# IMPLEMENTATION OF OFDM AND CHANNEL ESTIMATION USING LS AND MMSE ESTIMATORS

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# IMPLEMENTATION OF OFDM AND CHANNEL ESTIMATION USING LS AND MMSE ESTIMATORS

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***Abstract -*** ***During the past few years, the developments in digital communication are rapidly increasing to meet the ever increasing demand of higher data rates. Orthogonal Frequency Division Multiplexing (OFDM) has an edge over other frequency multiplexing techniques by using more densely packed carriers, thus achieving higher data rates using similar channels. This paper discusses the channel estimation in OFDM and its implementation in MATLAB using pilot based block type channel estimation techniques by LS and MMSE algorithms. This paper starts with comparisons of OFDM using BPSK and QPSK on different channels, followed by modeling the LS and MMSE estimators on MATLAB. In the end, results of different simulations are compared to conclude that LS algorithm gives less complexity but MMSE algorithm provides comparatively better results***

***Keywords*** **– OFDM, Channel Estimation, LS, MMSE**

## I. INTRODUCTION

Orthogonal Frequency Division Multiplexing (OFDM) is most commonly employed in wireless communication systems because of the high rate of data transmission potential with efficiency for high bandwidth and its ability to combat against multi-path delay. It has been used in wireless standards particularly for broadband multimedia wireless services.

An important factor in the transmission of data is the estimation of channel which is essential before the demodulation of OFD M signals since the channel suffers from frequency selective fading and time varying factors for a particular mobile communication system[1]. The estimation channel is mostly done by inserting pilot symbols into all of the subcarriers of an OFD M symbol or inserting pilot symbols into some of the sub-carriers of each OFDM symbol. The first method is called as the pilot based block type channel estimation and it has been discussed for a slow fading channel. This paper discusses the estimation of the channel for this block type pilot arrangement which is based on Least Square (LS) Estimator and Minimum Mean-Square Error (MMSE) Estimator. [2].

The second method is the comb-type based channel estimation in which pilot symbols are transmitted on some of the sub carriers of each OFDM symbol. This method usually uses different interpolation schemes such as linear, low-pass, spline cubic, and time domain interpolation. In [3] [4], it is shown that second-order interpolation performs better than the linear interpolation.

This paper aims to compare the performance of the pilot based block type channel estimation by using Binary Phase Shift Keying (BPSK) modulation scheme in a slow fading channel. In Section II, the basic system model of OFDM is discussed .In Section III, the estimation of the slow fading channel is performed, based on block-type pilot arrangement. In Section IV, the simulation parameter sand results are discussed. Section V concludes the findings.

## II. SYSTEMDESCRIPTION FOR OFDM

The basic OFDM system block diagram under the assumption of frequency domain equalizationis shown inFig.1.

The binary information is being generated from uniformly distributed random integers with equal probability of either 0 or 1 given as [5]:

$$d_k = [d_0, d_1, d_2, \dots, d_{N-1}] \quad k = 0, \dots, N-1 \qquad (1)$$

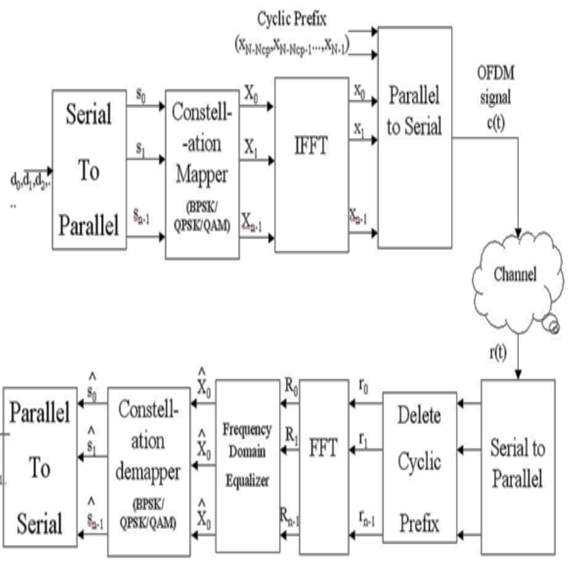

Figure 1: Basic OFDM system.

$d_k$isconverted from serial bit stream to parallel and mapped according to the modulation in the block of constellation mapper. The BPSK/QPSK symbols are then superimposed on orthogonal subcarriers using IDFT given as:

$$x(k) = \sum_{n=0}^{N-1} S(n)\sin\left(\frac{2\pi kn}{N}\right) - j\sum_{n=0}^{N-1} S(n)\cos\left(\frac{2\pi kn}{N}\right) \quad (2)$$

where, *S(n)* is the BPSK/QPSK symbols and N is the length IDFT. After the IFFT block, cyclic prefix of length D, which is considered to be greater than the impulse response of the channel, it is used to combat inter-symbol interference and inter-carrier interference (ICI). It is given as:

$$x(k) = \left[x_{cp}(k) \quad x(k)\right] \quad (3)$$

The OFDM signal is the constructed by applying the symbol along with CP to parallel to serial converter. It is then transmitted on channel given as:

$$y(k) = x(k) \circledast h(l) + n(k) \quad (4)$$

where,$h(l)$is the channel impulse response. The length of channel should be less than the cyclic prefix. For OFDM system, noise is generated in terms of symbols, so it is given as:

$$n(k) = 10^{\frac{-E_S}{20N_0}} * AWGN$$
(5)

Where $\frac{E_S}{N_0}$ is symbol to error ratio (SER) given as

$$\left(\frac{E_S}{N_0}\right)_{dB} = \left(\frac{N}{N_{cp}+N}\right)_{dB} + \left(\frac{N_{st}}{N}\right)_{dB} + \left(\frac{E_b}{N_0}\right)_{dB} \quad (6)$$

Here, $N_{cp}$ represents the length of cyclic prefix, $N_{st}$ is the no. of used subcarriers and $N$ is the length of FFT or no. of sub-carriers [5]. Since the OFDM signal has overhead in terms of CP, so to compensate for it, we have to scale it so that resultant OFDM signal that is received is given as:

$$r(k) = \sqrt{\frac{N_{cp}+N}{N}} \times y(k) \quad (7)$$

At the receiver the reverse steps are involved and since the OFDM symbols were circularly convolved with channel IR, so after FFT at the receiver [6], the received data is equalized by using the frequency domain equalizer and the equation given as:

$$\hat{X}(k) = \frac{Y(k)}{H(k)} \quad (8)$$

Where, $H(k)$ is the response of the channel in frequency domain.

The frequency domain equalization is useful for equalizing the symbols that were faded as a result of experiencing multipath. The results are discussed in Section V [7].

## III. SYSTEMDESCRIPTION FOR CHANNEL ESTIMATION

The OFDM system for pilot based block type channel estimation is shown in Fig. 2

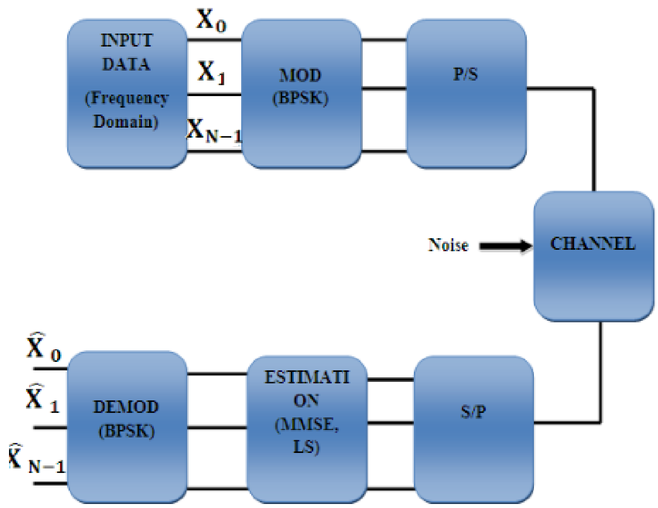


Figure 2: Channel Estimation using LS/MMSE algorithm

In block-type pilot based channel estimation, each subcarrier in an OFDM symbol is used in such a way that all sub-carriers are used as pilots. The estimation of the channel is then done using Least Square Estimator and Minimum Mean Square Error Estimator. [8],[9].

The system shown in Fig. 2 is modeled using the following equation:

$$\boldsymbol{y} = DFT_N\left(IDFT_N(\boldsymbol{X}) \circledast \frac{\boldsymbol{h}}{\sqrt{N}} + \widetilde{\boldsymbol{w}}\right) \qquad (9)$$

Where,

$$\boldsymbol{x} = [x_0 \quad x_1 \quad \dots \quad x_{N-1}]^T$$

$$\boldsymbol{y} = [y_0 \quad y_1 \quad \dots \quad y_{N-1}]^T,$$

$$\widetilde{\boldsymbol{w}} = [\widetilde{w}_0 \quad \widetilde{w}_1 \quad \dots \quad \widetilde{w}_{N-1}]^T$$

$$\boldsymbol{h} = [h_0 \quad h_1 \quad \dots \quad h_{N-1}]^T$$

The vector $\boldsymbol{h}/\sqrt{N}$ is the observed channel impulse response when the frequency response of g(t) is sampled and it is given as:

$$h_k = \frac{1}{\sqrt{N}} \sum_m e^{-j\frac{\pi}{N}(k+(N-1)\gamma_m)} \frac{\sin(\pi\gamma_m)}{\sin\left(\frac{\pi}{N}(\gamma_m - k)\right)} \qquad (10)$$

Where, m is the length of taps, N is the no of sub carriers, and $\gamma_m$ is the value of the tap.

If inter symbol interference is eliminated by the cyclic prefix, then the system shown in the Fig. 2 can be modeled using the equation given as [6]:

$$y_k = H_k X_k + w_k, \qquad k = 0 \dots \dots N-1 \qquad (11)$$

Where $H_k$ is the Frequency response of $h$ given by:

$$\boldsymbol{H} = [H_0 \quad H_1 \quad \dots \quad H_{N-1}]^T = DFT_N(h)$$
$$\boldsymbol{w} = [w_0 \quad w_1 \quad \dots \quad w_{N-1}]^T = DFT_N(\widetilde{w})$$

Now writing the (11) in Matrix form, it becomes:

$$\mathbf{y} = \mathbf{XFh} + \mathbf{w} \qquad (12)$$

Here,

$$\mathbf{X} = \text{diag}\{x_0 \quad x_1 \quad \dots \quad x_{N-1}\}$$

$$\boldsymbol{y} = [y_0 \quad y_1 \quad \dots \quad y_{N-1}]^T$$

$$\boldsymbol{w} = [w_0 \quad w_1 \quad \dots \quad w_{N-1}]^T$$

$$\boldsymbol{h} = [h_0 \quad h_1 \quad \dots \quad h_{N-1}]^T$$

$$\boldsymbol{F} = \begin{bmatrix} W_N^{00} & \cdots & W_N^{0(N-1)} \\ \vdots & \ddots & \vdots \\ W_N^{(N-1)0} & \cdots & W_N^{(N-1)(N-1)} \end{bmatrix}$$

$\boldsymbol{F}$is the matrix of DFT with corresponding weights given as:

$$W_N^{nk} = \frac{1}{\sqrt{N}} e^{-j2\pi\frac{nk}{N}}$$

If the channel vector h is Gaussian and is it not correlated with the noise of the channel w, then the frequency domain MMSE estimates of h becomes [5].

$$\widehat{H}_{MMSE} = F R_{hy} R_{yy}^{-1} y \qquad (13)$$

Where,

$$R_{hy} = E\{hy^H\} = R_{hy} F^H X^H$$

$$R_{yy} = E\{yy^H\} = XFR_{hh}F^H X^H + \sigma_n^2 I_N$$

Here$\boldsymbol{R_{hy}}$is the cross correlation matrix between $\boldsymbol{h}$and $\boldsymbol{y}$, $\boldsymbol{R_{yy}}$ is the auto correlation matrix of $\boldsymbol{y}$ with itself and $\boldsymbol{R_{hh}}$ is the auto correlation matrix of the $\boldsymbol{h}$with itself. Since, $\sigma_n^2$ denotes the noise variance [8]. The factors

$\boldsymbol{R_{hh}}$and $\sigma_n^2$are considered to be known. The LS estimate of the channel is given as:

$$\hat{h}_{LS} = X^{-1}y \tag{14}$$

Which minimizes$(y - XFh)^H(y - XFh)$.

Both estimators suffer from different drawbacks. The MMSE usually suffers from a high complexity, where LS estimator suffers from mean-square error which is high. The MMSE estimator requires to calculate an$N \times N$ matrixwhich results in a high complexity when$N$ becomes large [9].It should be noticed that both the estimators are derived under the assumption [8] of known channel correlation and noise variance. In actual scenario these quantities $\boldsymbol{R_{gg}}$and$\sigma_n^2$, are either considered to be fixed or estimated most commonly in an adaptive way[8].

## IV SIMULATIONS AND RESULTS

This section discusses the results of the simulation that were performed based on the information and mathematics discussed in the Section II & III respectively.

For the simulation of basic OFDM system, we used the following parameters as shown in Table I.

TABLE: SIMULATION PARAMETERS

| Parameters | Specification |
|---|---|
| FFT size | 64 |
| No of used Subcarriers ($N_{st}$) | 52 |
| Cyclic Prefix ($N_{cp}$) | 16 |
| No. of OFDM symbols | 100 |
| Constellation | BPSK/QPSK |
| Channel Model | AWGN, FNS, Multipath |
| No of taps/multipath | 8 |

The Fig.3 and Fig. 4 shows the comparison of BER with different SNR's on BPSK and QPSK constellation using 3 different channel models described in the Table I.

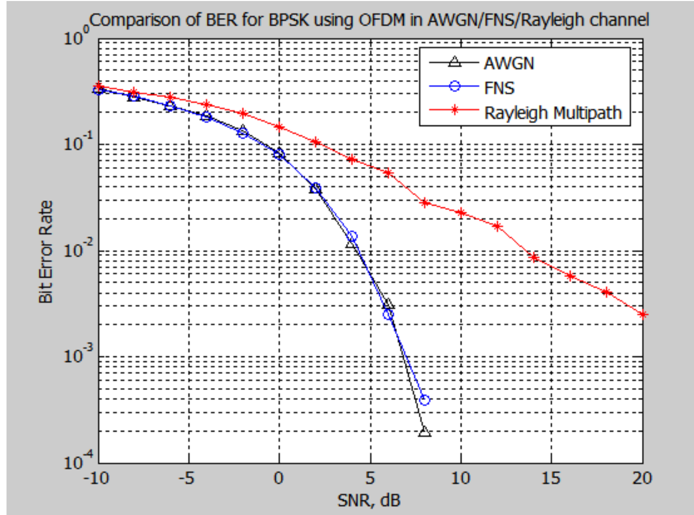


Figure 3: Comparison of BER for BPSK in AWGN/FNS/Rayleigh channel

The above figure shows comparison of BER at the three different channels. For small SNR values the calculated BER is quite large due to relative high power of noise. As SNR is increased the BER decreases as shown.

Similarly for QPSK, again the BER determines how many of the received bits are in error, and then computes it by the number of bits in error divided by the total no of bits in the transmitted signal.

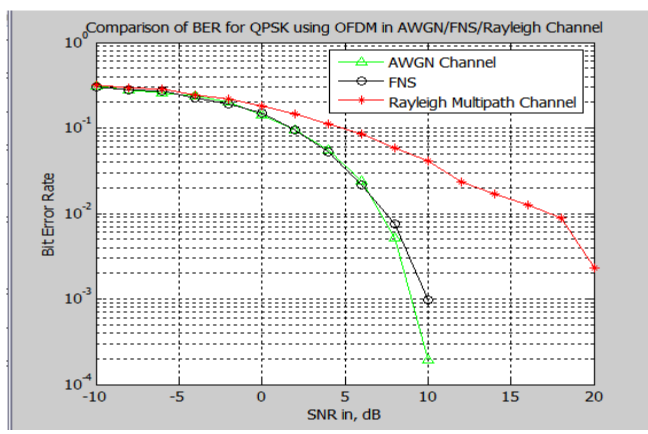


Figure 4: Comparison of BER for QPSK in AWGN/FNS/Rayleigh channel

The Fig. 4 shows the comparison of BER at three different channels. For small SNR values the calculated BER is quite large due to relative high power of noise. As the SNR increases, the BER decreases.

As the BER for the Multipath fading is simulated for the (no. of taps) = 8, which is less than the length of the CP, however if we increase the no of taps for the multipath fading then the resultant BER curve would show that the performance is getting worse and more errors would occur. Following Fig. 5 shows this effect of performance degradation by increasing the no. of taps in the multipath channel. The result shows that as we increase the no. of taps, the signal that is transmitted would go under high degradation because of the no. of time it would be reflected by the multipath (no. of taps).

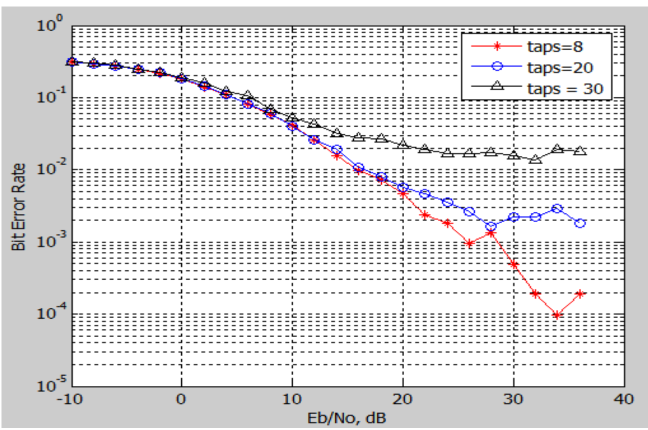


Figure 5: Comparison of BER of QPSK for different no of taps

Similarly if we want to make sure that frequency domain equalization is performed correctly, the constellation is plotted for un-equalized and equalized data and it shows that for QPSK the equalization was performed correctly as shown in the Fig. 6 and Fig. 7

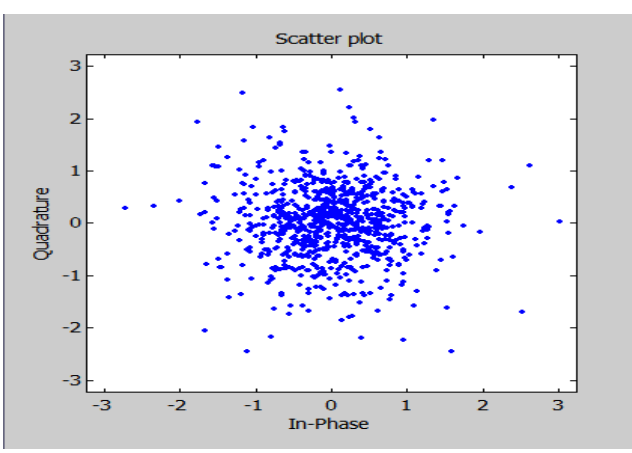


Fig. 6: Un-Equalized Constellation

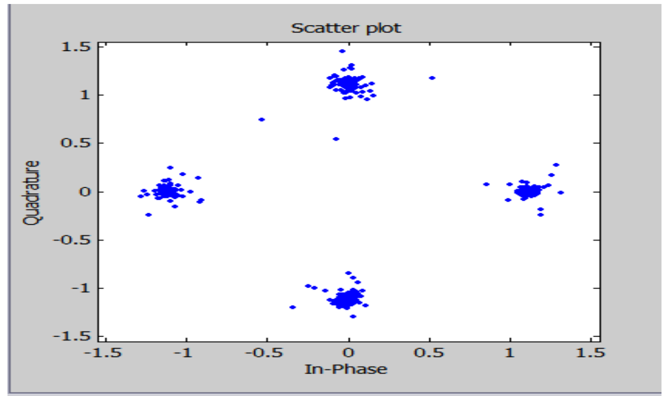


Fig. 7: Equalized Constellation

For the simulation of channel estimation, the following parameters were considered as shown in Table II.

TABLE II: Channel estimation parameters

| **Parameters** | **Specifications** |
|---|---|
| No of subcarriers (N) | 64 |
| No of OFDM symbols | 100 |
| Channel | Slow fading |
| K | 0, … , N-1 |
| $\gamma_m$ | Value of Taps |
| Constellation | BPSK |

Corresponding to the block diagram shown in Section III, simulations were performed for the channel estimation in OFDM using BPSK modulation in the channel with slow fading environment using the LS and MMSE algorithm.

The Fig. 8 shows the Mean Square Error MSE versus SNR for the LS and MMSE Estimators. For low SNR's channel noise effect is higher than the approximation effect, while it becomes dominant for large SNR's.

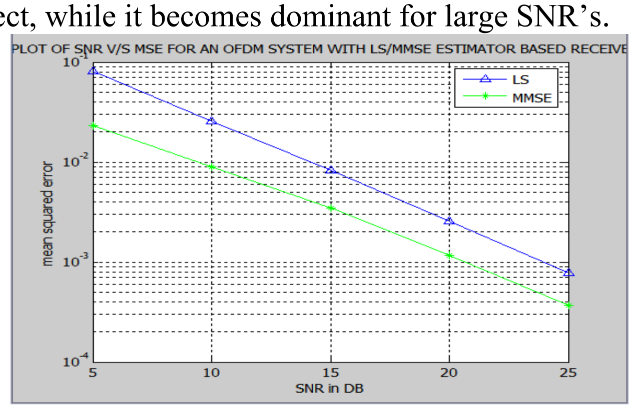

Figure 8: Mean Square Error MSE versus SNR for the LS and MMSE Estimators

Bit Error rate (BER) curves are based on the mean square errors of the channel estimation. For the calculation of BER, the simulation makes use of the formulae calculated earlier. In the simulation we first transmitted the training symbols just to estimate the behavior of the channel so that these results can be used again for the actual transmission in the simulation code. Fig.9 shows the BER of the OFDM system using LS and MMSE estimation for 3-taps.

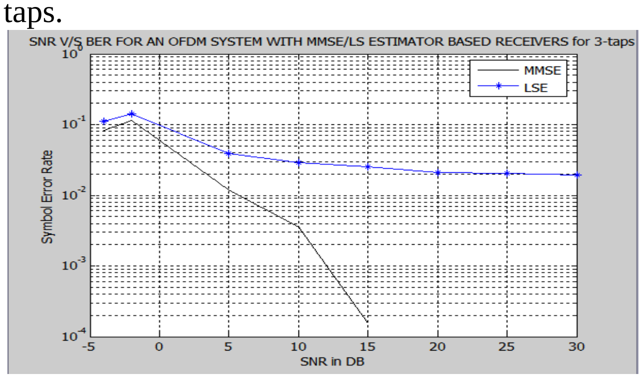


Figure 9: BER for MMSE/LS Estimator based receiver for 3-taps

## V. CONCLUSIONS

The paper highlights the channel estimation technique based on pilot aided block type training symbols using LS and MMSE algorithm. The Channel estimation is one of the fundamental issues of OFDM system design. The transmitted signal under goes many effects such reflection, refraction and diffraction. Also due to the mobility, the channel response can change rapidly over time. At the receiver these channel effects must be canceled to recover the original signal.

In section IV, the BER of AWGN channel is approximately $10^{-4}$ which is better than Rayleigh fading and flat fading channel at SNR=10dB using BPSK & QPSK on different number of taps. The MMSE is compared with LS and the MMSE performs better than the LS using 3 taps where the performance metric is mean square and symbol error rate.

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