University Of Bourgogne

Digital Signal Processing

Convolution and Correlation

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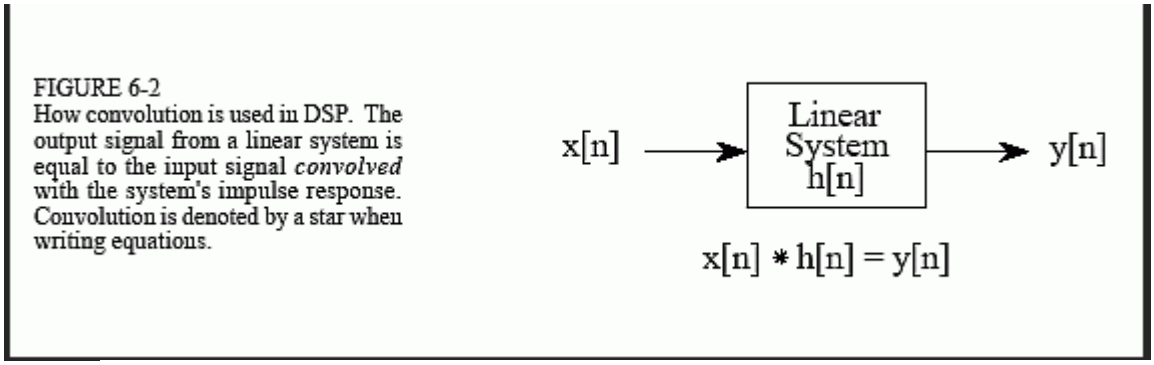
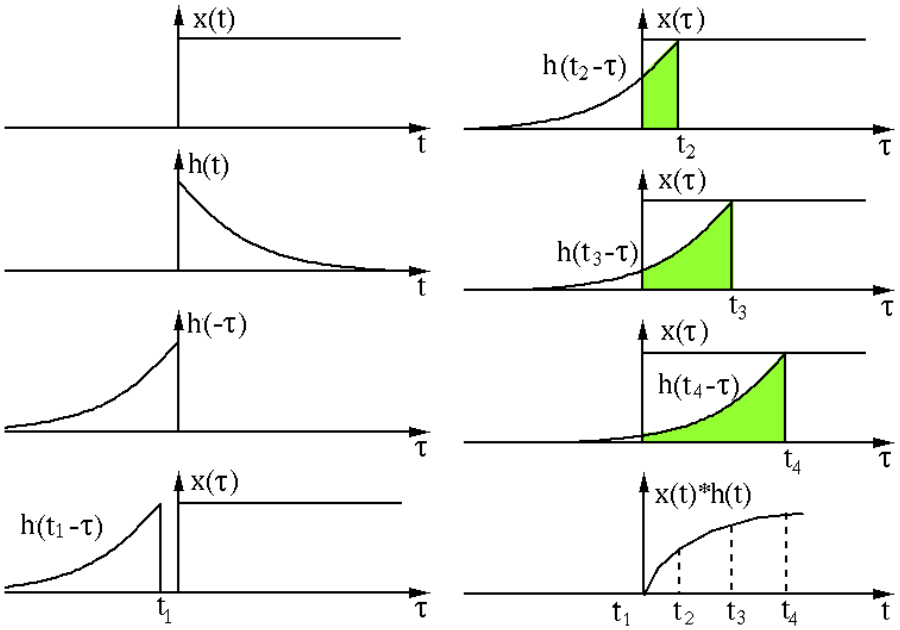
Vibot

Convolution：

Convolution is a mathematical way of combining two signals to form a third signal. It is the single most important technique in Digital Signal Processing. Using the strategy of impulse decomposition, systems are described by a signal called the *impulse response*. Convolution is important because it relates the three signals of interest: the input signal, the output signal, and the impulse response.





Formula of Convolution:

$$(f * g)[n] = \sum_{m=-\infty}^{\infty} f[m]g[n-m]$$

$$= \sum_{m=-\infty}^{\infty} f[n-m]g[m].$$

Auto Correlation:

The auto-correlation function measures the correlation of a Signal x(t) with itself shifted by some time delay j:

Its maximum value is achieved at lag =0, N, 2N (complete overlap with itself) where N is the sequence length.

The auto-correlation function can be used to detect repeats or periodicity in a signal or if it is pure random. Moreover, it can identify signals that are embedded in noise.

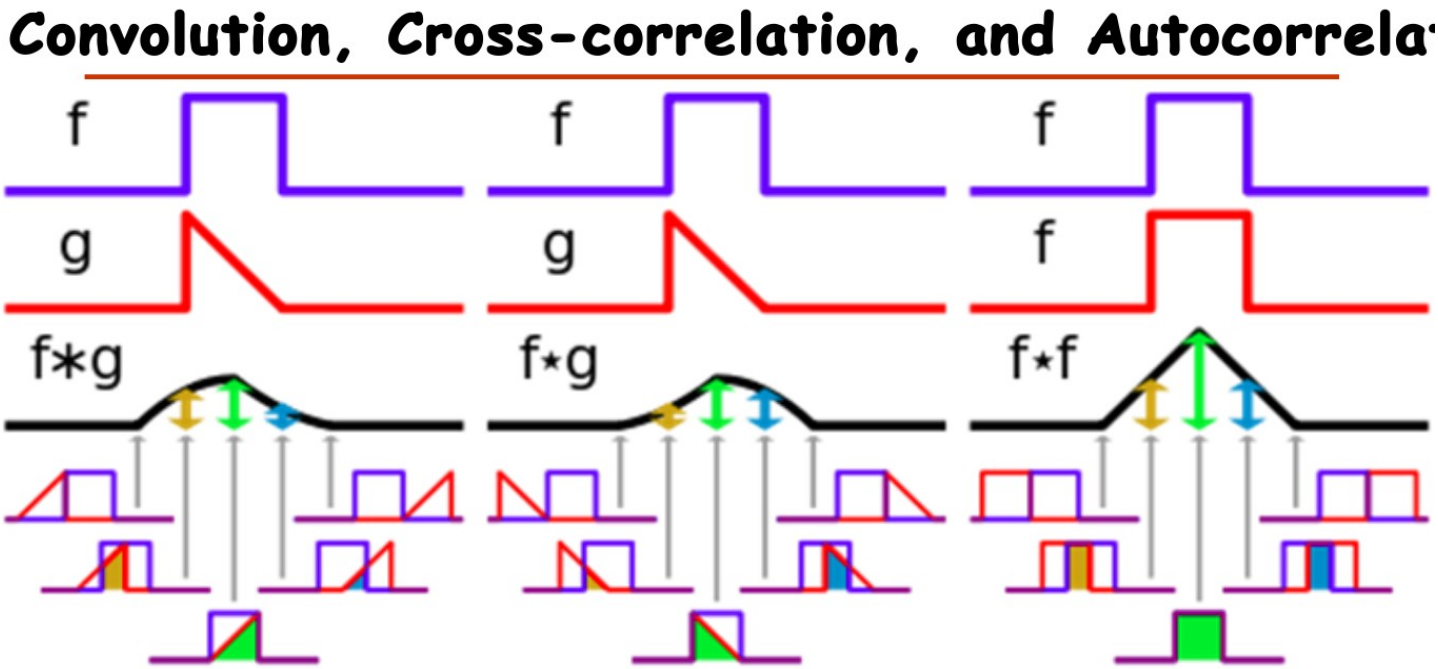
$$R_{yy}(\ell) = \sum_{n \in Z} y(n)\,\overline{y(n-\ell)} \qquad \textbf{(Eq.7)}$$

Cross Correlation:

In signal processing, cross-correlation is a measure of similarity of two series as a function of the displacement of one relative to the other. This is also known as a *sliding dot product* or *sliding inner-product*. It is commonly used for searching a long signal for a shorter, known feature. It has applications in pattern recognition, single particle analysis, electron tomography, averaging, cryptanalysis, and neurophysiology. The cross-correlation is similar in nature to the convolution of two functions. In an autocorrelation, which is the cross-correlation of a signal with itself, there will always be a peak at a lag of zero, and its size will be the signal energy.

$$R_{xy} = \sum_{n=0}^{N-1} x(n)y(n-k)$$

Convolution Vs. Cross-Correlation Vs. Auto-Correlation:



Convolution describes the response of a linear and time-invariant system to an input signal.

The inverse Fourier transform of the pointwise product in frequency space.

Cross-correlation is a measure of similarity of two signals.

It can be used for finding a shift between two signals.

Auto-correlation is the cross-correlation of a signal with itself.

It can be used for finding periodic signals obscured by noise.

http://en.wikipedia.org/wiki/Convolution

DSP System (Convolution & Correlation):

Add to The application the following features:

1. The ability to convolve two signals.
2. The ability to compute normalized cross-correlation of two signals or normalized auto-correlation of a signal.
3. The ability to compute the normalized cross-correlation of periodic signals (not necessary of same length)
4. Give the user the option to choose fast convolution\ correlation.

Notice:

1. For auto correlation only the computation of positive or negative lags is enough since it is symmetric.
2. The high similarity in computation between correlation and convolution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Auto
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Periodic
Cross
Fast
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signal2
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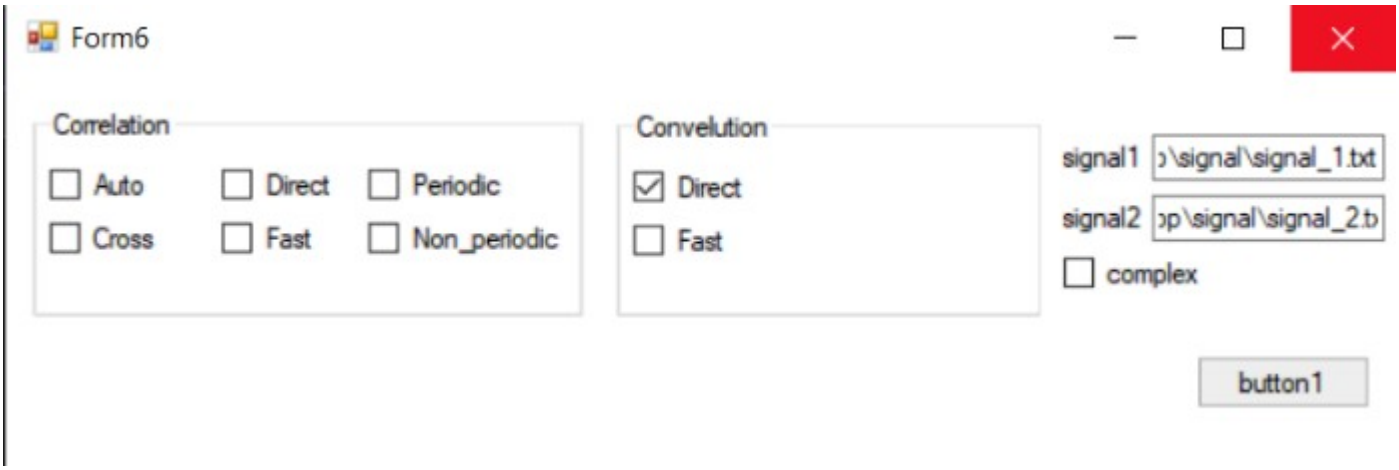
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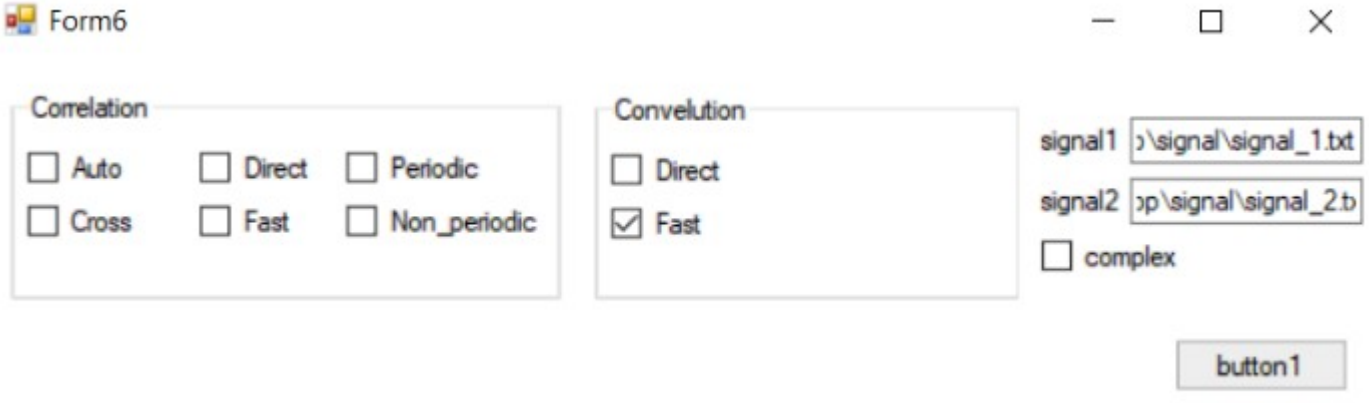
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