FINANCIAL ECONOMETRICS

WITH R

[QUANTITATIVE ANALYSIS]

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STYLIZED FACTS

- company name : **CarMax, Inc.** ;
  ticker : **KMX**
- length of the sample : days = **5030**;
  months = **240**; years = **20**
- date interval : **[2000-12-31; 2020-12-31]**
- stock market : **NYSE** ; country = **The United States of America**

1. Prices are non-stationary

Times series are either stationary or non-stationary, that is to say that their statistical properties either remain the same or change over time.

In light of the graphic representation of our empirical time serie of prices we can provide some unrigorous assumptions :

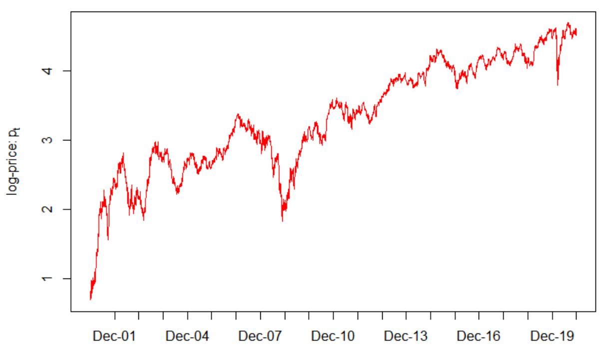

Figure 1. Log price pt: daily "adjusted closing" of CarMax stock from Yahoo.com.

1. It has a noticeable positive trend
2. Some changing levels can be identified

This visualization method gives us a somewhat strong first impression on the non-stationarity of this time series. However, we need to provide further justifications. As we voluntarily confuse stationarity with weak stationary, we can for instance use ACF (Autocorrelation Function) :

$$\rho(k) = \frac{Cov\,(p_t,\ p_{t-k})}{\sqrt{V(p_t)\,V(p_{t-k})}}$$

Where k represents 'lags'

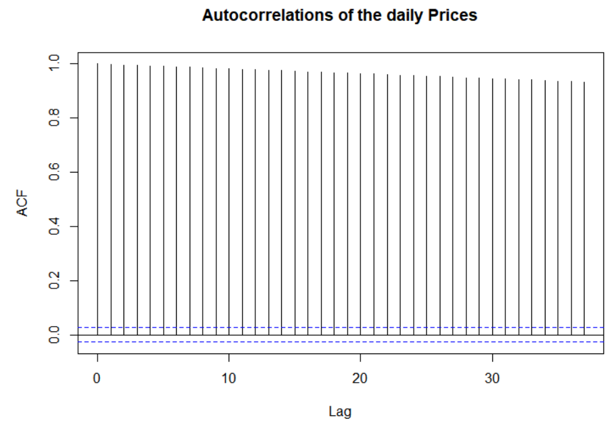

Figure 2. Empirical Autocorrelation of pt, the daily "adjusted closing" of CarMax stock from Yahoo.com.

Plotting the empirical value of the ACF for increasing lags demonstrates that it is slowly decaying and remains above the significance range. **Prices of CarMax can be considered as non-stationary.**

2. Returns are stationary

We can conduct the same method to prove that the time serie of returns is stationary, or in other words, it keeps the same statistical properties over time.

By visualization, one could easily assume that returns are stationary :

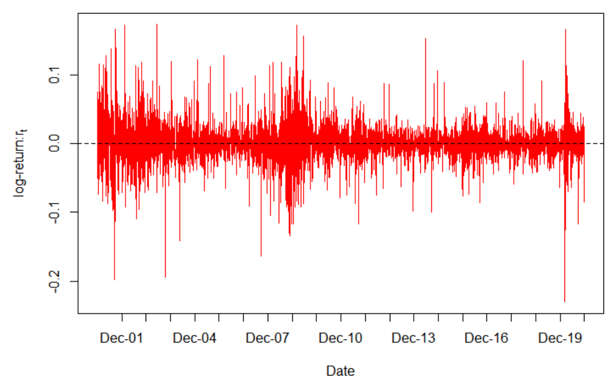

Figure 3. Log returns (rt := pt − pt−1) : daily "adjusted closing" of CarMax stock from Yahoo.com

1. It is likely to have a constant mean
2. Variation looks pretty much the same over time

As we did in the first stylized fact, we plot empirical ACF to verify the early hypothesis :

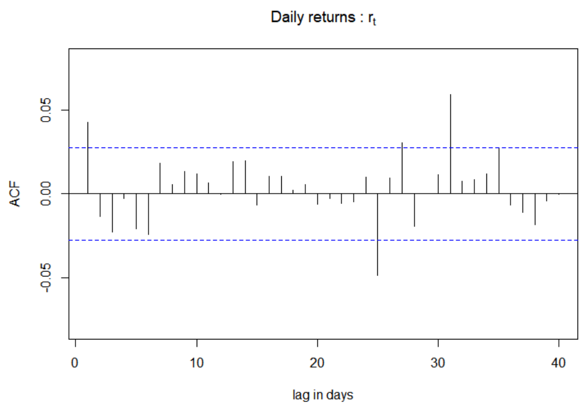

Figure 4. Empirical Autocorrelation of rt : daily “adjusted closing” of CarMax stock from Yahoo.com.

The values of ACF tend to degrade to zero quickly and somewhat remain in the Bartlett Interval. These computations allow us to confidently say that **returns of CarMax are stationary.**

Distribution of Returns

We voluntarily plot only graphs based on daily log-return for aesthetic consideration :

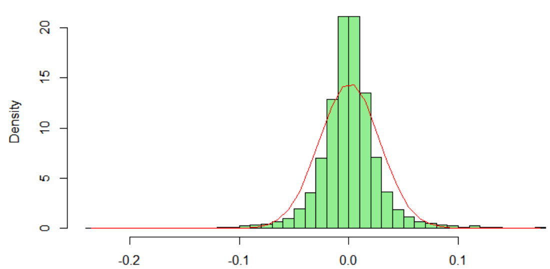

Figure 5. Log returns : histogram of daily “adjusted closing” of CarMax stock from Yahoo.com.

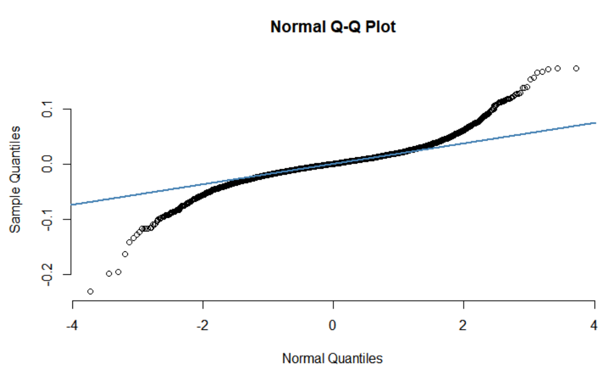

Figure 6. QQ plot of daily “adjusted closing” of CarMax stock from Yahoo.com, against quantiles of normal distribution with same mean and variance as the empirical distribution of returns

	Daily	Weekly	Montly	Annual
Mean	0.07667	0.35982	1.53307	11.15206
St.Deviation	2.75356	6.11809	12.36003	42.55344
Diameter.C.I.Mean	0.0761	0.3713	1.567	19.13401
Skewness	0.18026	0.02964	0.21723	0.04063
Kurtosis	10.04129	8.75417	5.09401	4.24351
Excess.Kurtosis	7.04129	5.75417	2.09401	1.24351
Min	-23.0433	-44.6549	-48.3634	-91.8826
Quant.5%	-3.91671	-8.9118	-15.7413	-36.711
Quant.25%	-1.21735	-2.77183	-6.30791	-7.98984
Median.50%	0.04064	0.30969	1.53662	7.62892
Quant.75%	1.28414	3.20952	7.98982	30.41511
Quant.95%	4.18997	10.23087	21.97703	70.76675
Max	17.3382	31.35785	48.65329	112.4089
Jarque.Bera.stat.X-squared	10418.32	1439.079	45.54552	1.2294
Jarque.Bera.pvalue.X100	0	0	0	54.08032
Lillie.test.stat.D	0.08363	0.08152	0.08134	0.13144
Lillie.test.pvalue.X100	0	0	0.0598	52.4605
N.obs	5030	1043	239	19

Figure 7. : Log returns : summary statistics of daily, weekly, monthly, annual “adjusted closing” of CarMax stock from Yahoo.com.

As we interpret that returns rt are similar to a random variable (or realisation), their distribution can be portrayed by their empirical (estimated) moments : mean, variance, skewness and kurtosis.

3. Returns are asymmetric

Computing sample skewness in Figure 7, we find that the skewness of returns rt is significantly different from zero (from daily : 0.18 to annual : 0.04)), and tends to have positive values. That is to say we observe large upward movements in CarMax stock but less large drawdown movements. This observation is not completely in line with the stylized fact presented in Lecture 2 as the skewness is not negative.
Nevermind, **one can conclude that the distribution of returns is asymmetric.**

4. Returns are leptokurtic

Computing sample kurtosis in Figure 7, we also find that the kurtosis of returns rt is large enough to be considered as leptokurtic (from daily : 10 to annual : 4.2). That means that the distribution of returns exhibits heavy tails or at least heavier than those of a Gaussian distribution.
Further ad-hoc evidence is observable with the QQ plot of Figure 6, which highlights the fact that it has a leptokurtic shape. The lower (resp. higher) quantiles of the distribution of returns are much smaller (resp. larger) than those of a Normal distribution with the same empirical mean and standard deviation than the sample.

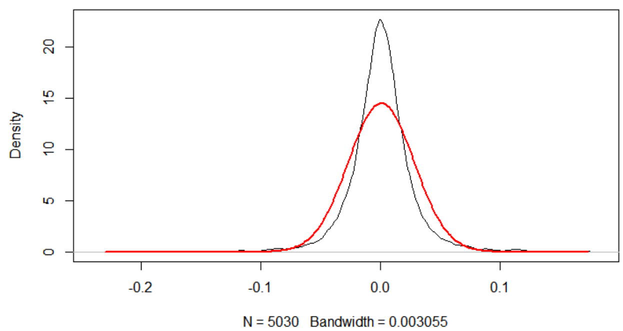

Figure 8. Log returns : Kernel density of the empirical distribution compared to a Normal distribution (red line) with the same empirical mean and standard deviation of the data.

The same can be observed with a kernel density, the peak around zero appears clearly, which indicates a thinner and fat-tailed distribution curve compared to a normal distribution. This signifies more weight in the tails and so more extreme values. Consequently, **one can conclude that the distribution of returns is leptokurtic**

5. Aggregational Gaussianity

As seen previously, mainly from daily frequency data, distribution of CarMax stock returns is likely to imply non-Gaussianity (with slight positive skewness and large kurtosis excess). In order to prove this conjecture, we need to use normality test (Lilliefors test) or tools that partially replicate it (Jarque Bera test)

Using the Jarque Bera test only allows us to tell if some time series do not follow a normal distribution => it is not a normality test. With the JB test statistics that we computed in Figure 7, we can easily reject the normality of the distribution with :

- ➔ Daily Returns : JB test statistic (=10418) > critical value at 10% (=4.61)
- ➔ Weekly Returns : JB test statistic (=1439) > critical value at 10% (=4.61)
- ➔ Monthly Returns : JB test statistic (=45.5) > critical value at 10% (=4.61)

The test fails to reject the null hypothesis of Annual returns (JB test statistic = 1.2), that only means that the distribution is symmetric and mesokurtic.

We need to adopt a true normality test to provide a precise statement on the normality of the distribution of returns in different frequencies. To do so, we use the Lilliefors test, a derivative of the Kolmogorv-Smirnov test. With the Lillie test statistics computed in figure 7, we can reject the normality distribution of :

- ➔ Daily Returns : Lillie test statistic (=0.083) > critical value at 10% (= $\frac{0.805}{\sqrt{5030}}$ = 0.012)
- ➔ Weekly Returns : Lillie test statistic (=0.081) > critical value at 10% (= $\frac{0.805}{\sqrt{1043}}$ = 0.027)
- ➔ Monthly Returns : Lillie test statistic (=0.081) > critical value at 10% (= $\frac{0.805}{\sqrt{239}}$ = 0.052)

And we can't reject the null hypothesis of the Normality of the returns at a 10% confidence level for :

- ➔ Annual Returns : Lillie test statistic (=0.131) < critical value at 10% (= $\frac{0.805}{\sqrt{19}}$ = 0.203)

Therefore, we have good evidence against the normality of high frequency returns (daily, weekly and to a lesser extent monthly) thanks to Jarque Bera and Lilliesfors tests. On the other hand, we have some evidence of normality distribution for annual returns thanks to Lilliefors test.

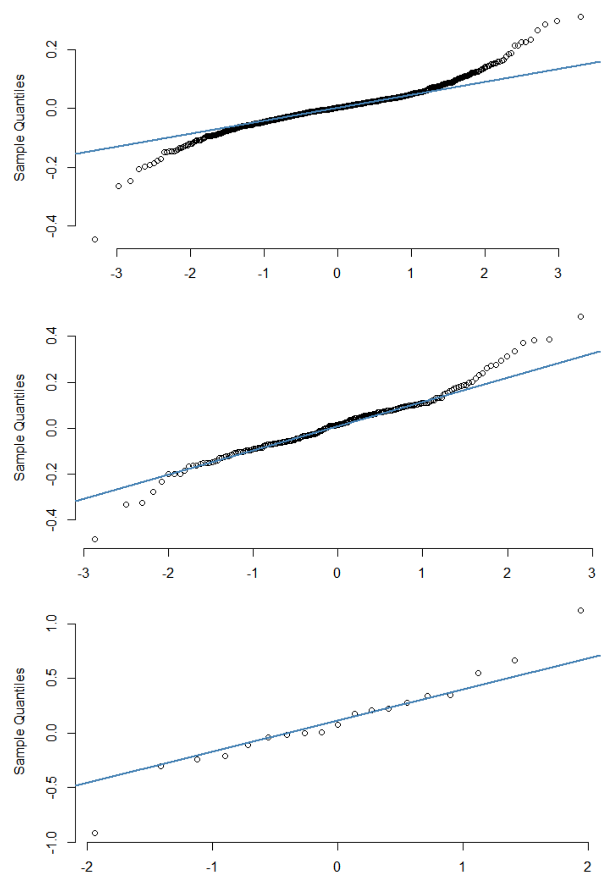

Figure 9. QQ plots of weekly, monthly and annual "adjusted closing" returns of CarMax stock from Yahoo.com, against quantiles of normal distribution with same mean and variance as the empirical distribution of returns

In view of figures 6 and 9, one can observe that aggregated returns (mostly annual) have some similarities with normal distribution contrary to higher frequent data (daily, weekly and to a lesser extent monthly). Which means that if we lower the frequency, the distribution of returns looks more and more like a normal distribution.

QQ plots and normality test provide evidence in favor of aggregational Gaussianity of returns of CarMax stock.

The time dependence of Returns

6. Returns are not autocorrelated

Autocorrelations of returns are often insignificant (except for very high intraday frequency). One can establish the level of autocorrelation by computing the empirical ACF as we did in the first two stylized facts.

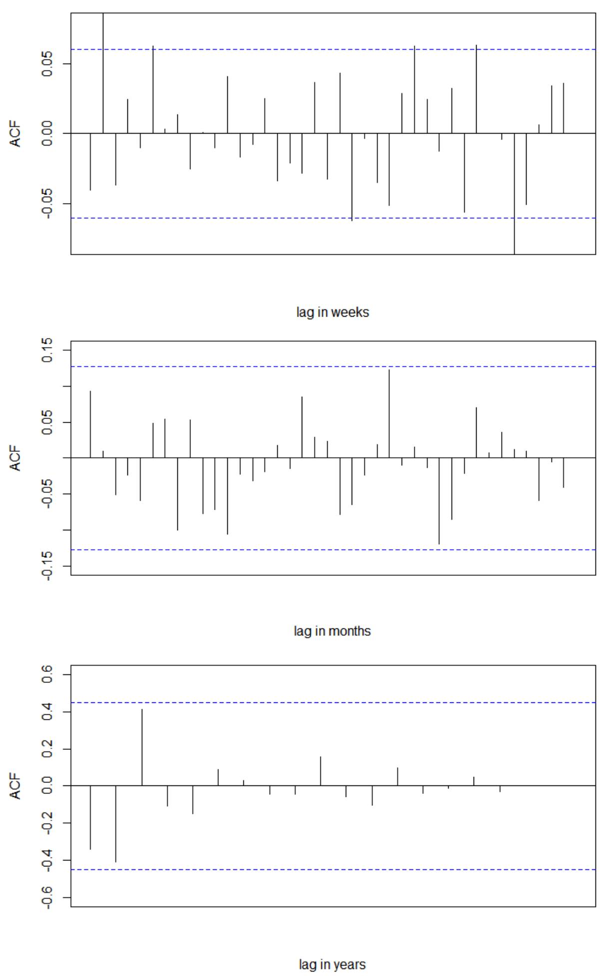

Figure 10. Empirical Autocorrelation of rt : weekly, monthly and annual return from the “adjusted closing” of CarMax stock from Yahoo.com.

Empirical autocorrelation (figures 4 and 10) provides us with a strong indication about the insignificance of autocorrelation between lagged returns thanks to the Bartlett Intervals for weekly, monthly and annual returns. In spite of this, we need to test the insignificance of autocorrelation of daily returns with other methods such as Box-Pierce and Ljung-Box tests.

lag	ACF	ACF diam.	acf test	BP stat	BP pval	LB stat	LB pval	crit
1	0.04283	0.02764	3.03761	9.22705	0.00238	9.23256	0.00238	3.84146
2	-0.01348	0.02764	-0.95579	10.14059	0.00628	10.14682	0.00626	5.99146
3	-0.02279	0.02764	-1.61605	12.75222	0.0052	12.76104	0.00518	7.81473
4	-0.00255	0.02764	-0.18074	12.78488	0.01238	12.79375	0.01233	9.48773
5	-0.02092	0.02764	-1.48386	14.98673	0.01042	14.99866	0.01037	11.0705
6	-0.02398	0.02764	-1.70055	17.87861	0.00654	17.89515	0.0065	12.59159
7	0.0182	0.02764	1.29106	19.54544	0.00664	19.56496	0.00659	14.06714
8	0.00533	0.02764	0.37771	19.6881	0.01158	19.70791	0.0115	15.50731
9	0.01335	0.02764	0.94697	20.58485	0.01463	20.60663	0.01452	16.91898
10	0.01175	0.02764	0.83315	21.279	0.01923	21.30243	0.01908	18.30704
11	0.00645	0.02764	0.45713	21.48797	0.02865	21.51194	0.02844	19.67514
12	-0.00035	0.02764	-0.02513	21.4886	0.04367	21.51258	0.04336	21.02607
13	0.01942	0.02764	1.37732	23.38562	0.03727	23.41527	0.03695	22.36203
14	0.01948	0.02764	1.38178	25.29494	0.03176	25.33068	0.03144	23.68479
15	-0.00672	0.02764	-0.47671	25.5222	0.04336	25.55871	0.04293	24.99579
16	0.01048	0.02764	0.74345	26.07491	0.05298	26.11341	0.05245	26.29623
17	0.01055	0.02764	0.74829	26.63486	0.06364	26.67547	0.063	27.58711
18	0.0022	0.02764	0.15625	26.65927	0.08561	26.69998	0.0848	28.8693
19	0.0058	0.02764	0.41146	26.82857	0.10871	26.86999	0.10772	30.14353
20	-0.00611	0.02764	-0.43368	27.01665	0.1348	27.0589	0.13362	31.41043
21	-0.00271	0.02764	-0.1922	27.05359	0.16909	27.09601	0.1677	32.67057
22	-0.00564	0.02764	-0.40032	27.21385	0.20322	27.25703	0.20163	33.92444
23	-0.00482	0.02764	-0.34164	27.33057	0.24216	27.37434	0.24037	35.17246
24	0.01019	0.02764	0.7227	27.85286	0.2663	27.89934	0.26431	36.41503
25	-0.04838	0.02764	-3.43129	39.62658	0.03184	39.73658	0.03103	37.65248

Figure 11. : Empirical Autocorrelation (ACF), ACF “diameter”, Box-Pierce (BP) test and Ljung-Box test (LB) : statistics and p-values. Data : daily log-return from the “adjusted closing” of CarMax.

We have evidence in favor of insignificance of autocorrelation of daily (log-)returns as we can observe in figure 11: Box-Pierce and Ljung-Box p-values are quite different from zero so the null hypothesis is not rejected => ACF are not significantly different from zero. Consequently, **one can conclude that there is an absence of autocorrelation for returns**

7. Volatility clustering

There is empirical evidence that periods with large (resp. low) volatility are likely to be followed by periods of large (resp. low) volatility. To verify this statement, we can use the ARCH effect, which implies that squared returns are significantly autocorrelated. As we did for the previous stylized fact, we can establish the level of autocorrelations of squared returns with correlograms.

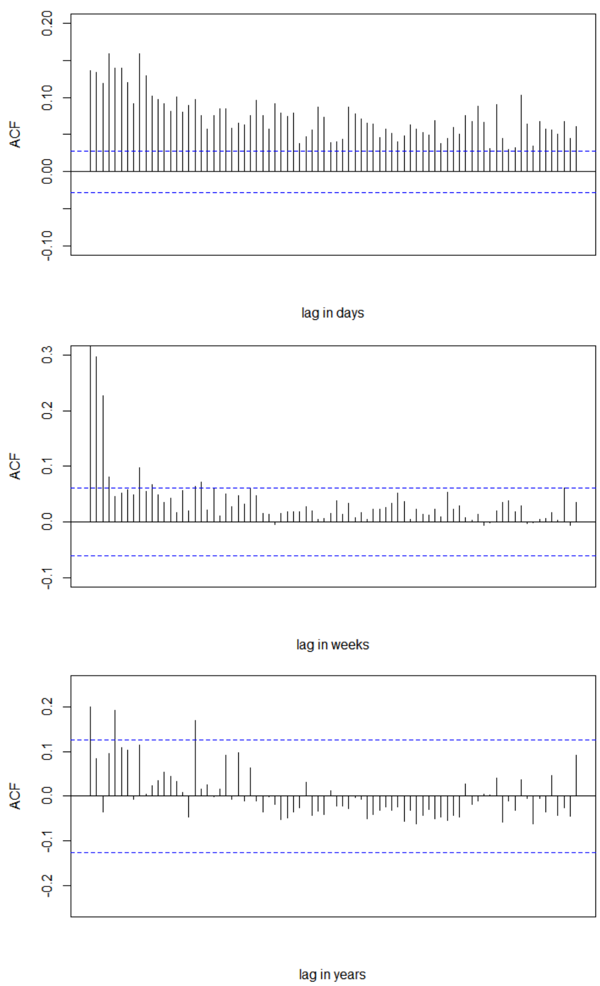

Figure 12. Empirical Autocorrelation of the daily, weekly and annual SQUARED log-return from the “adjusted closing” of CarMax stock from Yahoo.com.

The first correlograms (daily) of figure 1 exhibits significant autocorrelations, which are very consistent as lag increases, indeed it decays very slowly to zero. The second one shows less significant autocorrelations, which decay to zero. The last one presents a few weak autocorrelations, which decay quickly to zero. That means that the ARCH effect is observable with CarMax stock as autocorrelations are persistent and significant for high frequency (and to a lesser extent, lower frequency) squared returns.

These are indications of volatility clustering.

8. Leverage effect

There is empirical evidence that evolution of volatility and returns are negatively correlated, this is called leverage effect. One can verify if CarMax stock respects this leverage effect by computing empirical cross-correlation between lagged returns and squared returns (evolution of volatility in some ways), and comparing daily returns of CarMax stock with the changes in the VIX index :

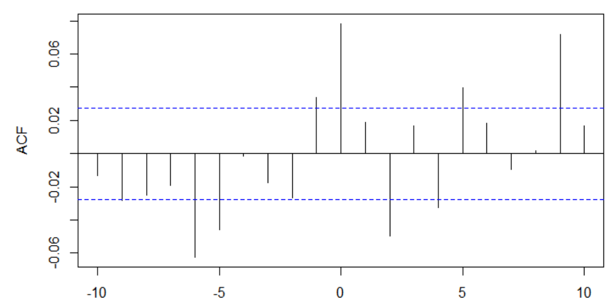

Figure 13. Empirical cross-correlation of lagged log-returns, with squared returns. Daily log-return from the “adjusted closing” of CarMax from Yahoo.com.

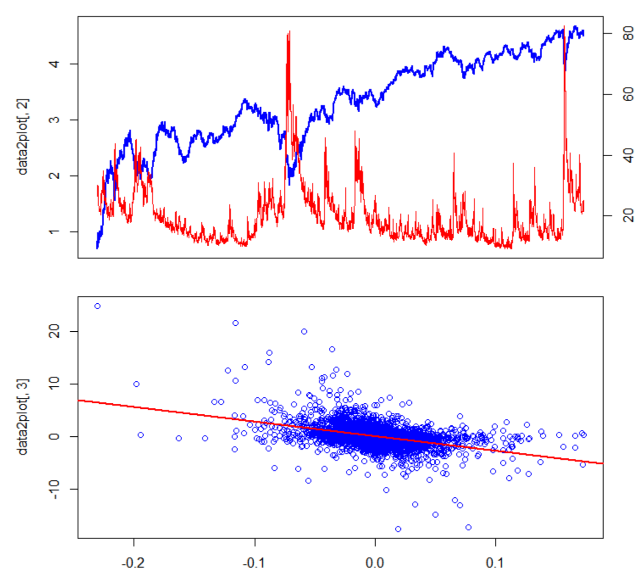

Figure 14. At the top : Time series plot of VIX (red) and CarMax stock (blue). At the bottom: scatter plot of daily CarMax log-returns against the daily changes of VIX for the same day.

- ➔ There is evidence of cross-correlation in figure 13
- ➔ We can see in the top panel of figure 14 that when price plummets, VIX explodes. In the bottom panel we observe with the regression line that negative returns implies high VIX.

There is a leverage effect in CarMax stock.

APPENDIX: R code

```
#########################################
# FINANCIAL ECONOMETRICS / FINAL EXAM
#
#########################################

# LOAD ALL THE PACKAGES/LIBRARIES (already installed)
############################################
library(quantmod)
library(xts)
library(readr)
library(latex2exp)
library(summarytools)
library(qwraps2)
library(normtest)
library(nortest)
library(moments)
library(xtable)
library(sm)
library(astsa)
library(portes)
library(forecast)
library(portes)
library(tseries)
library(sm)
library(nortest)
library(zoo)
############################################
rm(list=ls())
setwd("C:/Users/Dadou/Documents/Fin Econometrics [Final Exam]")
getwd()

############################################
# CarMax Stock
KMX    <- getSymbols("KMX",from="2000-12-31", to="2020-12-31", auto.assign=FALSE)
# VIX index
VIX      <- getSymbols("^VIX",from="2000-12-31", to="2020-12-31", auto.assign=FALSE)

############################################
#STYLIZED FACT 1
Pt.d  <- KMX$KMX.Adjusted
names(Pt.d)  <- "Pt.d"
pt.d  <- log(Pt.d)
names(Pt.d)  <- "pt.d"

# find end of month/week/year dates
last_day_of_month    <-    endpoints(pt.d,    on    = "months") # works on xts objects
last_day_of_week  <- endpoints(pt.d, on = "weeks")
last_day_of_year  <- endpoints(pt.d, on = "years")

# Compute weekly (w), monthly(m), and annual(y) log prices
pt.w  <-  pt.d[last_day_of_week]  ;  names(pt.w)    <- "pt.w"
pt.m  <-  pt.d[last_day_of_month];  names(pt.m)    <- "pt.m"
pt.y   <-  pt.d[last_day_of_year]   ;  names(pt.y)     <- "pt.y"

# compute log returns # for entire history of S&P 500
rt.d  <- diff(pt.d)     ; names(rt.d)  <- "rt.d"
rt.w  <- diff(pt.w)     ; names(rt.w)  <- "rt.w"
rt.m  <- diff(pt.m)     ; names(rt.m)  <- "rt.m"
rt.y  <- diff(pt.y)     ; names(rt.y)  <- "rt.y"
col2plot  <- c("red","blue","black")
lwd2plot  <- c(1    , 1     , 1    )
lty2plot  <- c(1    , 1     , 1    )

# convert prices int dataframes to produce nice plots
Pt.d.df    <-    cbind(index(Pt.d),    data.frame(Pt.d)); names(Pt.d.df)[1] <- "date"
pt.d.df    <-    cbind(index(pt.d),    data.frame(pt.d)); names(pt.d.df)[1] <- "date"
pt.w.df    <-    cbind(index(pt.w),    data.frame(pt.w)); names(pt.w.df)[1] <- "date"
rt.d.df    <-    cbind(index(rt.d),    data.frame(rt.d)); names(rt.d.df)[1] <- "date"
rt.w.df    <-    cbind(index(rt.w),    data.frame(rt.w)); names(rt.w.df)[1] <- "date"
rt.m.df    <-    cbind(index(rt.m),    data.frame(rt.m)); names(rt.m.df)[1] <- "date"
rt.y.df    <-    cbind(index(rt.y),    data.frame(rt.y)); names(rt.y.df)[1] <- "date"

###########################################
# PLOT: Prices, log-prices and log-returns of SP500
###########################################
# Daily Price
data2plot <- Pt.d.df;
plot(x = data2plot[,1], y = data2plot[,2], type = 'l', col=col2plot[1]        , lty = lty2plot[1], lwd = lwd2plot[1],
       xlab=""   , ylab=TeX('price: $P_t$'), main="", xaxt ="none")
seq_sel <- endpoints(data2plot$date, on = 'years'); date_seq = data2plot$date[seq_sel]; date_lab = format(date_seq,"%b-%y")
axis(1, at = date_seq, label = date_lab, las = 1, cex.axis=1.0); abline(0,0, lty = 2)
```

```
# Daily log price
data2plot <- pt.d.df;
plot(x = data2plot[,1], y = data2plot[,2], type = 'l', col=col2plot[1]	, lty = lty2plot[1], lwd = lwd2plot[1],
     xlab=""  , ylab=TeX('log-price: $p_t$'), main="", xaxt ="none")
seq_sel <- endpoints(data2plot$date, on = 'years'); date_seq = data2plot$date[seq_sel]; date_lab = format(date_seq,"%b-%y")
axis(1, at = date_seq, label = date_lab, las = 1, cex.axis=1.0)
abline(0,0, lty = 2)

# Daily log-returns
data2plot <- rt.d.df;
plot(x = data2plot[,1], y = data2plot[,2], type = 'l', col=col2plot[1]	, lty = lty2plot[1], lwd = lwd2plot[1],
        xlab=TeX("Date")   , ylab=TeX('log-return: $r_t$'), main="",  xaxt ="none")
seq_sel <- endpoints(data2plot$date, on = 'years'); date_seq = data2plot$date[seq_sel]; date_lab = format(date_seq,"%b-%y")
axis(1, at = date_seq, label = date_lab, las = 1, cex.axis=1.0); abline(0,0, lty = 2) # add zero line

###########################################
#ACF
#...of prices
acf(pt.d, main="Autocorrelations of the daily Prices")
#...of returns
lag.max.acf = 40;  lim.y.axes = c(-0.08,0.08)
data2plot = rt.d.df[-1,2]; # daily returns
Acf(data2plot, main=TeX('Daily returns : $r_t$'), lag.max = lag.max.acf, xlab = "lag in days", ylim=lim.y.axes, xlim = c(1,40))

data2plot = rt.w.df[-1,2]; # weekly returns
Acf(data2plot, main=TeX('Weekly returns : $r_t$'), lag.max = lag.max.acf, xlab = "lag in weeks", ylim=lim.y.axes, xlim = c(1,40))

lag.max.acf2 = 20;  lim.y.axes2 = c(-0.15,0.15)
data2plot = rt.m.df[-1,2]; # monthly returns
Acf(data2plot, main=TeX('Monthly returns : $r_t$'), lag.max = lag.max.acf, xlab = "lag in months", ylim=lim.y.axes2, xlim = c(1,40))

lim.y.axes3 = c(-0.60,0.60)
data2plot = rt.y.df[-1,2]; # year returns
Acf(data2plot, main=TeX('Annual returns : $r_t$'), lag.max = lag.max.acf2, xlab = "lag in years", ylim=lim.y.axes3, xlim = c(1,20))

##########################################
#Daily returns distribution vs normal distribution
data2plot = rt.d.df[,2];
hist_OUT <- hist(data2plot, freq = FALSE, breaks = 50, col="lightgreen",   xlab="", main=TeX('daily log-return'), )
norm_y <- dnorm(hist_OUT$mids, mean=mean(data2plot, na.rm=TRUE), sd=sd(data2plot, na.rm=TRUE));
lines(x=hist_OUT$mids, y=norm_y,col="red", lwd=1)

# QQ-plot vs quantiles of normal distribution
qqnorm(rt.d.df[,2], pch = 1, frame = FALSE, xlab="Normal Quantiles")
qqline(rt.d.df[,2], col = "steelblue", lwd = 2)
# QQ plot of weekly data
qqnorm(rt.w.df[,2], pch = 1, frame = FALSE, xlab="Normal Quantiles")
qqline(rt.w.df[,2], col = "steelblue", lwd = 2)
# QQ plot of annual data
qqnorm(rt.m.df[,2], pch = 1, frame = FALSE, xlab="Normal Quantiles")
qqline(rt.m.df[,2], col = "steelblue", lwd = 2)
# QQ plot of annual data
qqnorm(rt.y.df[,2], pch = 1, frame = FALSE, xlab="Normal Quantiles")
qqline(rt.y.df[,2], col = "steelblue", lwd = 2)

##########################################
#    Create function (named 'multi.fun' which computes the statics that we want on the input 'x')
multi.fun <- function(x) {
  c(Mean = mean(x)*100,
    St.Deviation = sd(x)*100,
    Diameter.C.I.Mean = qnorm(0.975)*sqrt(var(x)/length(x))*100,
    Skewness=moments::skewness(x),
    Kurtosis=moments::kurtosis(x),
    Excess.Kurtosis=moments::kurtosis(x)-3,
    Min    = min(x)*100,
    Quant  = quantile(x, probs = 0.05)*100,
    Quant  = quantile(x, probs = 0.25)*100,
    Median = quantile(x, probs = 0.50)*100,
    Quant  = quantile(x, probs = 0.75)*100,
    Quant  = quantile(x, probs = 0.95)*100,
    Max    = max(x)*100,
    Jarque.Bera.stat = jarque.bera.test(x)$statistic,
    Jarque.Bera.pvalue.X100 = jarque.bera.test(x)$p.value*100,
    Lillie.test.stat = lillie.test(x)$statistic,
```

```
                     Lillie.test.pvalue.X100      =
lillie.test(x)$p.value*100,
   N.obs = length(x)
  )}
# GENERATES TABLE 1 in slide 91
X <-list("daily" = rt.d.df[-1,2],
       "weekly" = rt.w.df[-1,2],
       "montly" = rt.m.df[-1,2],
       "annual" = rt.y.df[-1,2])
a <- sapply(X, multi.fun) # apply function to all
elements of list X,
# PRINT TABLE 1 ON R console
print(a)
round(a, digits = 5)

write.csv(round(a,          digits          =
5),file='table_STEP2.csv')

############################################
#KERNEL density
data2plot <- rt.d.df[-1,2]; # daily returns
mean.data <- mean(data2plot, na.rm=TRUE)
sd.data   <- sd(data2plot, na.rm=TRUE)
density.eval.points <- seq(from = min(data2plot), to
= max(data2plot), length.out = 300)
sm.density.out <- sm.density(data2plot, h = 0.0020,
eval.points = density.eval.points, dispaly="none")

# h is the 'bandwidth parameter': --> the larger the
bandwidth, the smoother the histogram
sm.density.out.default    <-    sm.density(data2plot,
eval.points = density.eval.points, dispaly="none")
# different bandwidth can produce different kernel
densities.....

# R default function is:
d <- density(data2plot)
plot(d)
norm_y    <-     dnorm(sm.density.out$eval.points,
mean=mean(data2plot,             na.rm=TRUE),
sd=sd(data2plot, na.rm=TRUE));
lines(x=sm.density.out$eval.points,
y=norm_y,col="red", lwd=2, lty=1)

############################################
#ACF, BOX tests
my.data <- rt.d.df[-1,2]
lags.all <- seq(1,25,1)
my.max.lag     <- 25
lags.all       <- seq(1,my.max.lag,1)
my.acf         <- acf(my.data, lag.max = my.max.lag,
plot = FALSE)
my.acf.diameter                               <-
qnorm(0.975)/sqrt(length(my.data))
my.acf.tstat.0          <-       (my.acf$acf[-1]     -
0)/sqrt(1/length(my.data))
my.LjungBox    <- LjungBox(my.data, lags=lags.all)
my.BoxPierce   <- BoxPierce(my.data, lags=lags.all)
crit.value.5.BP <- qchisq(0.95,lags.all)

my.table <- cbind(my.BoxPierce[,1],
              my.acf$acf[-1],
              my.acf.diameter,
              my.acf.tstat.0,
              my.BoxPierce[,2],
              my.BoxPierce[,4],
              my.LjungBox[,2],
              my.LjungBox[,4],
              crit.value.5.BP)
my.table.df <-as.data.frame(my.table)
names(my.table.df)  <- c("lag","acf","acf diam.","acf
test","Box-Pierce   stat","BP   pval","LB   stat","LB
pval","crit")
rownames(my.table.df) <-c()
options(scipen = 999)
b <- data.matrix(my.table.df)
round(b, digits = 3)
write.csv(round(b,          digits          =
5),file='ACF_test_table.csv')

############################################
#Cross-correlation
ret   = (rt.d.df[-1,2])   ; # daily returns
ret2  = (rt.d.df[-1,2])^2 ; # daily SQUARED returns
ccf(ret, ret2, lag.max = 10, type = "correlation",  plot
= TRUE)
#VIX and CarMax comparision
par(mfrow=c(1,1))
data2plot <- a.comp;
plot(x  =  data2plot[,1],  y = data2plot[,2], type ="l",
xaxt ="none",   col = "blue", lwd =2)
par(new = TRUE)
plot(x = data2plot[,1], y = data2plot[,3], type = "l",
xaxt = "n", yaxt = "n",
    ylab = "", xlab = "", col = "red", lty = 1)
axis(side = 4)
mtext("VIX", side = 4, line = 3)
seq_sel <- endpoints(data2plot$date, on = 'years');
date_seq   =   data2plot$date[seq_sel];  date_lab   =
format(date_seq,"%b-%y")
axis(1,  at  =  date_seq,  label  =  date_lab,  las  =  1,
cex.axis=1.0)

par(mfrow=c(1,1))
data2plot <- b.comp;
plot(x   =   data2plot[,2],   y   =   data2plot[,3],   col   =
"blue", lwd =1)
abline(lm(data2plot[,3]~data2plot[,2]),      col="red",
lwd=2) # regression line (y~x)
```