1. [3.0 points] Order the following expressions using their related asymptotic group functions:

$$2^n, n!, (\log n)!, n^3, e^n, 2^{\log_2 n}, n\log n, 2^{2^n}, n^{\log\log n}$$

**$2^{\log_2(n)}$ , nlogn , $n^3$ , (log(n))! , $n^{\log(\log(n))}$ , $2^n$ , $e^n$ , n! ,$2^{2\text{^}n}$**

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2. [3.0 points] Express the following function in terms of notation\

$$n^3/1000 - 100\,n^2 - 100\,n + 3$$

$n^3$ / 1000 ➔ highest oreder

$n^3$ / 1000 ➔ $n^3$ *(1 / 1000) ➔ 1/1000 constant doesn't affect

F(n) = O($n^3$)

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3. [5.0 points] Insertion sort can be expressed as a recursive procedure as follows. In order to sort A[1..n], we recursively sort A[1..n -1] and then insert A[n] into the sorted array A[1..n - 1]. Write a recurrence for the running time of this recursive version of insertion

**T(n)=T(n−1)+Θ(n)**

4. [5.0 points] Use iteration method to give asymptotic bounds for the following recurrence relation

$$T(n) = \begin{cases} 1 & \text{if } n = 1 \\ T(n-1) + n(n-1) & \text{if } n \geq 2 \end{cases}$$

Iter 1 : T(n) = T( n -1 ) + n(n-1)

Iter 2 : T(n) = T(n-2) + (n-2)(n-1)

Iter 3 : T(n) = T(n-3) + (n-3)(n-2)

Iter K : T(n) = T(n-k) + $\sum_{i=n-k+1}^{n} i(i-1)$

n – k =1 , k = n -1 ≈ n

$\sum_{i=2}^{n} i^2 - i = \sum_{i=2}^{n} i^2 - \sum_{i=2}^{n} i$

$= [\frac{i(i+1)(2i+1)}{6} - \frac{i(i+1)}{2}]+1$

$= \frac{i^3}{3} - \frac{i^2}{2} = \frac{n^3}{3} - \frac{n^2}{2}$

T(n) = $O(n^3)$

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5. [5.0 points] Use the iterative method to give asymptotic bounds for the following recurrence relation

$$T(n) = 7\,T\left(\frac{n}{2}\right) + n^2$$

Iter 1 : $7T(n/2) + n^2$

Iter 2 : $7^2\, T(n/2^2) + (n/2^2)^2 + (n/2)_2 + n^2$

Iter 3 : $7^3 T(n/2^3) + (n/2^2)^2 + (n/2)^2 + n^2$

Iter K : $7^k\, T(n/2^k) + (n/2^{k-1})^2 + (n/2^{k-2})^2 + n^2$

$(n/2^k) = 1$ , $n = 2^k$ , k = Log(n) , T(1) = 1

T(n) = $O(7^{Log(n)}) = O(n^{Log(7)})$

6. [4.0 points] Use Master Theory to give asymptotic bounds for the following recurrence relation

$$T(n) = 8\,T\left(\frac{n}{2}\right) + n^2$$

a = 8 , b = 2 , $F(n) = n^2$

$n^{\text{Log}_b(a)} = n^{\text{Log}_2(8)} = n^3$ ➔ $F(n) < n^{\text{Log}_b(a)}$

Case 1 : $T(n) = O(n^3)$