Magnetic Levitation System Project Report

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Linear Systems (ENGR 6131)

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Fall 2021

Abstract. This document represents the process of controlling Magnetic Levitation system. In the first part, the dynamic equations of systems are derived and linearized. The state-space representation of the system obtained from the linearized equations and the system is implemented in MATLAB. One controller is designed in each modern control theory approach and classical control theory approach. The results of designing the controller are used to compare these two approaches and to verify the advantages and disadvantages of each over the other one.

1 Introduction

The modern control theory is one of the most important advancements in control theory. The main difference of it compare to classical control theory is that the modern control theory focuses on development of models to represent the internal dynamics of a system through state space methods while classical control theory focuses only on capturing the input/output representation of systems. Therefore, not only there is more observation for the system, also there is more control. In this report, the aim is to compare these 2 methods of controlling the systems, by trying to control a physical system, Magnetic Levitation System, in both ways.

1.1 Statement of the Problem

In this project, the aim is controlling Magnetic Levitation System with classical control theory approach and modern control theory approach, and comparing the result from them to obtain and to verify the advantages and disadvantages of both approaches in designing controller for physical systems.

1.2 Design Specifications

There are two side in design specifications. First is the transient response specifications and second is the steady state response specifications. Design specifications here are assumed as the maximum overshoot less than 10%, the settling time less than 2sec, and the steady state error less than 5%.

2 Methodology and Results (Main body)

2.1 Equations of system

The simplified equations of motion according to the description paper are as follows:

$$\mathrm{m}\ddot{\mathrm{y}}_1 + \mathrm{F}_{\mathrm{m}_{12}} = \mathrm{F}_{\mathrm{u}_{11}} - \mathrm{mg} \tag{1}$$

$$\mathrm{m}\ddot{\mathrm{y}}_2 - \mathrm{F}_{\mathrm{m}_{12}} = \mathrm{F}_{\mathrm{u}_{22}} - \mathrm{mg} \tag{2}$$

where

$$F_{u_{11}} = \frac{u_1}{a(y_1 + b)^4} \tag{3}$$

$$F_{u_{22}} = \frac{u_2}{a(-y_2 + b)^4} \tag{4}$$

$$F_{m_{12}} = \frac{c}{(y_{12} + d)^4} \tag{5}$$

So

$$\mathrm{m}\ddot{\mathrm{y}}_1 + \frac{c}{(y_{12} + d)^4} = \frac{u_1}{a(y_1 + b)^4} - \mathrm{mg} \tag{6}$$

$$\mathrm{m}\ddot{\mathrm{y}}_2 - \frac{c}{(y_{12} + d)^4} = \frac{u_2}{a(-y_2 + b)^4} - \mathrm{mg} \tag{7}$$

The equations are non-Linear, so they are linearized using the first order Taylor expansion around the operating condition. ($y_1 = 2.00cm$, $y_2 = -2.00cm$, and $y_c = 12.00cm$)

The inputs u_1 and u_2 have to find at operating conditions. They can be found from the eq. (1) and eq. (2) and using other operating conditions, and putting $\ddot{\mathrm{y}}_1$ and $\ddot{\mathrm{y}}_2$ equal to zero.

$$u_{1o} = a(y_{1o} + b)^4 * \left(mg + \frac{c}{(y_{12o} + d)^4}\right) = 2.9242 * 10\text{^}3 \tag{8}$$

$$u_{2o} = a(y_{2o} + b)^4 * \left(mg - \frac{c}{(y_{12o} + d)^4}\right) = 2.8111 * 10\text{^}3 \tag{9}$$

Substituting the non-linear terms with the first order Taylor expansion it yields:

$$\mathrm{m}\ddot{y}_1 + \left(\frac{4c}{(y_{12\_o}+d)^5} + \frac{4u_{1\_o}}{a(y_{1\_o}+b)^5}\right)(y_1 - y_{1\_o}) - \frac{4c}{(y_{12\_o}+d)^5}(y_2 - y_{2\_o}) = \frac{u_1 - u_{1\_o}}{a(y_{1\_o}+b)^4} \tag{10}$$

$$\mathrm{m}\ddot{y}_2 - \frac{4c}{(y_{12\_o}+d)^5}(y_1 - y_{1\_o}) + \left(\frac{4c}{(y_{12\_o}+d)^5} + \frac{4u_{2\_o}}{a(-y_{2\_o}+b)^5}\right)(y_2 - y_{2\_o}) -= \frac{u_2 - u_{2\_o}}{a(-y_{2\_o}+b)^4} \tag{11}$$

Assume:

$$k_1 = \frac{4u_{1\_o}}{a(y_{1\_o}+b)^5} = 0.7614 \tag{12}$$

$$k_2 = \frac{4u_{2\_o}}{a(-y_{2\_o}+b)^5} = 0.7320 \tag{13}$$

$$k_3 = \frac{4c}{(y_{12\_o}+d)^5} = 0.0075 \tag{14}$$

$$k_4 = \frac{1}{a(y_{1\_o}+b)^4} = 4.0491 * 10\text{^}(-4) \tag{15}$$

$$k_5 = \frac{1}{a(-y_{2\_o}+b)^4} = 4.0491 * 10\text{^}(-4) \tag{16}$$

The state-space matrices A and B can be extracted easily from the eq. (10) and eq. (11). Consequently, consider $x_1 = y_1$, $x_2 = \dot{y}_1$, $x_3 = y_2$, and $x_4 = \dot{y}_2$ as state variables, the state-space system is as follows:

$$\frac{d}{dt}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -(k_1+k_3)/m & 0 & k_3/m & 0 \\ 0 & 0 & 0 & 1 \\ k_3/m & 0 & -(k_2+k_3)/m & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ k_4/m & 0 \\ 0 & 0 \\ 0 & k_5/m \end{bmatrix}\begin{bmatrix} \mathrm{u}_1 \\ \mathrm{u}_2 \end{bmatrix} \tag{17}$$

$$y = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} \mathrm{u}_1 \\ \mathrm{u}_2 \end{bmatrix} \tag{18}$$

```
A =

         0    1.0000         0         0
   -6.4076         0    0.0624         0
         0         0         0    1.0000
    0.0624         0   -6.1623         0


B =

         0         0
    0.0034         0
         0         0
         0    0.0034


C =

     1     0     0     0
     0     0     1     0


D =

     0     0
     0     0
```

Since the system is a MIMO system, there is a matrix of transfer functions for open-loop system. The matrix is made of the transfer functions of each input and output peer to peer. The transfer functions are as follows:

```
sys_tf =

  From input 1 to output...
       0.003374 s^2 + 0.02079
   1:  -----------------------
       s^4 + 12.57 s^2 + 39.48

              0.0002107
   2:  -----------------------
       s^4 + 12.57 s^2 + 39.48

  From input 2 to output...
              0.0002107
   1:  -----------------------
       s^4 + 12.57 s^2 + 39.48

       0.003374 s^2 + 0.02162
   2:  -----------------------
       s^4 + 12.57 s^2 + 39.48

Continuous-time transfer function.
```

There are 2 outputs y1 and y2, and 2 inputs. Since the transfer function gain of the output 2 from the input 1, and the output 1 from the input 2 are small, it can be neglected. So, the system can be divided to an upper part system with the output 1 and the input 1, and a lower part system with the output 2 and the input 2. Some plots and result are obtained for both of these cases separately, in the rest of the report.

The Controllable Canonical form, the Observer Canonical form and the Jordan Canonical form of the system are obtained from MATLAB and they are as follows:

Observable Canonical Form:

```
Ao =

         0         0         0  -39.4817
    1.0000         0         0         0
         0    1.0000         0  -12.5699
         0         0    1.0000         0


Bo =

    1.0000  102.6328
         0         0
         0   16.0172
         0         0


Co =

         0    0.0034         0   -0.0216
         0         0         0    0.0002


Do =

     0     0
     0     0
```

Controllable Canonical Form:

```
Ac =

         0    1.0000         0         0
         0         0    1.0000         0
         0         0         0    1.0000
  -39.4817         0  -12.5699         0


Bc =

         0         0
    0.0034         0
         0         0
   -0.0216    0.0002


Cc =

    1.0000         0         0         0
  102.6328         0   16.0172         0


Dc =

     0     0
     0     0
```

A matrix of Jordan Canonical form:

```
J =

   0.0000 - 2.5343i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.0000 - 2.4794i   0.0000 + 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 2.5343i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 2.4794i
```

Considering initial conditions of zero, the step response and the impulse response of the system are obtained by using the “step” and “impulse” command in MATLAB.

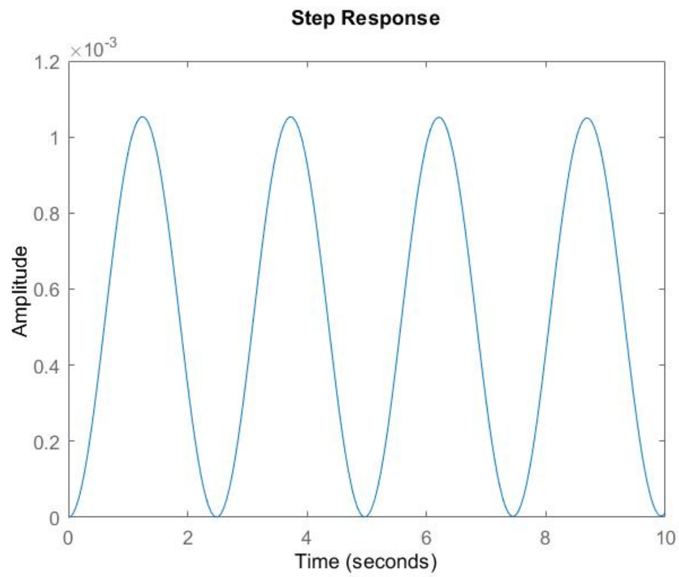

Fig. 1 The output 1 when the input 1 is a step function.

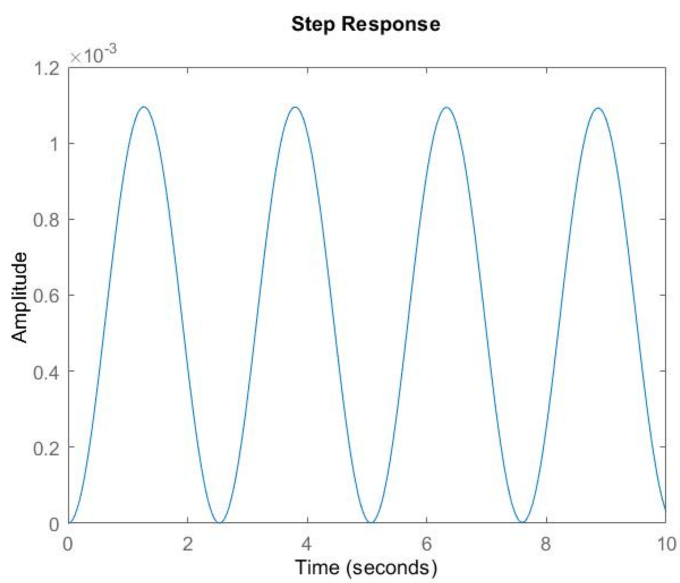

Fig. 2 The output 2 when the input 2 is a step function.

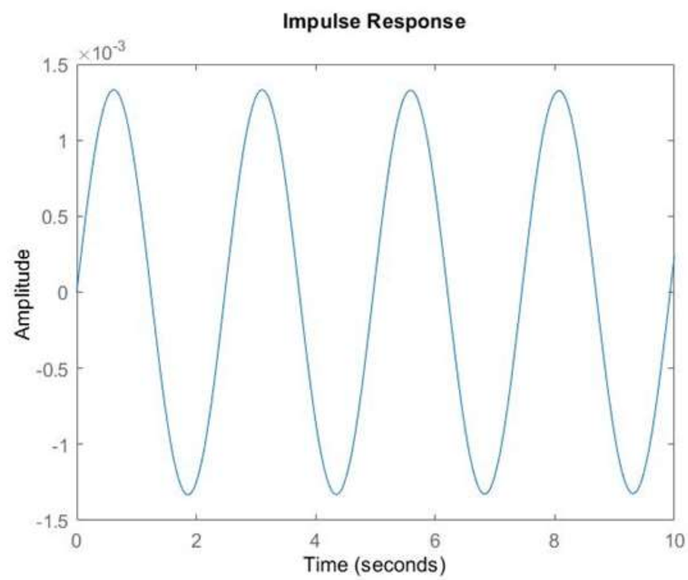

Fig. 3 The output 1 when the input 1 is a impulse function.

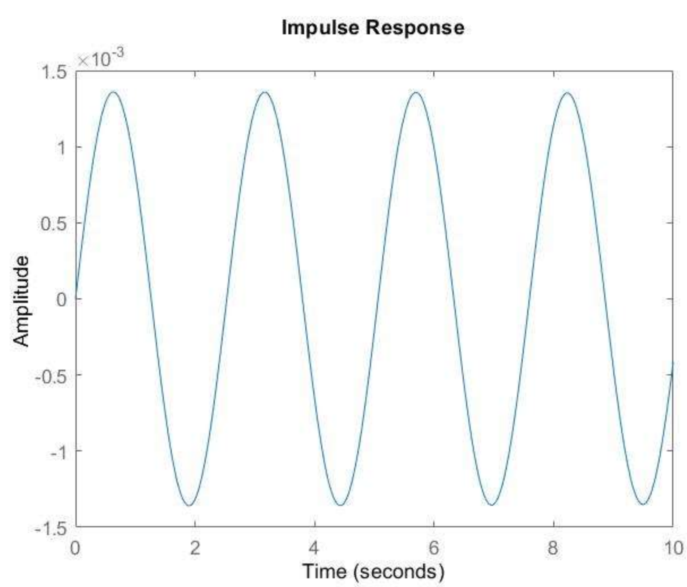

Fig. 4 The output 2 when the input 2 is a impulse function.

For the Bode plot and the Root Locus plot, again the plots are obtained for 2 cases as follows:

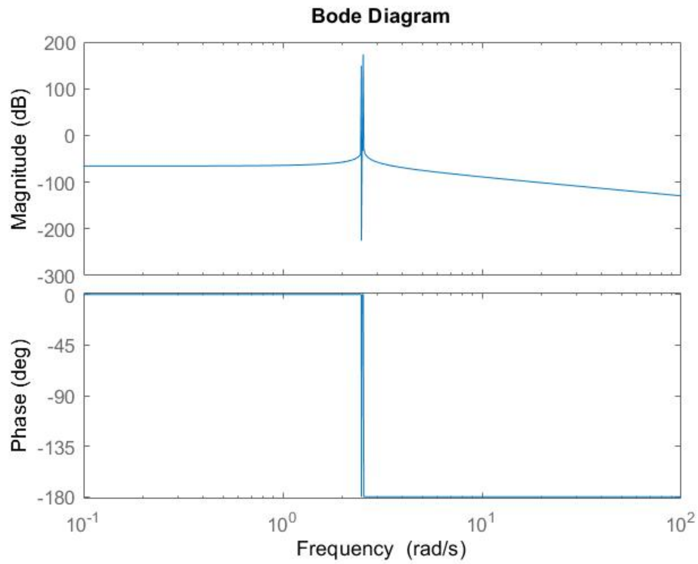

Fig. 5 The Bode Diagram of the upper system.

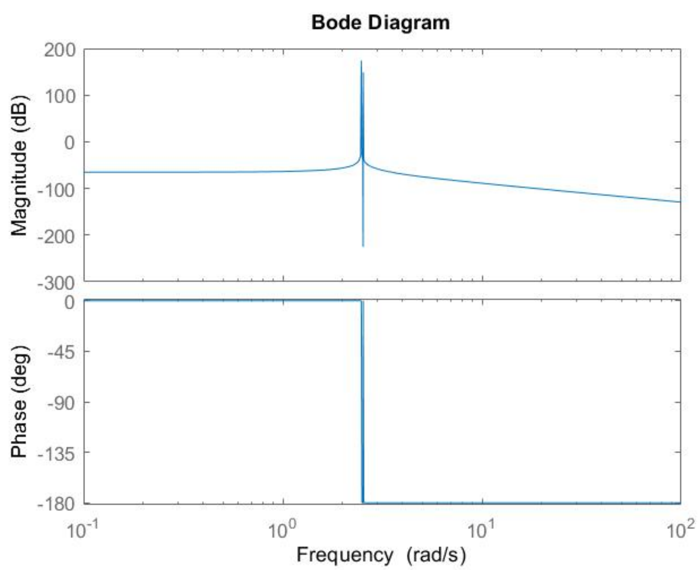

Fig. 6 The Bode Diagram of the lower system.

From the Jordan Canonical Form or transfer functions, it could be found that the poles of the system are on the imaginary axis, which they are shown correctly in following root Locus plots:

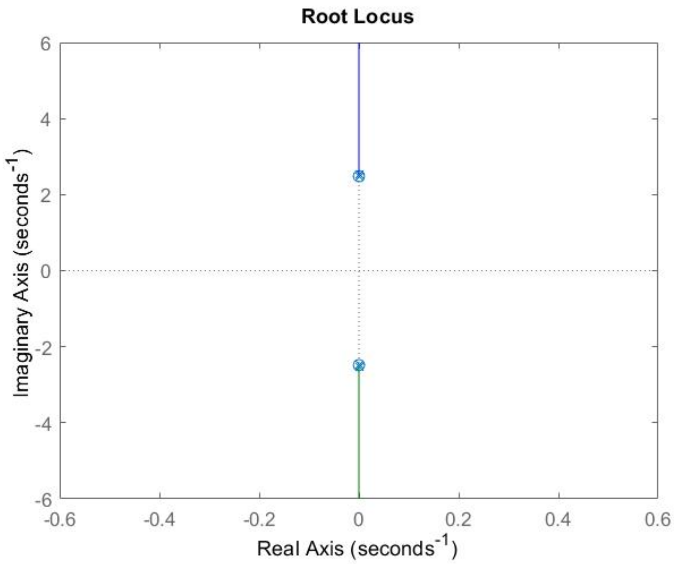

Fig. 7 The Root Locus of the upper system.

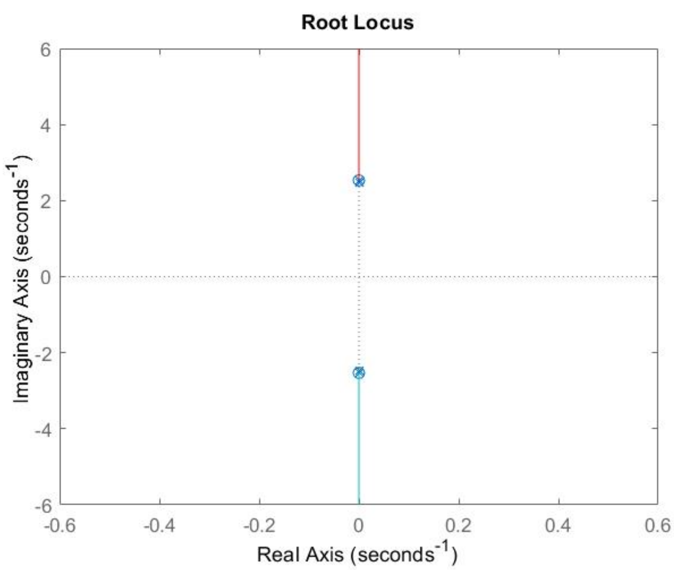

Fig. 8 The Root Locus of the lower system.

2.2 Tuning a PID controller

Design specifications are assumed as the maximum overshoot less than 10%, the settling time less than 2sec, and the steady state error less than 5%. To design a proper PID controller, I started with only a Proportional controller, then a PI, and when these didn't work (See appendix), a PID controller was tested and the result is as follows:

In designing the PID controller there were 2 case. In the first one, the steady state specification (steady state error) was fulfilled but not the transient specification (Overshoot).

In the second case, the steady state specification (steady state error) was not fulfilled however the transient specifications (Overshoot and Settling time) were as I wanted. Therefore, a PID controller which can address all the specifications couldn't be found. This is one of the biggest disadvantages of classical control theory which the proper controller can be found by trying and error, so you might not find the proper controller. However, in modern control theory, there is a unique specific solution for controller.

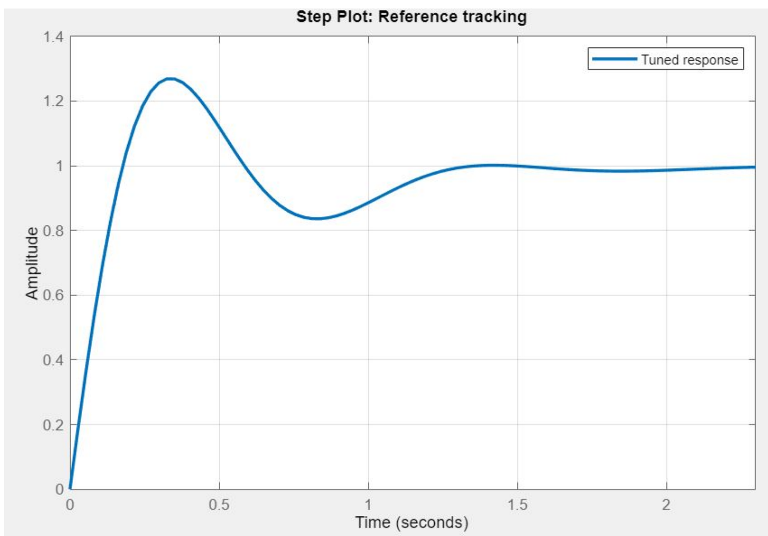

Fig. 9 Step response of the system with PID controller

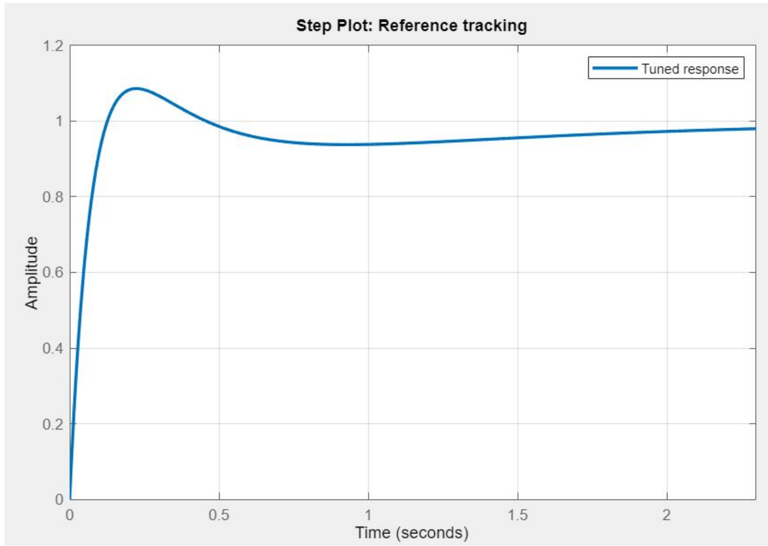

Fig. 10 Step response of the system with PID controller

To obtain the responses for other inputs such as Sin wave and square wave, the closed-loop system with PID controller found from pidTunner, implemented in Simulink. (see appendix)

The control input to the system is one of the important things that should be careful about, because although there is not any limitation in the Simulink, there are some limitations in building a real physical controller for a system since the actuator and the physical plant can't accept any inputs. The responses to each inputs and also the control inputs for each 3 inputs are as follows:

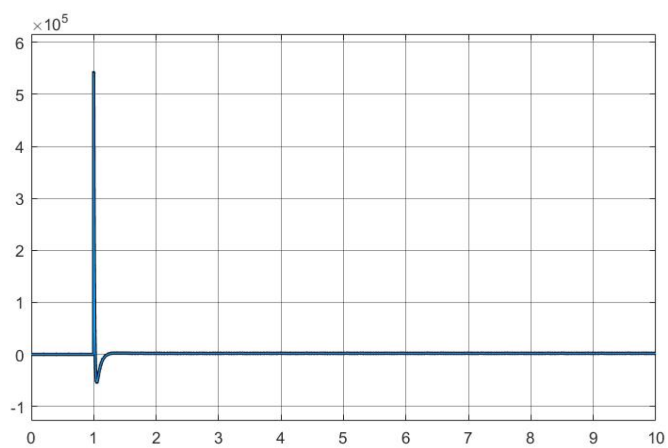

Fig. 11 Control input of the system with PID controller for the step input

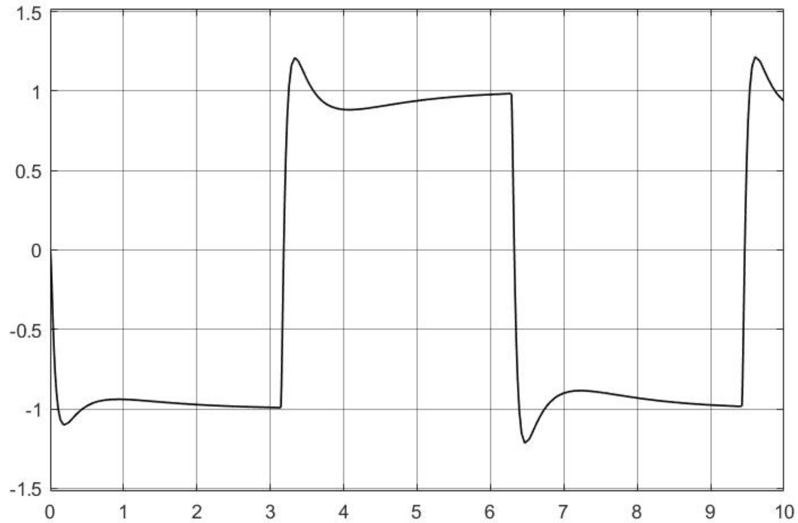

Fig. 12 Squad wave response of the system with PID controller

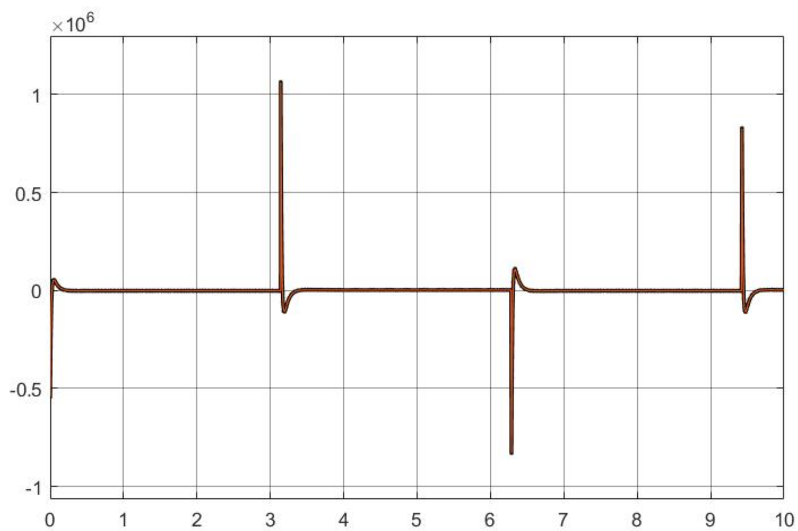

Fig. 13 Control input of the system with PID controller for the squad wave input

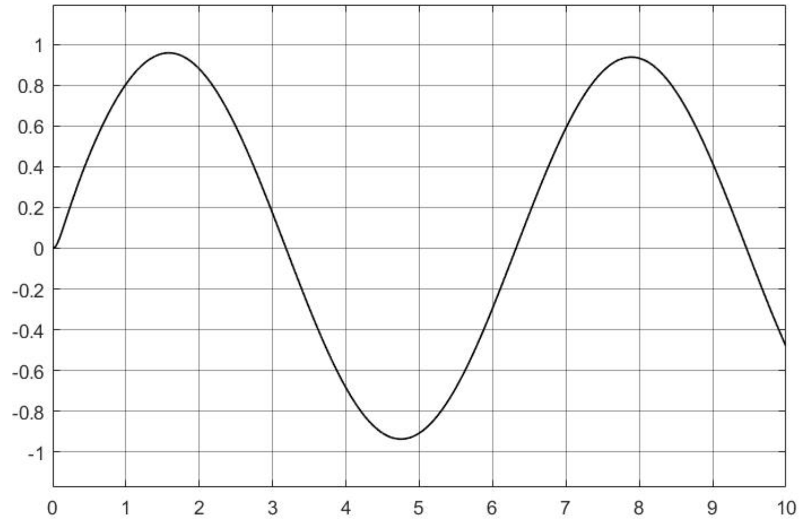

Fig. 14 Sinusoidal response of the system with PID controller

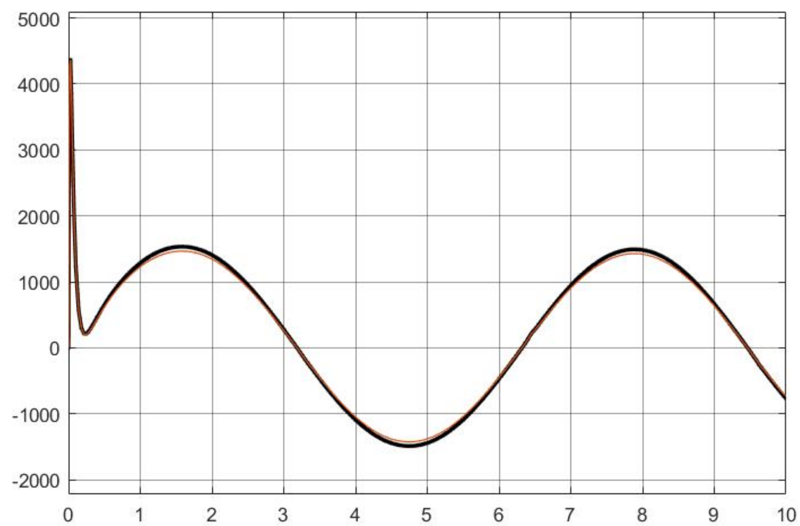

Fig. 15 Control input of the system with PID controller for the Sin input

As it is obvious from the fig.11, fig13, and fig15, the controller gives large signals to the actuators. Although in Simulink there is no problem with this, but in real implementation it might be a problem. This is also one of the disadvantages of PID controller.

The robustness of the system can be shown by adding noise to the output. If let the output be the output plus a noise signal in Simulink (see appendix), the PID controller not only couldn't remove the noise effect, but also increased the effect since the gains of the PID controller was big numbers. The system with noise step response is as follows:

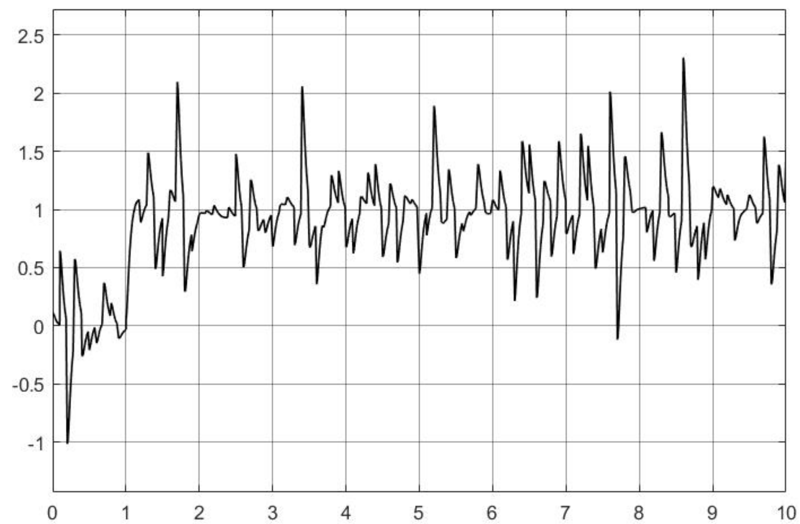

Fig. 16 Step response of the system with PID controller when the output is noisy

Another disadvantage of PID controller is shown in fig.16. The classical control theory couldn't compensate the effects of the noise.

2.3 Tuning a state feedback controller

Design specifications are assumed as the maximum overshoot less than 10%, the settling time less than 2sec, and the steady state error less than 5%. To design a proper state feedback controller, first the desired poles or Eigen values are found using the design specifications as follows:

- Overshoot = 10%, so damping ratio ζ=0.6
- Settling time = 2 sec, so $\mathrm{w_n}$ = 4

The desired poles or Eigen values of the (A-Bk) matrix is as follows:

$$s^* = -\zeta \mathrm{w_n} \pm \mathrm{j}\mathrm{w_n}\sqrt{1-\zeta^2} = -2.4 \pm \mathrm{j}3.2$$

The step responses of the closed-loop system with state feedback controller are as follows:

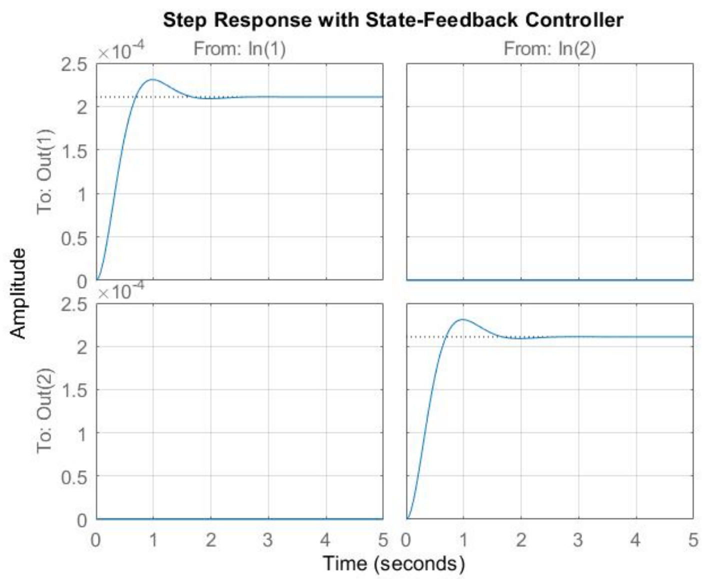

Fig. 17 Step responses of the system with state-feedback controller

As it is shown in the figure.17, the design specifications (Overshoot, settling time, and Steady state error) are perfectly met with state-feedback controller.

To obtain the responses for other inputs such as Sin wave and square wave, the closed-loop system with state-feedback controller found from coding, implemented in Simulink. (see appendix)

The responses to each inputs and also the control inputs for each 3 inputs are as follows:

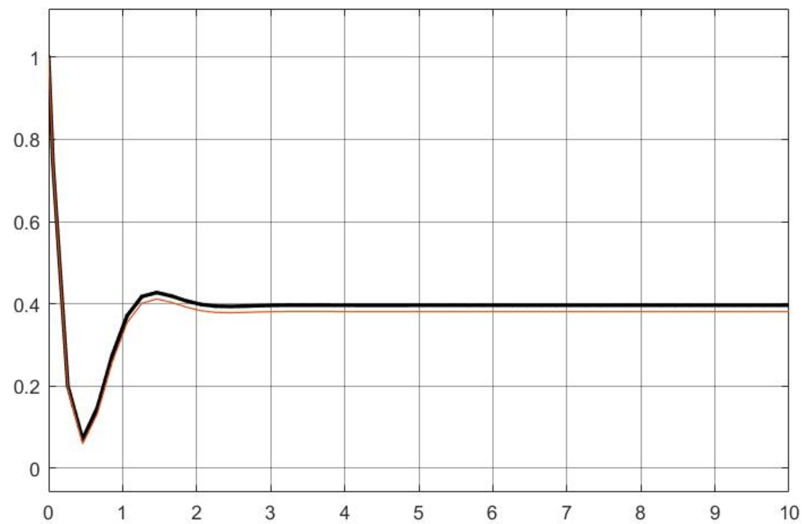

Fig. 18 Control input of the system with state-feedback controller for the step input

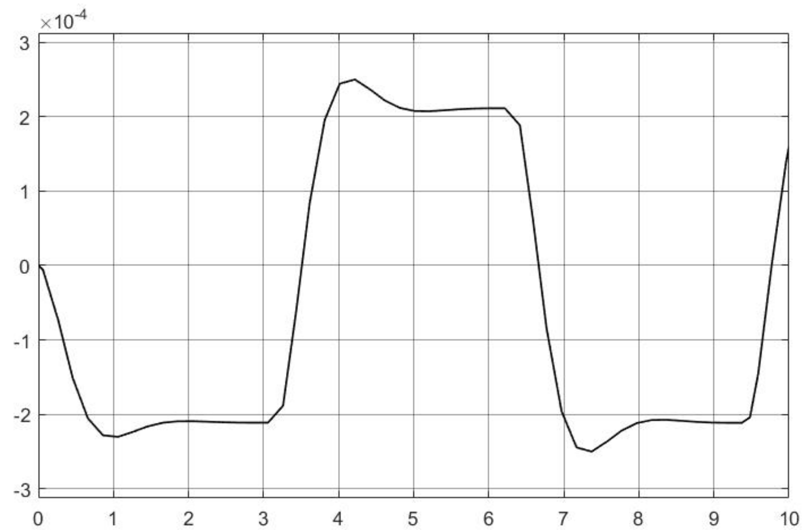

Fig. 19 Square wave response of the system with state-feedback controller

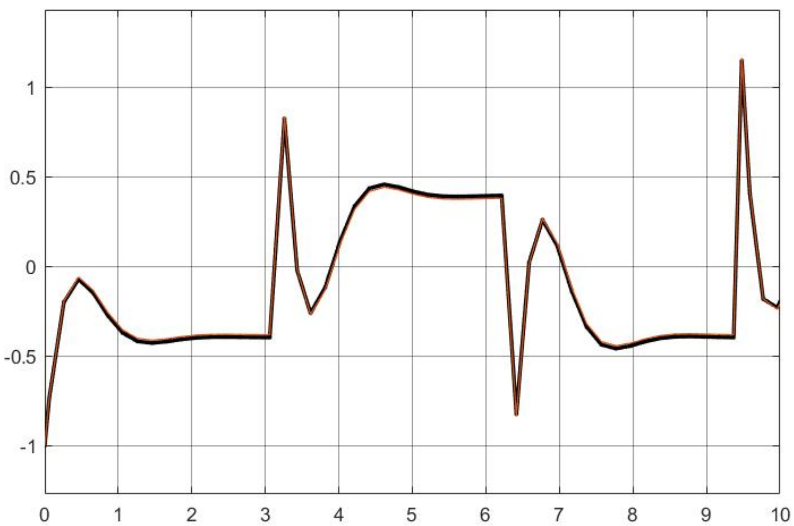

Fig. 20 Control input of the system with state-feedback controller for the square wave input

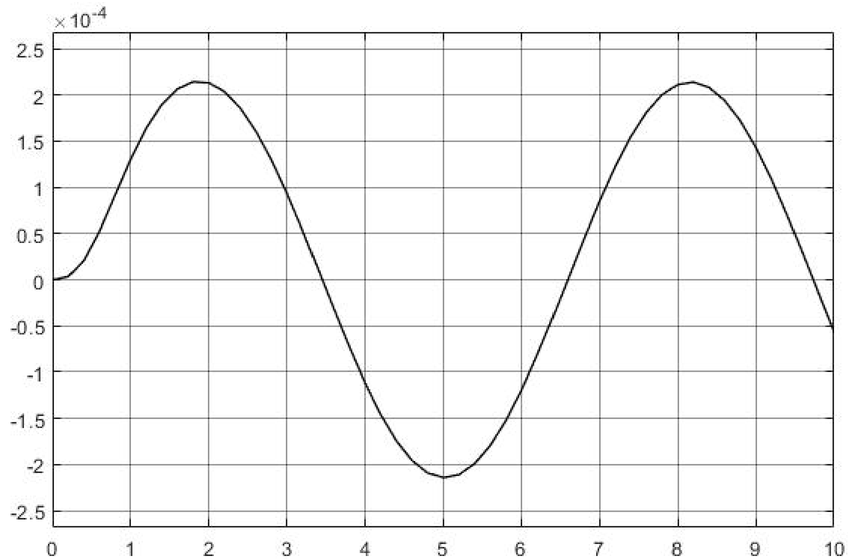

Fig. 21 Sinusoidal response of the system with state-feedback controller

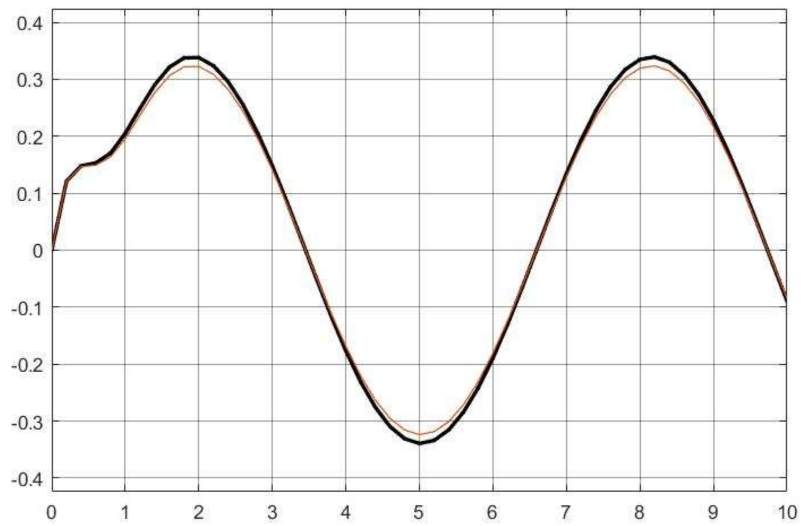

Fig. 22 Control input of the system with state-feedback controller for the Sin input

As it is obvious from the fig.18, fig20, and fig22, the state-feedback controller gives signals with smaller altitude than PID controller to the actuators. This is one of the advantages of the state-feedback controller, because the control will be done with much less than energy than the PID controller. Also the actuators are more likely to support these control inputs.

3 Discussion

To control this system, we had two choices. First to use classical control theory (PID controller), and second to use modern control theory (state-feedback controller). Each of these approaches has advantages and disadvantages over the other one. The advantages of modern control theory over classical control theory are as follows:

- It is possible to observe and control the inner states of the system and not merely input-output relations.
- It is possible to include initial conditions.
- We have insight to what is happening in the system.
- To control a system using modern control theory approach, unique solutions exist. Therefore, we don't need to do a try and error process like classical control theory approach to find the proper controller.
- It is possible to analyze time-varying or time-invariant, linear or non-linear, single or multiple input-output systems.
- The control inputs of the state-feedback controller are more rational than the PID controller.

The disadvantages of modern control theory over classical control theory are as follows:

- The modern control theory has more complex techniques.
- Many computations may be required to design a controller.

4 Appendix

First the codes used to obtain the result in main body are as follows:

```
clc;
clear all;
%% Variables
a = 1.65;
b = 6.2;
c = 2.69;
```

```
[V,J] = jordan(A);
%% Step Response & Impulse Response
t=0:0.01:10;
figure
step(sys_tf(1,1),t)
hold on;
figure
step(sys_tf(2,2),t)
hold on;
figure
impulse(sys_tf(1,1),t)
hold on
figure
impulse(sys_tf(2,2),t)
hold on
%% Checking Controllability and Obervability
sys_OL = sys;
sys_order = order(sys_OL)
ctrb_rank = rank(ctrb(A,B))
obsv_rank = rank(obsv(A,C))
%% Bode Diagram
figure
bode(sys_OL(1,1))
hold on;
figure
bode(sys_OL(2,2))
hold on;
%% Root locus
figure
rlocus(sys(1,1))
hold on;
figure
rlocus(sys(2,2))
hold on;
%% Check open loop eigenvalues
E=eig(A)
%Solve for K using pole placement
P=[-2.4+3.2i -2.4-3.2i -2.4+3.2i -2.4-3.2i];    %Put 4 poles in desired place
Kc = place(A,B,P);
%Create closed loop system
sys_cl = ss(A-B*Kc,B,C,D);
%Check closed loop eigenvalues
EE=eig(A-B*Kc)
%Check step response
t = 0:0.01:5;
figure
step(sys_cl,t)
grid
title('Step Response with State-Feedback Controller')
```

Here are the Simulink block diagrams:

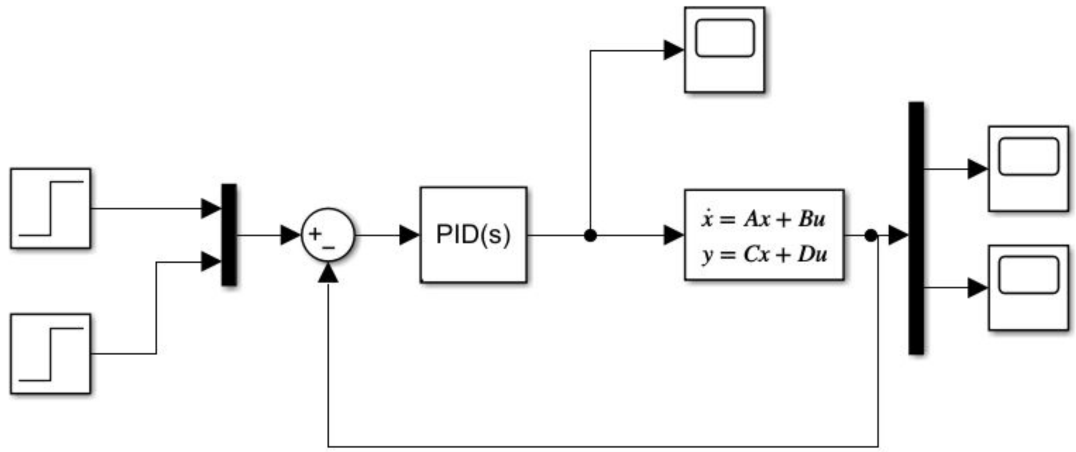

Fig. 23 Block diagram of the system with PID controller and step inputs

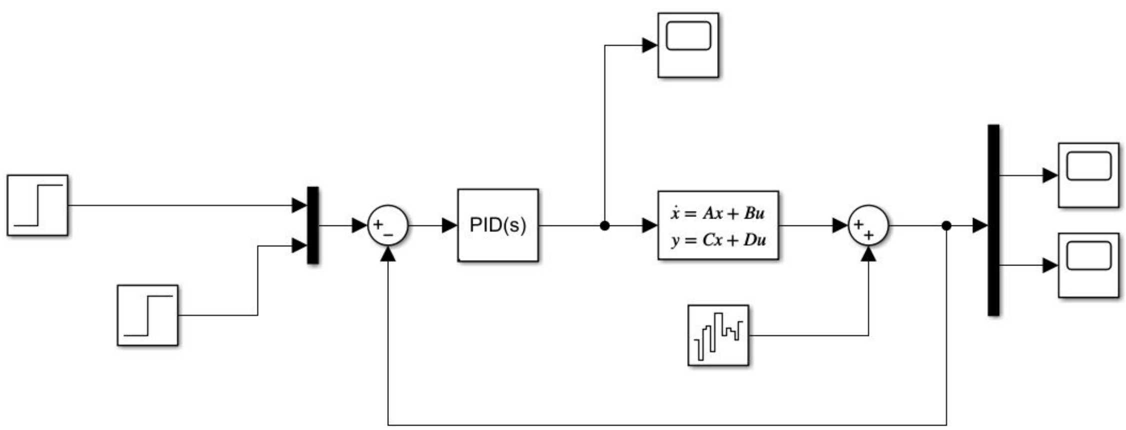

Fig. 24 Block diagram of the system with PID controller and step inputs and with noisy output

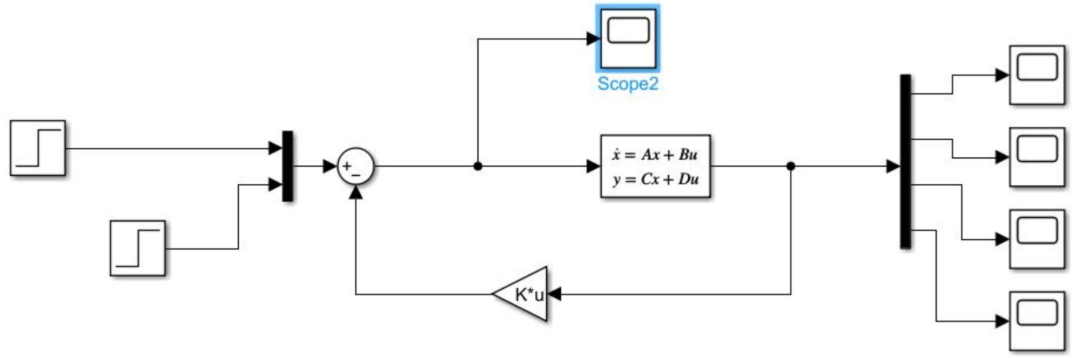

Fig. 25 Block diagram of the system with state-feedback controller and step inputs

Checking the observability and controllability of the system is one of important steps. The order of the system and the ranks of observability and controllability matrix are as follow which show that the system is both observable and controllable:

```
sys_order =

     4


ctrb_rank =

     4


obsv_rank =

     4
```