GRAPH THEORY

LECTURE 3

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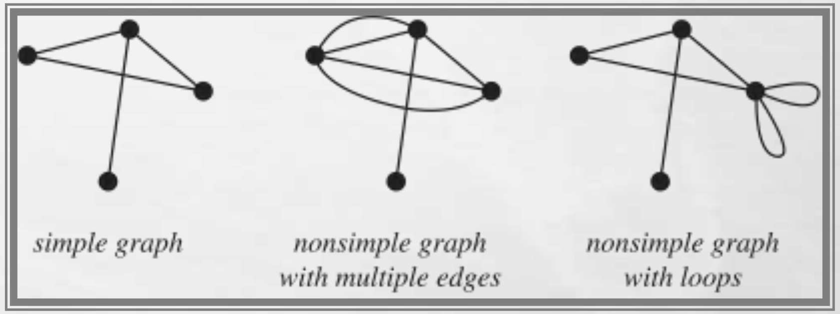
SIMPLE GRAPH

- A SIMPLE GRAPH, ALSO CALLED A STRICT GRAPH (TUTTE 1998, P. 2), IS AN UNWEIGHTED, UNDIRECTED GRAPH CONTAINING NO GRAPH LOOPS OR MULTIPLE EDGES (GIBBONS 1985, P. 2; WEST 2000, P. 2; BRONSHTEIN AND SEMENDYAYEV 2004, P. 346). A SIMPLE GRAPH MAY BE EITHER CONNECTED OR DISCONNECTED.
- UNLESS STATED OTHERWISE, THE UNQUALIFIED TERM "GRAPH" USUALLY REFERS TO A *SIMPLE* GRAPH. A SIMPLE GRAPH WITH MULTIPLE EDGES IS SOMETIMES CALLED A MULTIGRAPH (SKIENA 1990, P. 89).



The ***maximum number of edges*** possible in a simple graph with n vertices is ${}^{n}C_{2}$

where ${}^{n}C_{2} = \frac{n(n-1)}{2}$

The maximum number of edges with n=3 vertices is

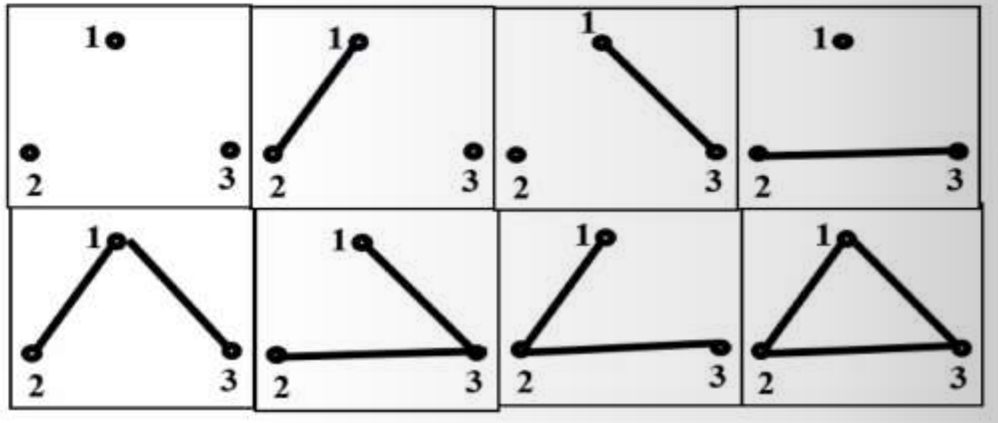
$${}^{n}C_{2} = \frac{n(n-1)}{2} = \frac{3(3-1)}{2} = \frac{3(2)}{2} = 3 \text{ edges.}$$

The ***number of simple graphs possible with n vertices*** $= 2^{^nC_2} = 2^{\frac{n(n-1)}{2}}$

The maximum number of simple graphs with n=3 vertices is

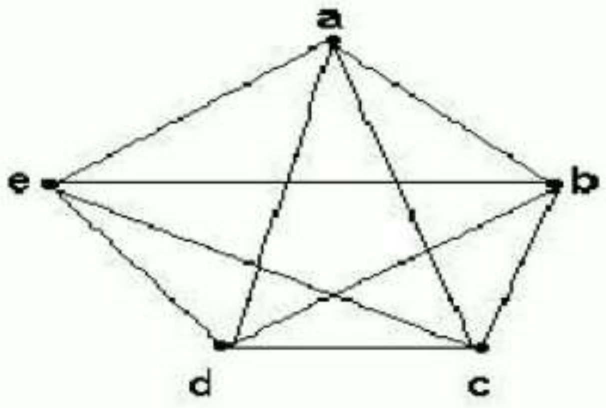
$$2^{^nC_2} = 2^{\frac{n(n-1)}{2}} = 2^{\frac{3(3-1)}{2}} = 2^{\frac{3(2)}{2}} = 2^3 = 8$$

These 8 graphs are as shown in the alongside figure.



Complete graph K_n

- Let $n \geq 3$
- The *complete graph* K_n is the graph with n vertices and every pair of vertices is joined by an edge.
- The figure represents K_5



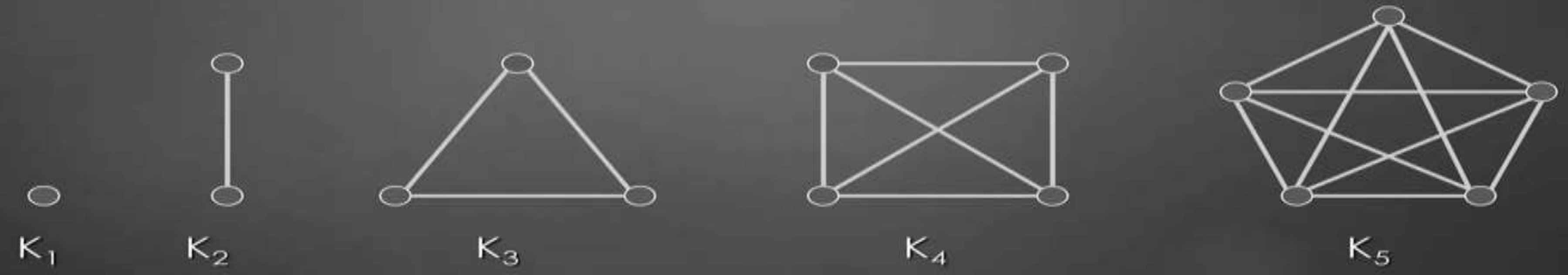
Complete Graph

Definition: Let G be simple graph on n vertices. If the degree of each vertex is (n-1) then the graph is called as **complete graph.**

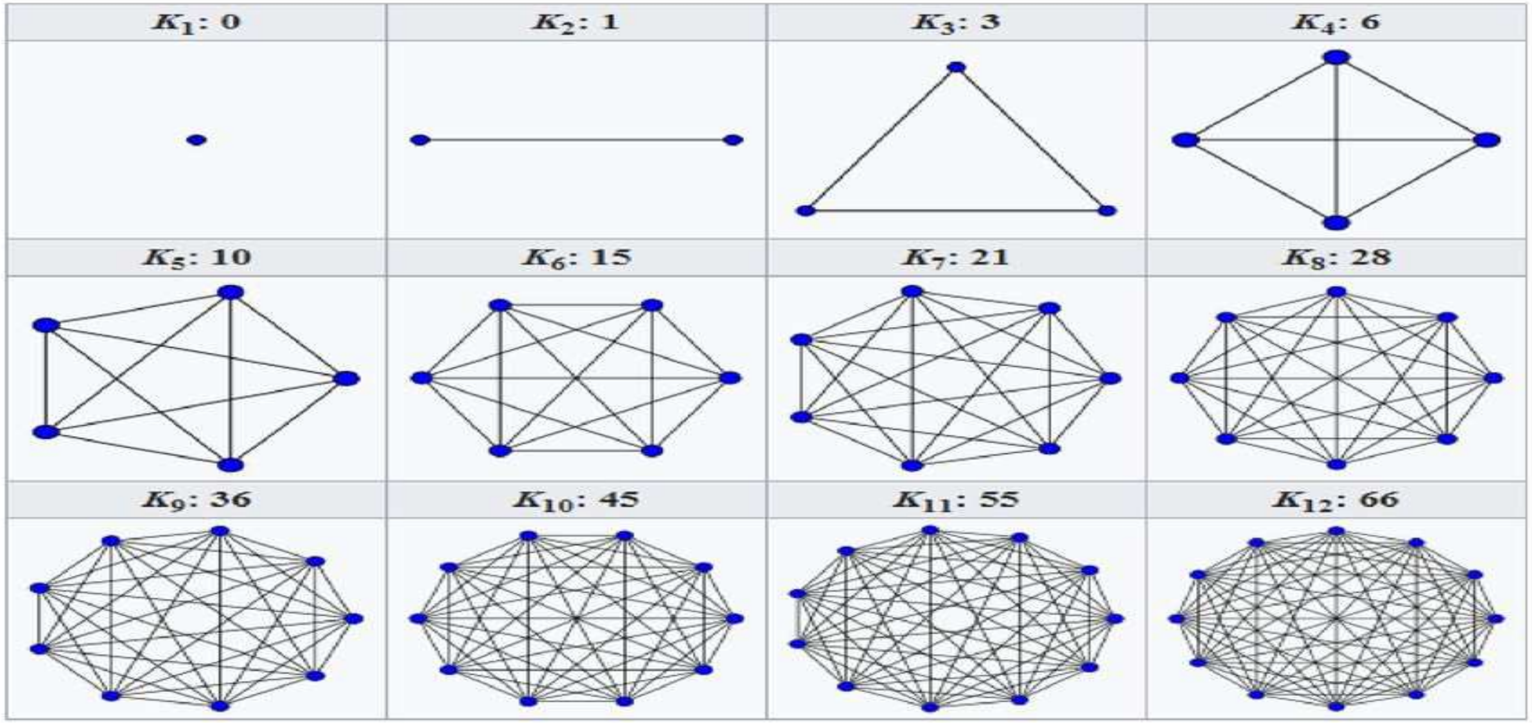
Complete graph on n vertices, it is denoted by K_n.

In complete graph K_n, the number of edges are

n(n-1)/2,For example,



Complete graphs on n vertices, for n between 1 and 12, are shown below along with the numbers of edges:



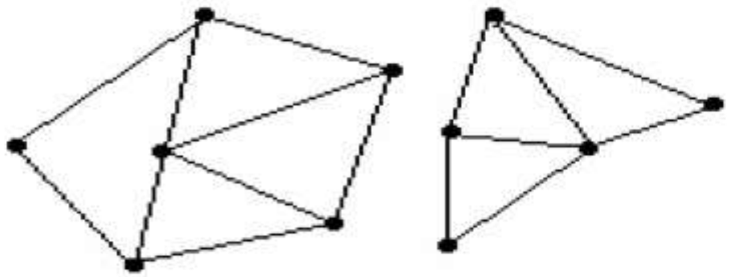
Null Graph

Definition: If the edge set of any graph with n vertices is an empty set, then the graph is known as **null graph.**

It is denoted by N_n **For Example,**

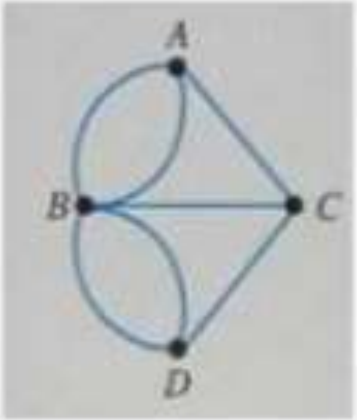
Connected graphs

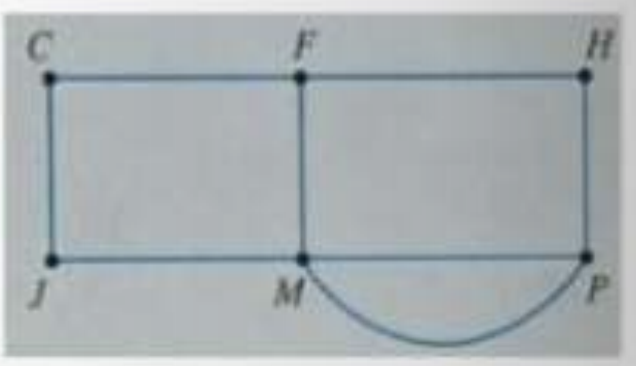
- A graph is *connected* if every pair of vertices can be connected by a path
- Each connected subgraph of a non-connected graph G is called a *component* of G



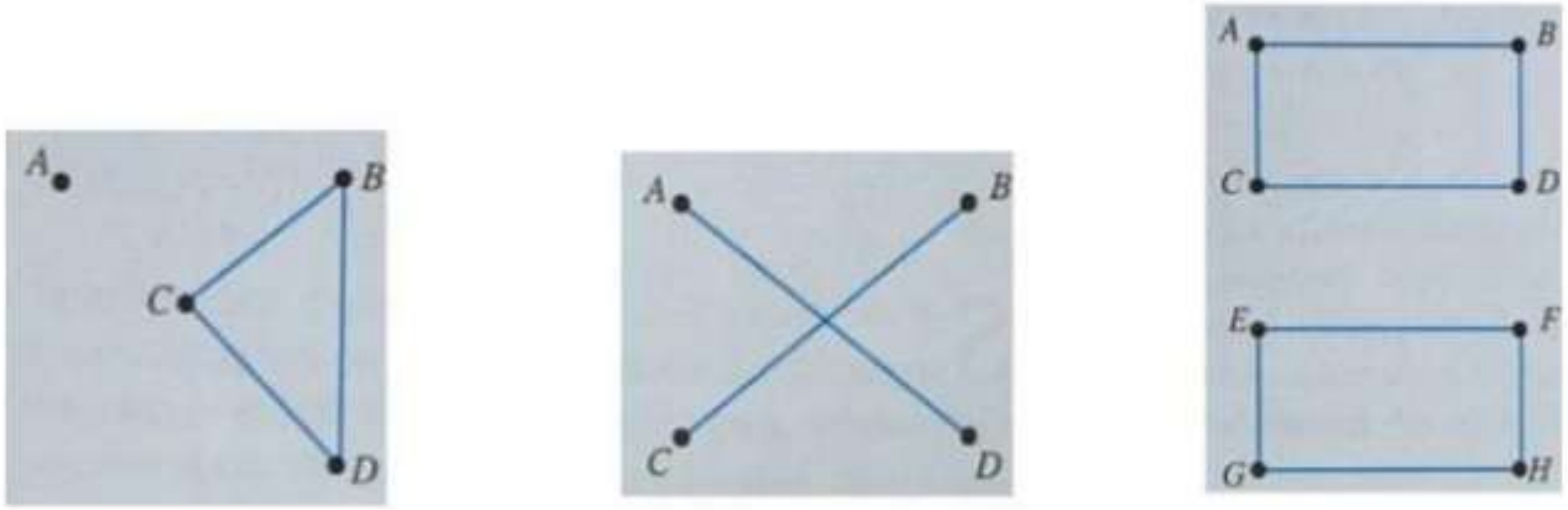
2 connected components

On a *connected graph*, you can draw a path from one vertex to any other vertex.





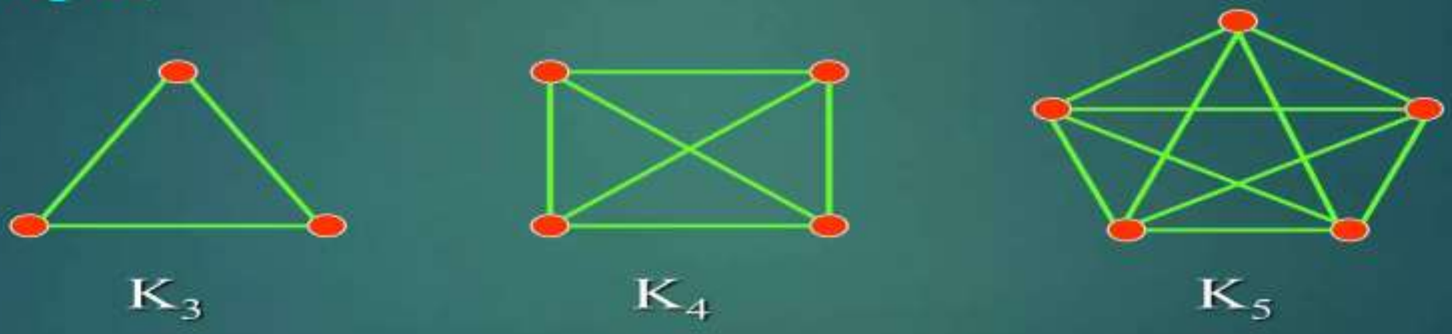
If a graph is not connected, it is ***disconnected***.



Regular Graph

Definition: If the degree of each vertex is same say 'r' in any graph G then the graph is said to be a **regular graph** of degree r.

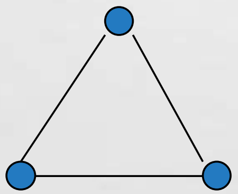
For example,



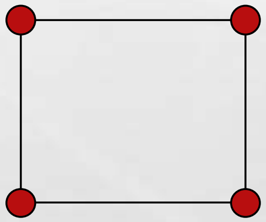
SIMPLE GRAPHS – SPECIAL CASES

Cycle: C_n, $n \geq 3$ consists of n vertices $v_1, v_2, v_3 \ldots v_n$ and edges $\{v_1, v_2\}, \{v_2, v_3\}, \{v_3, v_4\} \ldots \{v_{n-1}, v_n\}, \{v_n, v_1\}$

Representation Example: C_3, C_4



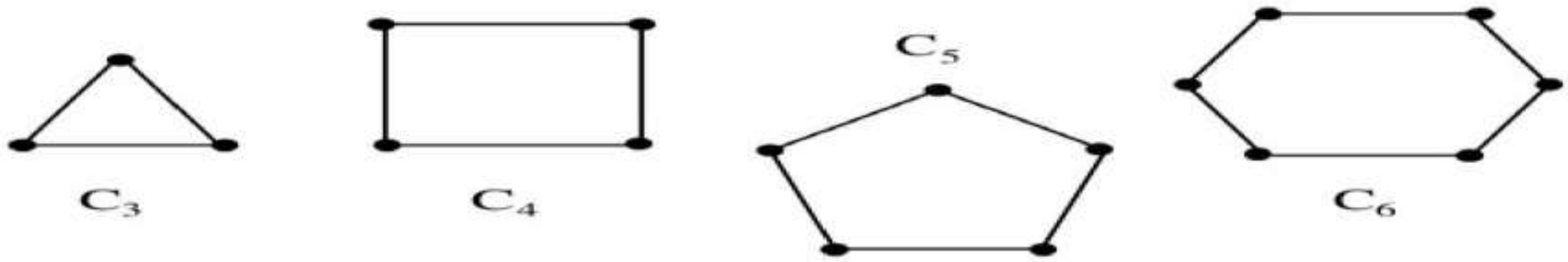
C_3



C_4

Cycle Graph

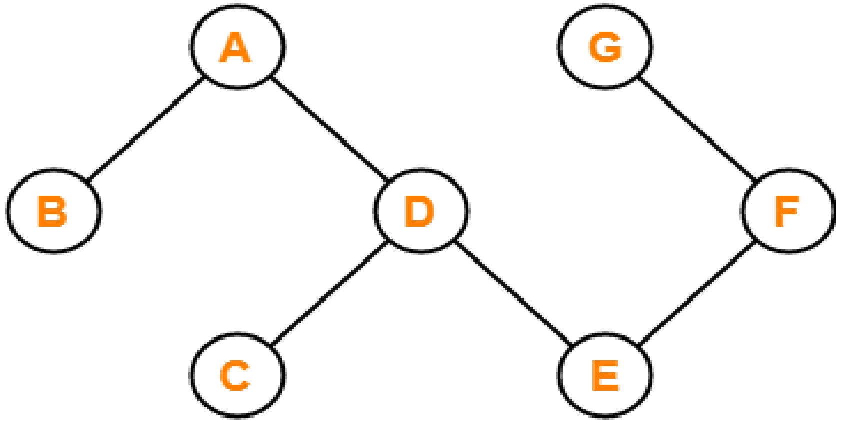
- A graph in which all the edges forms a cycle is called Cycle graph.
- The cycle graph with n vertices is denoted as C_n
- The number of vertices in C_n *is* equal to the number of edges.



11. Acyclic Graph-

- A graph not containing any cycle in it is called as an acyclic graph.

Example-



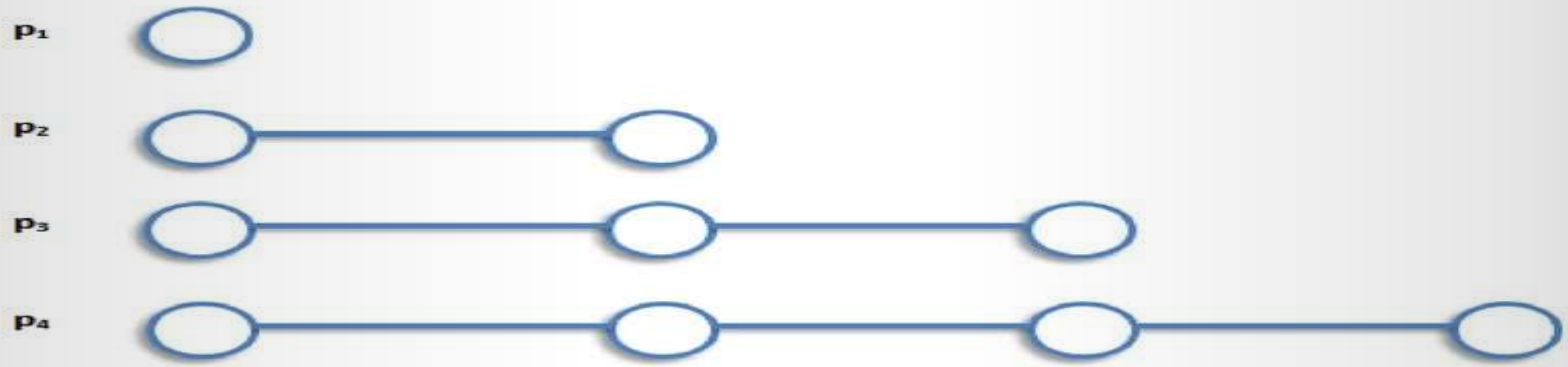
Example of Acyclic Graph

Here,

- This graph do not contain any cycle in it.
- Therefore, it is an acyclic graph.

Path Graph

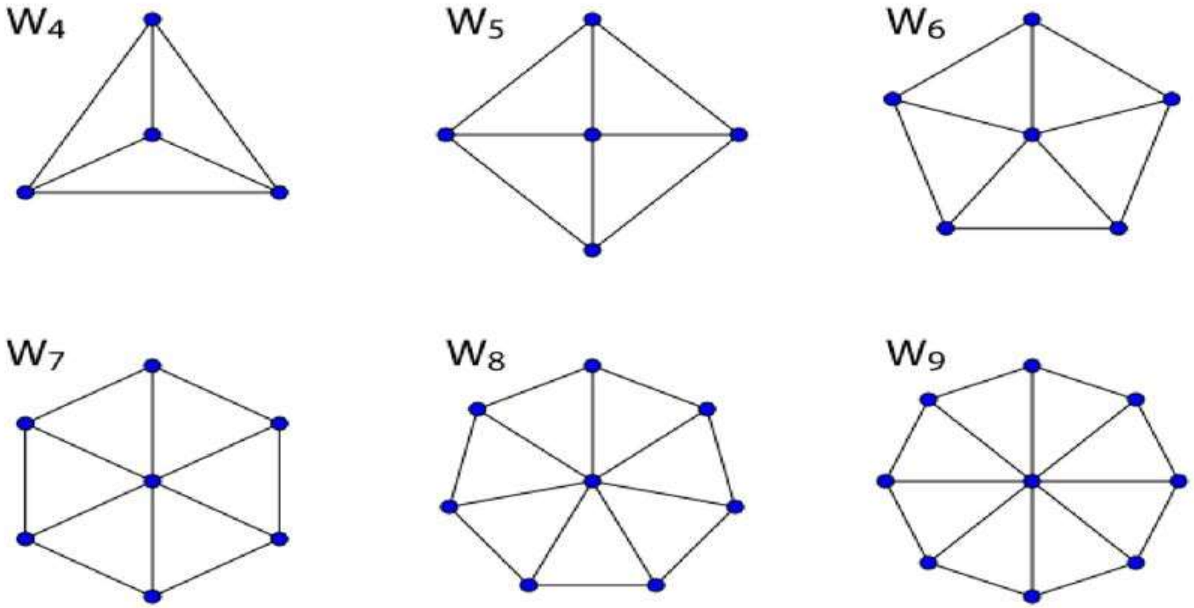
- A graph whose edges forms a path is called a path graph.
- A path graph with n vertices will have n-1 edges.



Wheel graph

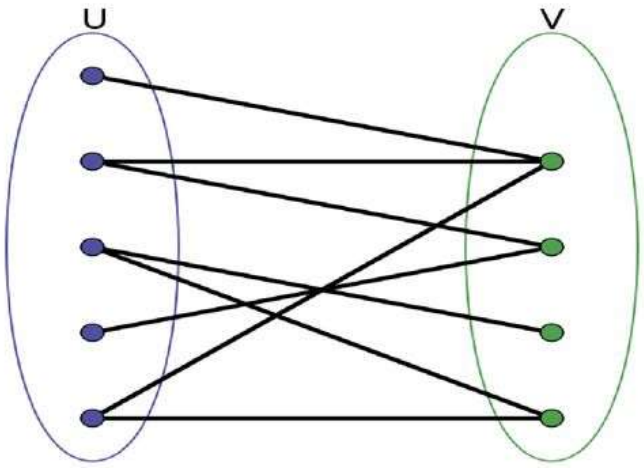
- A graph obtained from a cycle graph by joining a single new vertex(the hub) to each vertex of the cycle is called Wheel graph.
- W_n to denote a wheel graph with *n* vertices(n>=4).
- A wheel graph with n vertices contains 2(n-1) edges.

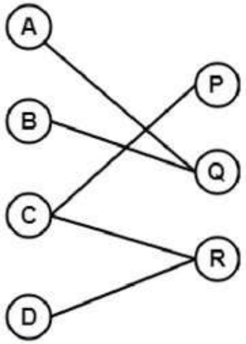
Examples



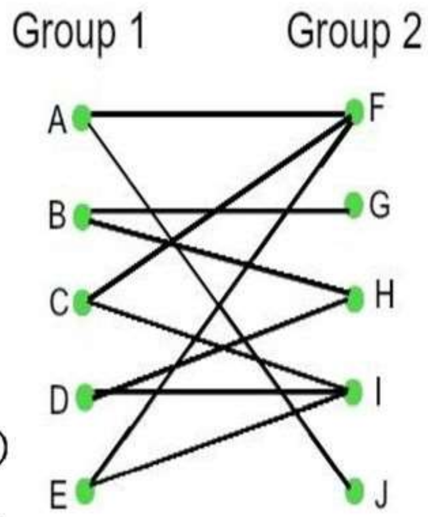
Bipartite Graph

- A graph in which the set of vertices can be partitioned into two sets M and N in such a way that each edge joins a vertex in M to a vertex in N is called a bipartite graph.
- No two graph vertices within the same set are adjacent.





Bipartite Graph



Is the following graph is bipartite?

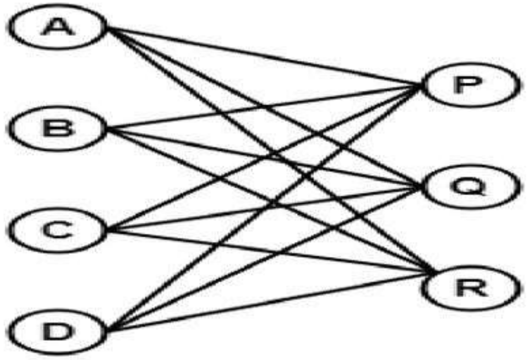
Cycle graphs with an even number of vertices are bipartite

A graph is bipartite if and only if it does not contain an odd cycle

Complete Bipartite Graph

- A Bipartite graph in which every vertex of M is adjacent to every vertex of N is called a Complete Bipartite graph.

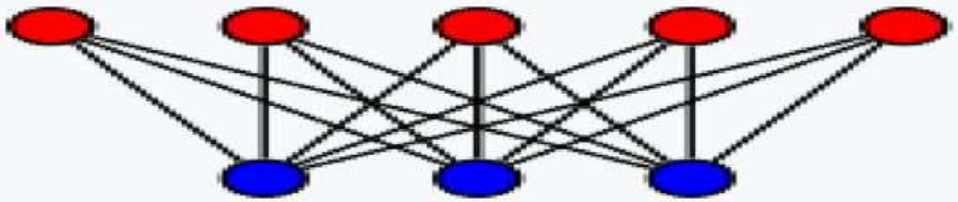
Complete Bipartite Graph = Complete Graph + Bipartite Graph



Complete Bipartite Graph

A complete bipartite graph with partitions of size $|V_1|=m$ and $|V_2|=n$, is denoted $K_{m,n}$

Complete bipartite graph

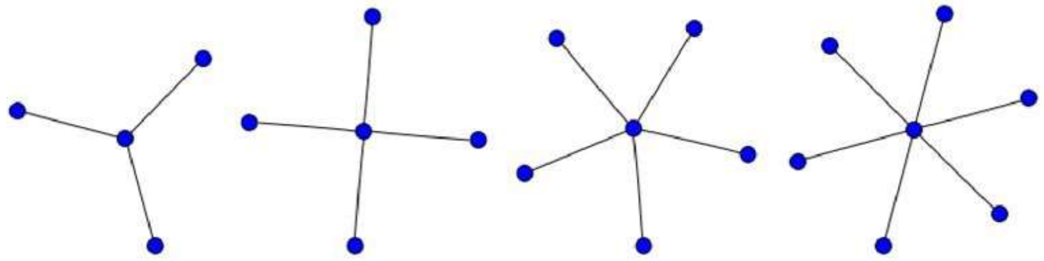


A complete bipartite graph with $m = 5$ and $n = 3$

Vertices	$n + m$
Edges	mn

Star Graph

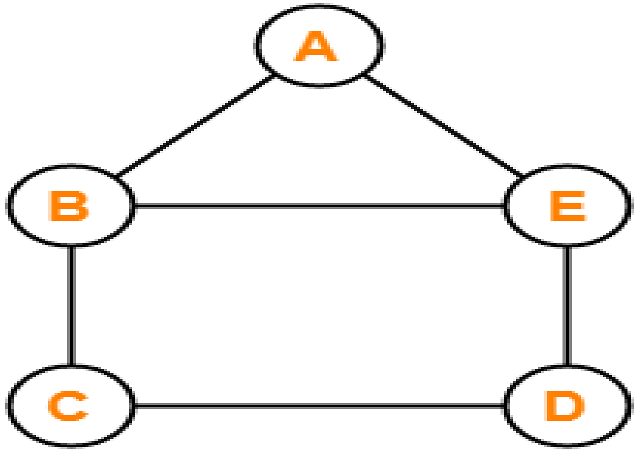
- A Complete Bipartite graph $\mathbf{k_{1,n}}$ is called a star graph.
- The star graphs $K_{1,3}$, $K_{1,4}$, $K_{1,5}$, and $K_{1,6}$.



15. Planar Graph-

- A planar graph is a graph that we can draw in a plane such that no two edges of it cross each other.

Example-

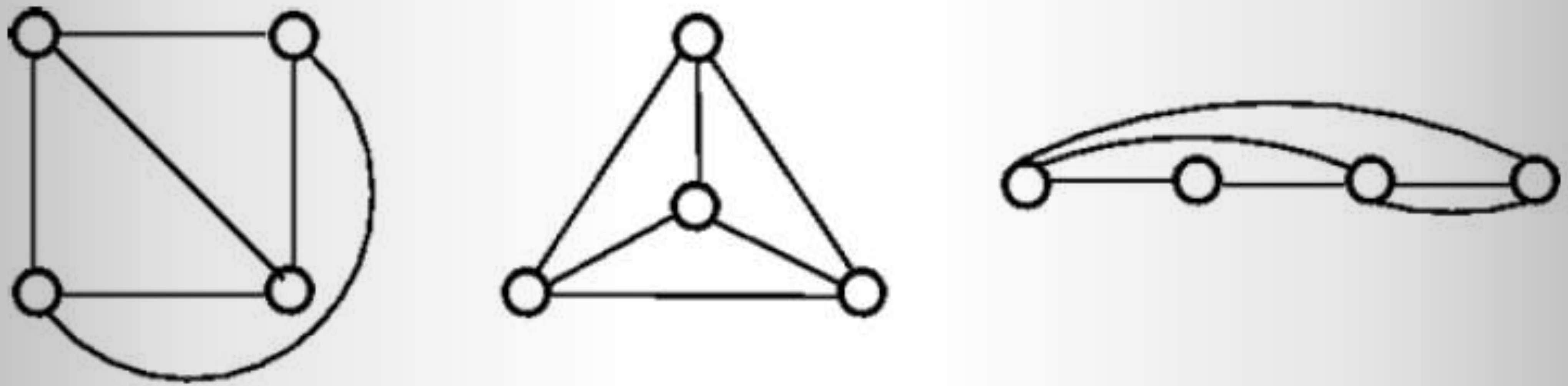


Example of Planar Graph

Here,

- This graph can be drawn in a plane without crossing any edges.
- Therefore, it is a planar graph.

The three plane drawings of the above graph are:

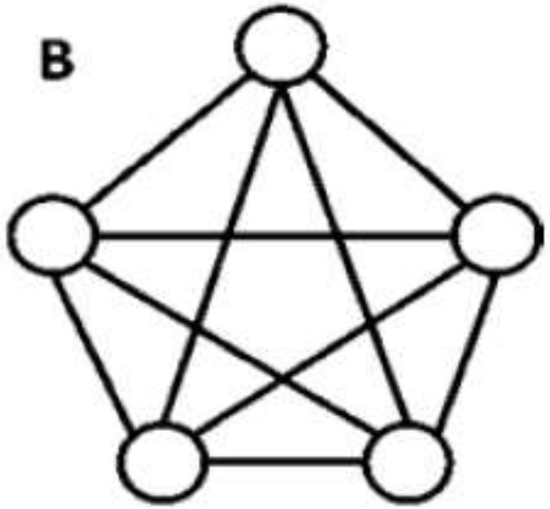


The above three graphs do not consist of two edges crossing each other and therefore, all the above graphs are planar.

18. Non - Planar Graph

A graph that is not a planar graph is called a non-planar graph. In other words, a graph that cannot be drawn without at least on pair of its crossing edges is known as non-planar graph.

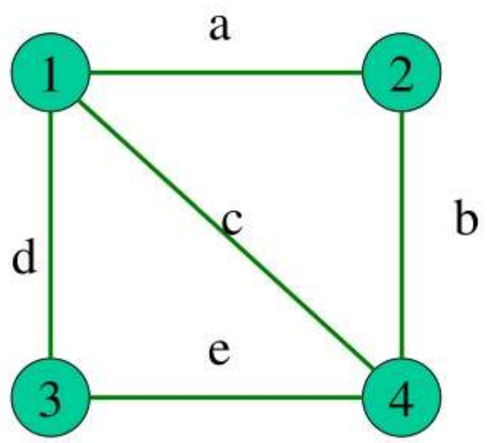
Example



Subgraphs

- Graph H=(U,F) is **subgraph** of graph G=(V,E), if U µ V and F µ E.
- **Warning!** It is important that (U,F) is indeed a graph! For each edge from F must have both of its endpoints in U.

Subgraphs - Example

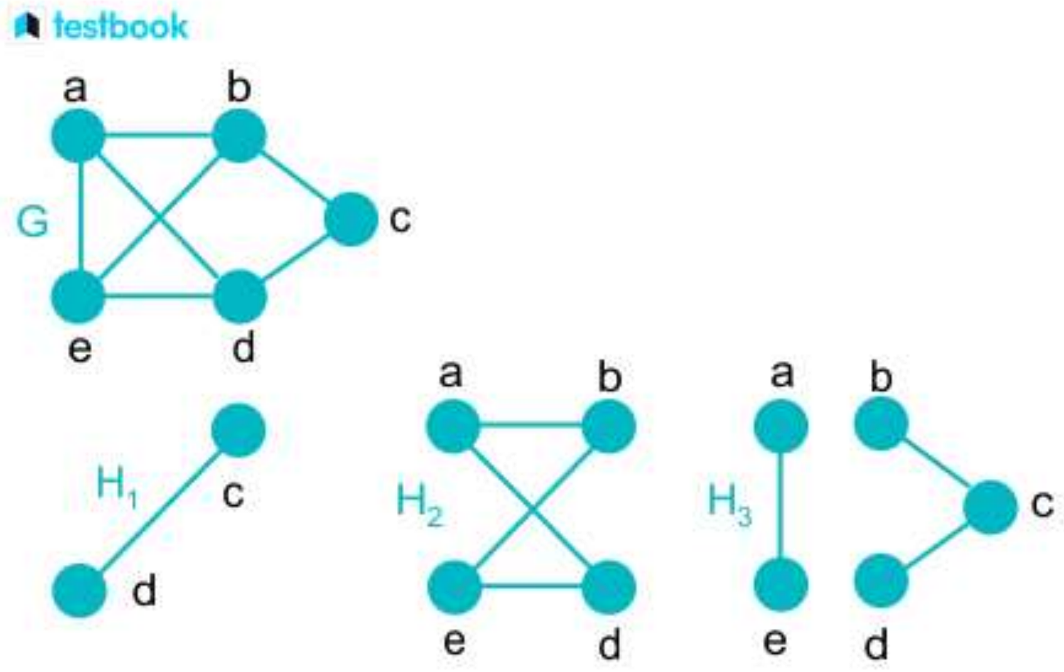


- G=(V,E)
- VG ={1,2,3,4}
- EG = {a,b,c,d,e}

Let: U = {1,2,3}, W = {2,3,4}, F = {b}, P = {a,d}. Then (U,P) and (W,F) are subgraphs while (U,F) and (W,P) are not.

Types of Subgraphs in Graph Theory

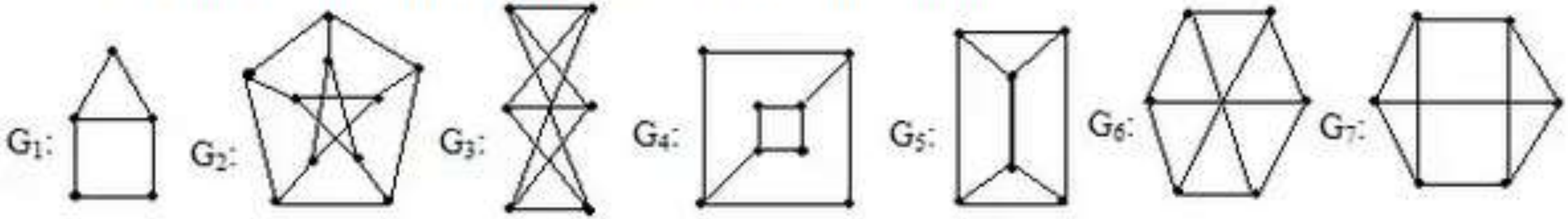
A subgraph G of a graph is graph G' whose vertex set and edge set subsets of the graph G. In simple words a graph is said to be a subgraph if it is a part of another graph.



In the above image the graphs $H_1,\ H_2,\ and\ H_3$ are different subgraphs of graph G.

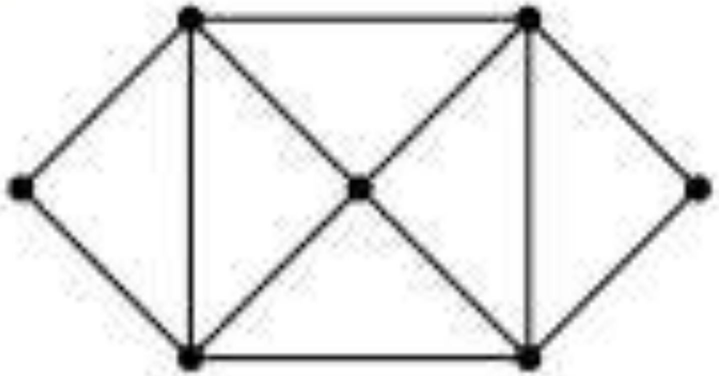
There are 2 different types of subgraph:

Exactly which of the graphs shown below are not planar?



(A) G3,G6,G7 (B) G2,G3,G7 (C) G2,G3,G6 (D) G2,G3,G6,G7

Consider the graph drawn below.



1. Find a subgraph with the smallest number of edges that is still connected and contains all the vertices.
2. Find a subgraph with the largest number of edges that doesn't contain any cycles.

Determine the number of vertices for a graph G, which has 15 edges and each vertex has degree 6. Is the graph G be a simple graph ?

- **Tree Graph:** A connected graph with no cycle. If a tree graph has m nodes, then there are m-1 edges.

Example:

- **Unweighted Graph:** A graph G is said to be un weighted if its edges are not assigned any value.

Example:

- **Weighted Graph:** A labeled graph where each edge is assigned a numerical value w(e).

Example: