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FPGA Implementation of ML, ZF and MMSE Equalizers for MIMO Systems

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Abstract

This paper presents an FPGA implementation of Maximum likelihood (ML), zero forcing (ZF) and minimum mean squared error (MMSE) equalizers applied to wireless multi-input multi-output (MIMO) systems with no fewer receive than transmit antennas. In spite of much prior work on this subject, we reveal several new and surprising analytical results in terms of output signal-to-noise ratio (SNR), by comparing the Bit Error Rate (BER) and the average detection time consuming. Results based on the platform of Xilinx Virtex 6. We discuss the case where there a multiple transmit antennas and multiple receive antennas resulting in the formation of a Multiple Input Multiple Output (MIMO) channel with Zero Forcing equalizer, MIMO with MMSE equalizer, MIMO with ZF Successive Interference Cancellation equalizer, MIMO with ML equalization, MIMO with MMSE SIC and optimal ordering.

1. Introduction

The large execution times demanded for solving complex optimization problems in embedded systems is one of the main challenges in the field of engineering optimization. One solution is the acceleration by a specialized hardware implementation. However, this is coming along with a loss of flexibility especially for the realization of the application-specific fitness function [1]. Multiple transmit and receive antennas are currently used to significantly enhance the performance of wireless communications of MIMO systems. MIMO systems have received a significant amount of attention in recent years for the promise of greatly increasing spectral efficiency and exploiting both transmits and receives diversity when the channel state information is known. However, the wireless channel uncertainties, especially in fast fading scenarios, pose challenges in achieving the benefits in the MIMO systems [2] [3] [4] [5]. It usually takes more resources like power and bandwidth to deal with the unknown channel coefficients in multiple antenna systems [6]. At present FPGAs and ASICs are playing a vital role in designing, simulating, testing and implementing the new communication techniques. Modulation scheme such as QAM is one of the widely used modulation techniques in cellular communication because of its high efficiency in power and bandwidth16-QAM (Quadrature Amplitude Modulation) is a kind of digital modulation scheme which transmits four bits per symbol on two orthogonal carriers [7]. The optimal detectors like ZF and MMSE have some bad performances. However, the ML has the best performances in terms of binary errors rates: it's optimal. But this method has the inconvenience to be complex when the number

of points of the constellation is big and when the number of antenna increases [8]. In this paper we present the performance of FPGA implementation of ML, ZF and MMSE equalizers for MIMO systems for different QAM modulations

2. Problem formulation

Let us consider a MIMO system with M transmits antennas and N receives antennas. Then the channel output is written as:

$$y = Hx + w \quad H \in R^{m \times n} \tag{1}$$

Where

$H \in R^{m \times n}$ is the MIMO channels matrix, $x \in A^n$ is the transmitted symbol matrix and

$W \in R^m$ is an additive noise matrix whose elements are assumed to be i.e. complex Gaussian random variables. The performance of MIMO communication system can be measured in term of binary success rate (BER).

The probability of mistake is generally calculated according to SNR (Signal to Noise Ratio) report γ.

$$\gamma = \varepsilon \left[\frac{|Hx|^2}{|W|^2} \right] \tag{2}$$

We choice $\quad s = Hx$,

$$\varepsilon \left[|s|^2 \right] = \varepsilon \left[\sum_{k=1}^m |s_k|^2 \right] = \varepsilon \left[\sum_{k=1}^m \sum_{j=1}^n |h_{kj} x_j|^2 \right]$$

$$= \sum_{k=1}^m \sum_{j=1}^n \varepsilon \left[|h_{kj}|^2 \right] . \varepsilon \left[|x_j|^2 \right] = E_S . n . m \tag{3}$$

Considering

$$\varepsilon \left[|w|^2 \right] = 2.m.N_0 \tag{4}$$

We obtain the medium SNR:

$$\gamma = \frac{E_s . n}{2.N_0} \tag{5}$$

The factor 2 in the middle power of the noise appears because the real and imaginary parts of w have each the N_0 variance; therefore, the variance of every component of w is $2N_0$ [9].

In this work, we focus on the model of the AWGN channel (Additive White Gaussian Noise) which is the most frequently used channel model for numerical simulation. This model represents the received signal as the sum of the transmitted signal and an additive white Gaussian noise. The latter models the noise which can be external sources such as noise or internal antenna such as thermal noise caused by the agitation of electrons in electronic equipment for reception.

The interference of the channel is represented as a linear symbolized by a multiplication of $\tilde{X}$ with a matrix $\tilde{H}$ of dimension NxM channel. With the sound channel is modeled by an additive white Gaussian noise $\tilde{W}$. The vector, $\tilde{Y}$ with Mx1 dimensional as shown in Figure 1, is obtained at the reception was written as

$$\tilde{Y} = \tilde{H} \tilde{X} + \tilde{W} \tag{6}$$

$$\tilde{X} \longrightarrow \boxed{\tilde{H}} \longrightarrow \oplus \longrightarrow \tilde{Y}$$
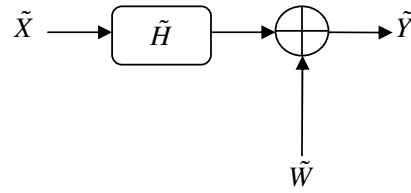
$$\tilde{W}$$

Fig 1.The AWGN channel model

3. Maximum Likelihood detector for MIMO systems

Assuming a linear time-invariant data transmission system, these data of the receiver are interfered by the additive white Gaussian noise. ML algorithm is the best joint detection for the symbol vector y , which is transmitted by each antenna in a symbol interval. When the transmitted information symbol vectors have equal probability, ML is equivalent to the following cost function presented in (7):

Consider the system of linear diagram shown in fig.2. To communicate on that channel, we are faced with the task of detecting a set of M symbols transmitted from a set of N signals observed. Our observations are corrupted by a non-ideal communication channel, generally modeled as a linear system followed by an additive noise vector.
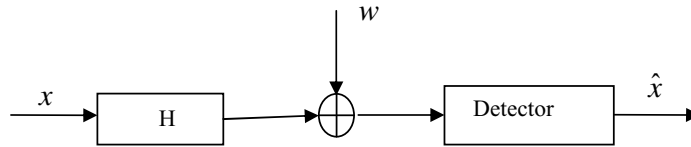
$$x \longrightarrow \boxed{H} \longrightarrow \oplus \longrightarrow \boxed{\text{Detector}} \longrightarrow \hat{x}$$

$$w$$

Fig 2. General principle of detection: symbol vector transmitted $x \in A^m$, channel matrix $H \in \mathbb{R}^{n \times m}$, additive noise vector $w \in \mathbb{R}^n$, observation vector $y \in \mathbb{R}^n$ and the detected symbol $\hat{x} \in \mathbb{R}^m$

For AWGN channel, the interference channel model become $y = x + w$ and y is a noisy version of x. The distance minimization criterion simplifies to

$$\hat{x} = \arg\min_{x \in A^m} |y - x|^2 \qquad (7)$$

4. Zero forcing detectors for MIMO Systems

ZF algorithm uses the pseudo-inverse matrix of the channel matrix H as a weight vector, and then we obtain the output before decision shown in (8). ZF detection algorithm introduced the concept of pseudo-inverse matrix to simplify the algorithm, and the channel matrix H is transformed into N_T parallel scalar channels together with the noise. Obviously, the noise component is enhanced due to the left multiplication. Thus, ZF algorithm reduces the complexity of the ML algorithm, but its performance declined at the same time.

The estimator of the vector transmitted by ZF is obtained at the receiver using the following process

$$G = H^+ = \left(H^H H \right)^{-1} H^H \qquad (8)$$

Where H + is the pseudo inverse matrix of H (also called inverse matrix as defined in Moore-Penrose)

Because the cost function is convex, this problem has a unique minimum :

$$\rho_{ZF} = H^+ y = \left(H^H H \right)^{-1} H^H y = x + \tilde{w} \tag{9}$$

With $\tilde{w} = H^+ w$ transformed noise

5. Minimum Mean-Square Error detector for MIMO systems

ZF algorithm can cancel the other antennas' interfere, but enlarges the background noise and its performance is relatively poor. Consequently, some scholars proposed the MMSE detection algorithm. [10] MMSE detection algorithm not only requires the calculation of the inverse matrix, but also needs an estimation of the SNR for all receiving antennas. Therefore, MMSE has the effective suppression of background noise interference. The MMSE algorithm, which is different from the ZF, uses the matrix G_{MMSE} instead of H + shown in (10) .

To account for the effect of noise, another linear approach to estimate the received vector is to minimize the mean squared error MMSE (Minimum Mean-Square Error) between the data vector x and its estimate $\hat{x}$

The error defined as follows:

$$\varepsilon^2 = E \left| \left(x - \hat{x} \right)^* \left(x - \hat{x} \right) \right| \tag{10}$$

The estimated vector $\hat{x}$ is obtained by the vector product and received a matrix G that minimizes the mean square error

$$E \left\{ \| Gy - x \|^2 \right\} \tag{11}$$

With

$$G = \left(H^H H + \sigma_w^2 I \right)^{-1} H^H \tag{12}$$

$$\rho_{MMSE} = (H^T H + \sigma^2 I)^{-1} Hy \tag{13}$$

From (11), where a value of $\gamma = 0$, the matrix G becomes a pseudo inverse matrix H, so it is concluded that the technique of zero forcing ZF is a particular case of the criterion of MMSE.

This criterion minimizes the mean square error due to both noise and interference between symbols and this in the setting, unlike the ZF receiver that only deals with inter-symbol interference. The MMSE receiver is less sensitive to noise but less separates the signals.

6. FPGA implementation :

We consider a multiple antenna system with n_T transmit and n_R, n_T receive antennas. The data is demultiplexed into n_T data substreams (called layers). These substreams are mapped on to M-QAM symbols and transmitted over the n_T antennas simultaneously [11].
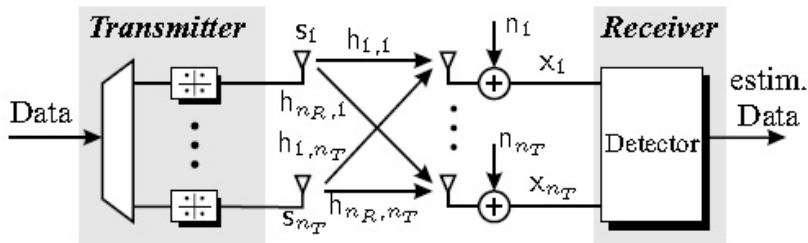

Fig. 3. Model of a MIMO system with nT transmit and nR receive antennas.

We study the assessment of the BER according to the SNR while modifying: the type of Equalizer, the mapping technique adopted and the number of antenna, We suppose that the channel is multi – fading to compare the characteristic BER according to the SNR for MMSE ,ZF and ML Equalizer, we use for simulation QAM-4 modulation for 3x5 MIMO system with 64 subcarriers and 16 the length of the Cyclic prefix. The result of this comparison is on the Fig 4
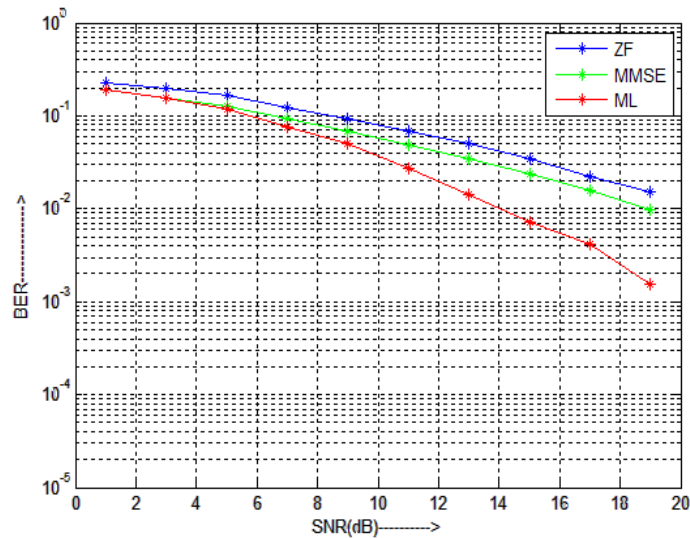

Fig 4.BER according to SNR for ZF, MMSE and ML detector using Rayleigh canal for 3x5 MIMO systems for modulation QAM-4

In this study a proposed MIMO system was simulated using MATLAB software. The different equalization schemes Zero Forcing (ZF) equalizer and Minimum Mean Square Error (MMSE) which aid in the elimination of Inter Symbol Interference (ISI) thus improving overall performance were compared to analyze the BER of the designed system. From the simulation results, the MMSE equalizer clearly had a better performance over the ZF equalizer in the region of about 3 db. MIMO transmission with MMSE equalization offers greater performance over ZF equalization [12].

In Fig 4, the bit error rate (BER) versus the signal-to-noise ratio (SNR) performance of symbol detectors in the 3×5 systems is shown. As it can be seen, the ML has better performance than ZF and MMSE algorithms. For optimal solution of ML detection, all $M N_{tx}$ possible combination of transmitted symbols must be searched. For this reason, the computational complexity increases with transmitter antenna.

This section is devoted to compare and interpret different results in ModelSim simulation mode with the results found in Matlab based on the shape of the curve BER = f (SNR) and the execution time.

To perform the simulation in ModelSim, one chooses the period of the clock CLK equal to 40 ns, Fig 5:

Fig 5 . BER as a function of SNR for ML detector, 3×5 MIMO system produced by ModelSim for QAM 4
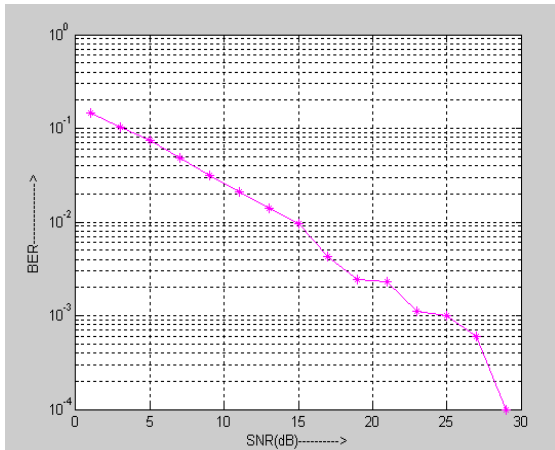
Fig .6 ML BER as a function of SNR for

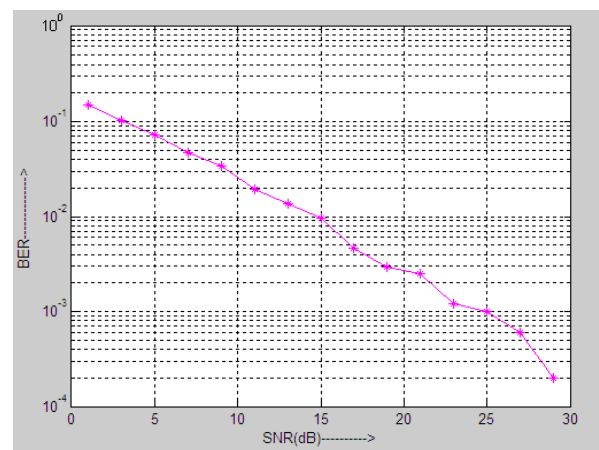MIMO 3×5 simulated by Matlab

(execution time =13,198 s)

Fig. 7 BER as a function of SNR for

MIMO 3×5 simulated in ModelSim

(execution time =280 ns)

Interpretation: The simulation with ModelSim gives a curve that represents the performance of the ML detector evaluating BER based on SNR virtually identical to that obtained with Matlab with a notable advantage of real-time assurance while reducing the execution time SINCE it was found that the execution time given by the simulation in ModelSim (worth 280 ns) is much lower than that given by Matlab (worth 13.198 s).

Summary: The following table 1 summarizes the different ML runtime found for each choice of clock period taken screen.

Table 1: Execution time for each clock period

Clock period (ns)	20	30	40
execution time (ns)	140	210	280

Interpretation: the execution time increases, however, increasing the clock period, so it is preferably to work with the smallest clock period to save runtime.

We use Virtex 6 XC6VLX75tl to develop the implementation of different detectors. The results of FPGA synthesis are presented in the table 2.

Table 2 resource utilization for different equalizer in Virtex 6

Virtex6	ML	ZF	MMSE
Slice (93120)	12% (11174)	16% (14899)	20% (18624)
Flip-flops (11640)	0% (4)	0%(29)	0%(35)
LUTs (11640)	12% (1396)	10% (1146)	19% (2211)
IOBs (240)	50% (120)	57% (136)	57% (136)
GCLKs (32)	3% (1)	3% (1)	3% (1)
Numbers of BRAM (156)	3% (5)	2.5 % (4)	2.5 % (4)

7. Conclusion:

The synthesis of new systems aims to transmission of digital information at higher bandwidths and service for a quantity more demanding. After the presentation of the basic principles of digital transmission and exposure characteristics of the linear model of the wireless channel, examples of systems studied in the literature and depicted as linear radio channels are given and an analysis of detection techniques under optimal suboptimal most popular is described. These sensors have no compromise between performance and complexity; note for example that the simple linear detectors have poor performance as maximum likelihood detectors which have a computational complexity much more complex. In this article, we focused on the different detection algorithms classical MIMO system such as ZF, MMSE and ML results show that ML is the optimal detector, but with a search space and the high computational complexity.. To experimentally validate the performance study, a real implementation on a FPGA algorithms developed is done.

In further work it's very important to study a heuristic approach PSO to reduce the computational complexity of ML detector. PSO is a swarm intelligence based optimization algorithm that has been shown to perform very well for a large number of applications. While PSO has been applied in a large number of applications, PSO is typically executed in software [13].

References

[1] Michael Rueckauer, Daniel M. Munoz, Timo Stripf, A Flexible Implementation of the PSO Algorithm for Fine- and Coarse-Grained Reconfigurable Embedded Systems, International Conference on Reconfigurable Computing and FPGAs (ReConFig), 2013 .

[2] B. M. Hochwald and T. L. Marzetta, "Unitary space-time modulation for multiple-antenna communication in Rayleigh flat fading," IEEE Trans. Inform. Theory , vol. 46, pp 543-564, Mar. 2000.

[3] T. L. Marzetta, "BLAST training: Estimating channel characteristics for high-capacity space-time wireless," i n Proc. 37th Annu. Allerton Conf. Communications, Control, and Computing , Sept. 22-24, 1999.

[4] B. Hassibi and B. M. Hochwald, "How much training is needed in multiple- antenna wireless links?," IEEE. Trans. Inform. Theory , vol. 49, pp. 951-963, Apr. 2003.

[5] L.Zheng and D. Tse, "Communicating on the Grassmann Manifold: A Geometric Approach to the Non-coherent Multiple Antenna Channel", IEEE Transactions on Information Theory , vol. 48(2), pp. 359-383, February 2002.

[6] M. Taherzadeh, A . M obas h er, and A. K. Khandani, "L L L reduction achieves the r eceive diver s ity in MIMO. decoding," IEEE Trans. Infor m . T heor y, vol. 53, no. 12, pp. 4801–4805, 2007.

[7]   Raghunandan Swain, Ajit Kumar Panda ,   Design of 16-QAM Transmitter and Receiver: Review of Methods of Implementation in FPGA, RESEARCH INVENTY: International Journal of Engineering and Science ISSN: 2278-4721, Vol. 1, Issue 9 (November 2012), PP 23-27

[8]   P. H. Tan and L. K. Rasmussen, Tabu search multiuser detection in CDMA, Radio Sci. and Comm. Conf., Stockholm, Sweden, pp. 744-748,   Jun 2002.

[9]   A.TRIMECHE , N. Boukid, A.SAKLY, A.MTIBAA, Performance Analysis of ZF and MMSE Equalizers for MIMO Systems, 7th International conference   on Design & Technology of Integrated Systems in Nanoscale Era (DTIS), 2012

[10] Golden GD, Foschini GJ, Valenzuelara. et al. "Detection algorithm and initiallaboratory resultsusingV-BLAST space-time communication architecture", Electronics Letters,pp.14-16, 1999.

[11] Dirk Wubben, Ronald Bohnke, Near-Maximum-Likelihood Detection of MIMO Systems using MMSE-Based Lattice-Reduction, 7th International Symposium on Turbo Codes and Iterative Information Processing (ISTC), 2012

[12] Bara'u Gafai Najashi and Tan Xiaoheng, A Comparative Performance Analysis of Multiple-Input Multiple-Output using MATLAB with Zero Forcing and Minimum Mean Square Error Equalizers, American J. of Engineering and Applied Sciences 4 (3): 425-428, 2011 ISSN 1941-7020

[13] P.Michael Palangpour , "FPGA   Implementation of PSO Algorithm and Neural Networks" page 4, 2010.