 1$
- (b)  $T(n) = 2T(n/4) + \sqrt{n}$
- (c)  $T(n) = 2T(n/4) + n$
- (d)  $T(n) = 2T(n/4) + n^2$
- (e)  $T(n) = 2T(n/3) + 1$
- (f)  $T(n) = 5T(n/4) + n$
- (g)  $T(n) = 7T(n/7) + n$
- (h)  $T(n) = 9T(n/3) + n^2$
- (i)  $T(n) = 8T(n/2) + n^3$
- (j)  $T(n) = 49T(n/25) + n^{3/2} \log n$

2. (15 points) Textbook problem 2.14.

3. (15 points) Textbook problem 2.16.

4. (15 points) Textbook problem 2.17.

5. (15 points) Quicksort: In class, we had completed the first partitioning step for array

$A = \langle 13, 19, 9, 5, 12, 8, 7, 4, 21, 2, 6, 11 \rangle$ . 11 was the pivot, and the array after first partition looked like this:  $\langle 9, 5, 8, 7, 4, 2, 6, \mathbf{11}, 21, 13, 12, 19 \rangle$ . Complete the rest of the partitions, until you get a fully sorted array.

6. (20 points) Counting sort-based problems:

- (a) Show the operation of counting sort on  $A = \langle 6, 0, 2, 0, 1, 3, 4, 6, 1, 3, 2 \rangle$ . Maintain flags on repeated elements to show that counting sort is stable.
- (b) Describe an algorithm that, given  $n$  integers in the range  $[0..k]$ , pre-processes its input and answers any query about how many of the  $n$  integers fall in a range  $[a..b]$  in  $O(1)$  time. Your algorithm can use up to  $O(n + k)$  pre-processing time.

How to submit: Upload your **pdf** file on Canvas.