

JPEG

In this notebook the JPEG is implemented as an assignment to **CIE 425 : Information Theory and Coding** course in **University of Science and Technology, Zewail City, Egypt**.

Prepared By

Name: Mahmoud Yasser

ID: 201700960

Mail: s-mahmoudyasser@zewailcity.edu.eg

Algorithm

The general algorithm can be illustrated by this block diagram:

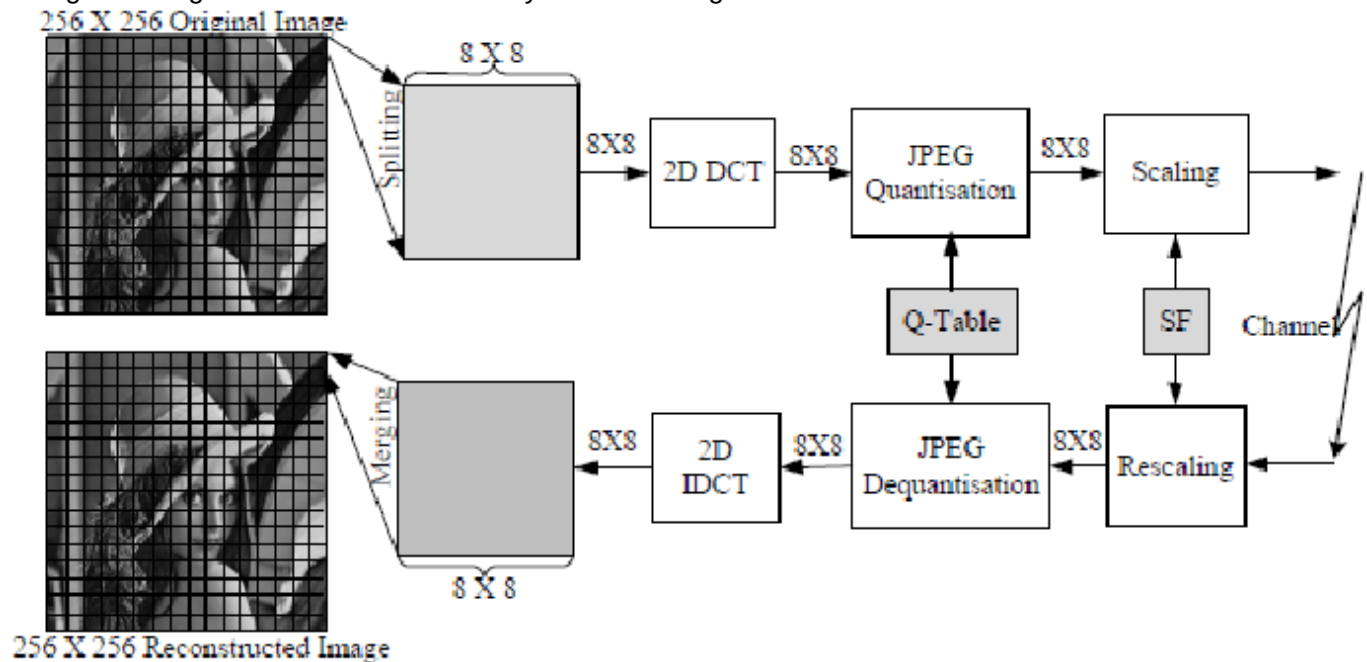


Figure 3: Block diagram of the JPEG based DCT

For the encoder

1. The image is read and segmented into 8x8 blocks.

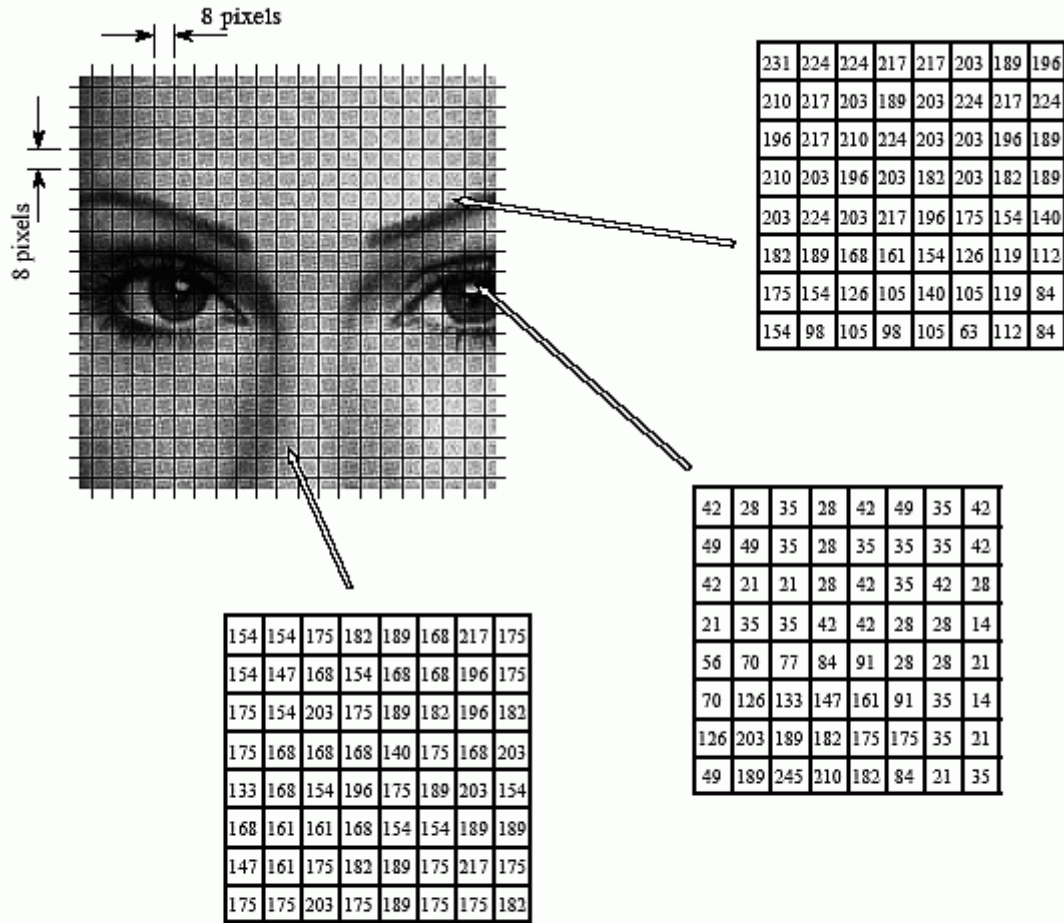


FIGURE 27-9

JPEG image division. JPEG transform compression starts by breaking the image into 8×8 groups, each containing 64 pixels. Three of these 8×8 groups are enlarged in this figure, showing the values of the individual pixels, a single byte value between 0 and 255.

2. Then for each block, 2D DCT is applied to the block. The DCT basis can be obtained by this formula:

$$C_u = \begin{cases} \frac{1}{\sqrt{2}} & \text{if } u = 0 \\ 1 & \text{else} \end{cases}$$

$$C_v = (\text{similar to the above})$$

$$F_{vu} = \frac{1}{4} C_v C_u \sum_{y=0}^{N-1} \sum_{x=0}^{N-1} S_{yx} \cos\left(v\pi \frac{2y+1}{2N}\right) \cos\left(u\pi \frac{2x+1}{2N}\right)$$

the basis can be visualized as follows

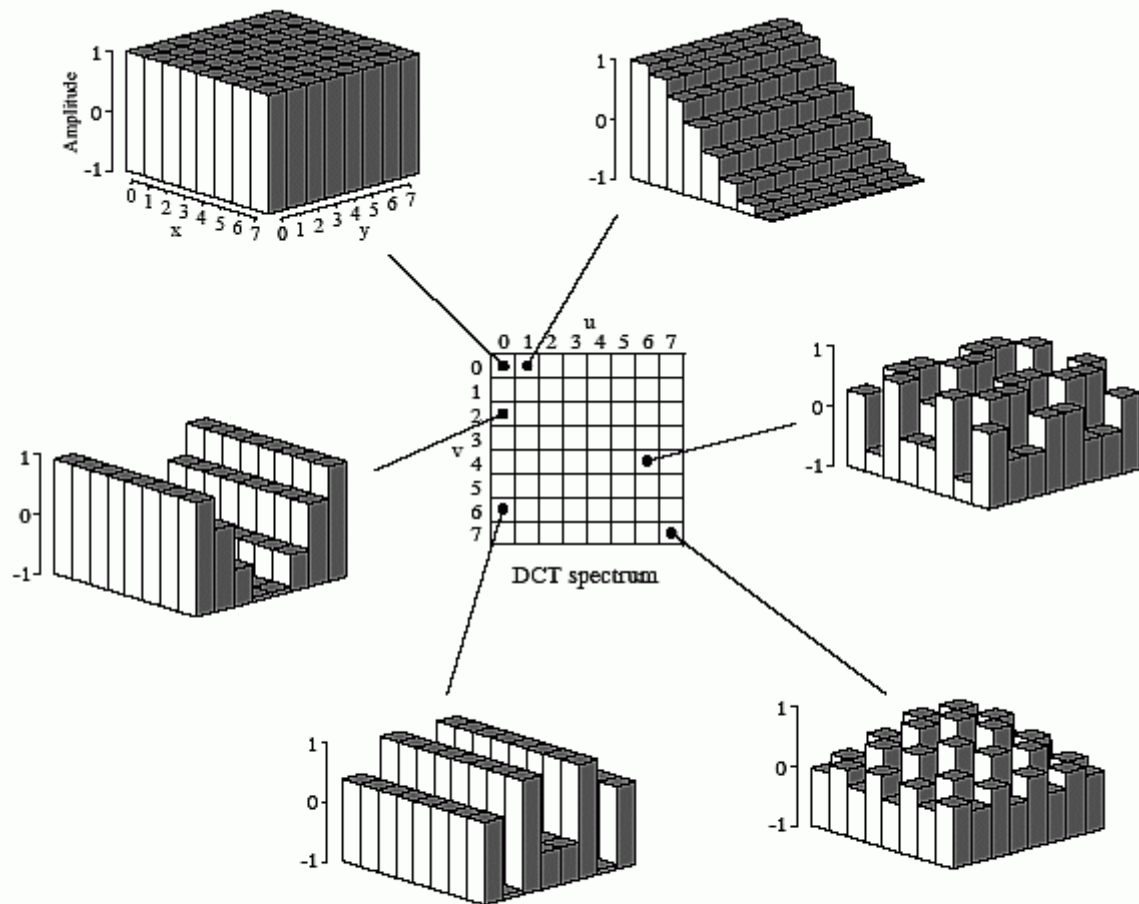


FIGURE 27-10

The DCT basis functions. The DCT spectrum consists of an 8×8 array, with each element in the array being an amplitude of one of the 64 basis functions. Six of these basis functions are shown here, referenced to where the corresponding amplitude resides.

1. After that a Quantization Table is applied to the output of 2D DCT. This step is curial to the compression as dividing (integer division) the 8×8 block by the quantization table is equvelant to assigning less number of bits to this frequency component.

a. Low compression								b. High compression							
1	1	1	1	1	2	2	4	1	2	4	8	16	32	64	128
1	1	1	1	1	2	2	4	2	4	8	16	32	64	128	128
1	1	1	1	2	2	2	4	4	8	16	32	64	128	128	128
1	1	1	1	2	2	4	8	8	16	32	64	128	128	256	256
1	1	2	2	2	2	4	8	16	16	32	64	128	128	256	256
2	2	2	2	2	4	8	8	32	32	64	128	128	256	256	256
2	2	2	4	4	8	8	16	64	64	128	128	256	256	256	256
4	4	4	4	8	8	16	16	128	128	128	256	256	256	256	256

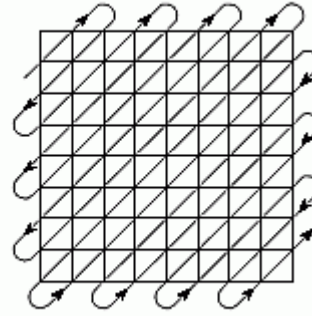
FIGURE 27-13

JPEG quantization tables. These are two example quantization tables that might be used during compression. Each value in the DCT spectrum is divided by the corresponding value in the quantization table, and the result rounded to the nearest integer.

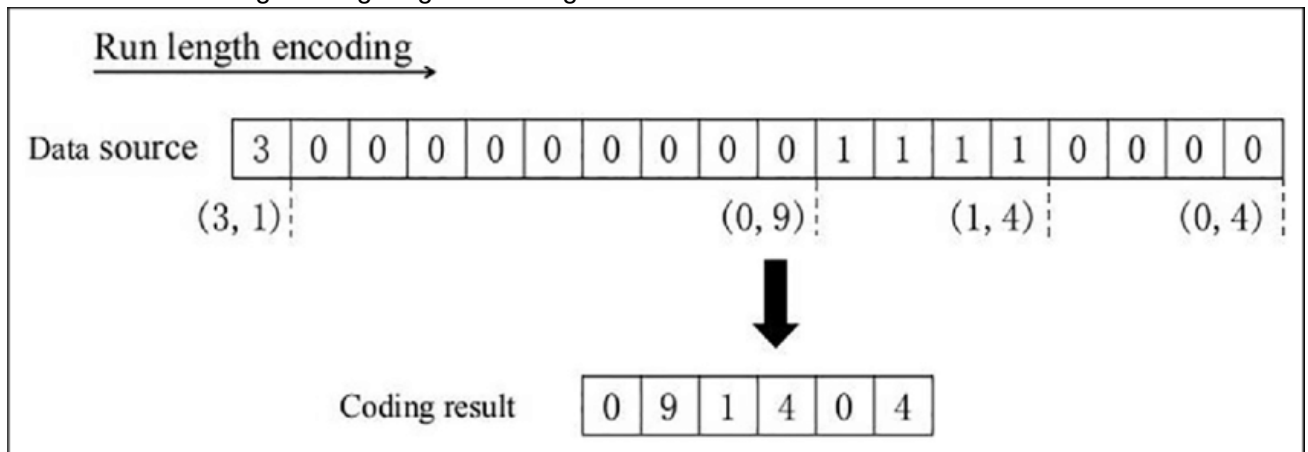
2. Converting 2D to 1D. This step is to make the high frequency components come one after the other as most of them will be zero after the quantization table as a preperation to running length coding.

FIGURE 27-14

JPEG serial conversion. A serpentine pattern used to convert the 8×8 DCT spectrum into a linear sequence of 64 values. This places all of the high frequency components together, where the large number of zeros can be efficiently compressed with run-length encoding.



3. After Converting to 1D it's expected for a natural images to have low values for high frequency components, So the reslut after the integer rounded division will be zero. So we can encode this last zeros of the 1D using running length encoding



4. The last step is to encode the the resulted sequence by entropy encoding (Huffman).

For the decoder

An optimized version of the JPEG is implemented after this version by calculating the DCT basis block once and call it when needed in the DCT

Specifications

It's required to achieve a dynamic range of compression ratio that is between 6:1 and 30:1. The JPEG user will have the ability to control over the needed compression ratio. So there will be 2 basic compression ratios that are achieved by the following quantization table:

a. Low compression								b. High compression							
1	1	1	1	1	2	2	4	1	2	4	8	16	32	64	128
1	1	1	1	1	2	2	4	2	4	4	8	16	32	64	128
1	1	1	1	2	2	2	4	4	4	8	16	32	64	128	128
1	1	1	1	2	2	4	8	8	8	16	32	64	128	128	256
1	1	2	2	2	2	4	8	16	16	32	64	128	128	256	256
2	2	2	2	2	4	8	8	32	32	64	128	128	256	256	256
2	2	2	4	4	8	8	16	64	64	128	128	256	256	256	256
4	4	4	4	8	8	16	16	128	128	128	256	256	256	256	256

FIGURE 27-13
JPEG quantization tables. These are two example quantization tables that might be used during compression. Each value in the DCT spectrum is divided by the corresponding value in the quantization table, and the result rounded to the nearest integer.

The compression ratios can be obtained by multiplying these two basic quantization tables.

For the image quality, I will use the root mean square and Structural Similarity Index as metrics to evaluate the quality of quantization table. A Structural Similarity Index of 0.2 is required

Importing the required Libraries

In [1]:

```
# Import functions and libraries
import numpy as np
import matplotlib.pyplot as plt
import scipy
import imageio
from numpy import pi
from numpy import sin
from numpy import zeros
from numpy import r_
from scipy import signal
from scipy import misc # pip install Pillow
import matplotlib.pyplot as pylab
from matplotlib import pyplot as plt
import matplotlib.cm as cm
from sympy import Matrix, init_printing ## it's always nice to have the matrix function and init_printing functions
from skimage import color
from skimage import io
from dahuffman import HuffmanCodec
```

Reading the Test Image

In [2]:

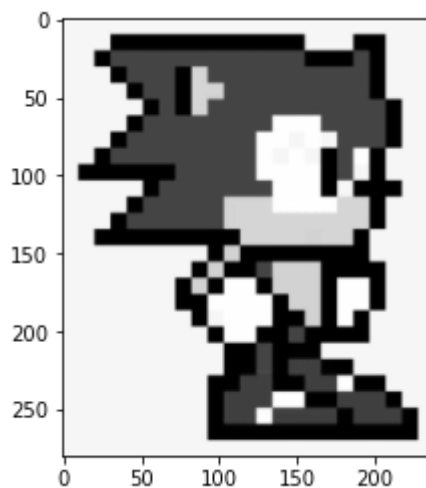
```
# Black and White (gray scale)  
  
Img= imageio.imread('test.png',as_gray=True)
```

In [3]:

```
Img=Img-128  
plt.imshow(Img, cmap=cm.gray)
```

Out[3]:

<matplotlib.image.AxesImage at 0x7f88097ec910>



Implementing the 2D DCT and 2D IDCT

In [4]:

```
def dct2_8(image):
    dct_block=[] #initialize empty list for the basis function

    for u in range(8): # Iterating over horizontal frequencies
        dct_block.append([]) # adding new row
        for v in range(8): # Iterating over vertical frequencies
            dct_block[-1].append([]) # adding new column in the current row
            #This completes the allocation of the cell that will contain a basis
function

            #Calculating the basis function for that frequency compoment :
            for x in range(8): # Iterating over the horizontal dimention
                dct_block[-1][-1].append([]) # adding new row inside the cell
                for y in range(8): # Iterating over the vertical dimention
                    #Calculating the basis
                    dct_block[-1][-1][-1].append(np.cos((2*x+1)*u*np.pi/16)*np.c
os((2*y+1)*v*np.pi/16))

            dct_block=np.array(dct_block)# 8*8*8*8 array that represent the bases functi
ons

            result=[]
            for i in range(8):
                result.append([])
                for j in range(8):
                    if (i==0 or j == 0):# first row and column to be divided by 32 the r
est is by 16
                        # Multiplying the image by the corresponding base function
                        result[-1].append(sum(sum(np.array(image)*np.array(dct_block[i,j
])))/32)
                    else:
                        result[-1].append(sum(sum(np.array(image)*np.array(dct_block[i,j
])))/16)

            result[0][0]=result[0][0]/2 # / 2 is for normalization so that the first ter
m to be divied by 64

            return np.array(result)
```

In [5]:

```
def idct2_8(image):
    dct_block=[] #initialize empty list for the basis function

    for u in range(8): # Iterating over horizontal frequencies
        dct_block.append([]) # adding new row
        for v in range(8): # Iterating over vertical frequencies
            dct_block[-1].append([]) # adding new column in the current row
            #This completes the allocation of the cell that will contain a basis
function

            #Calculating the basis function for that frequency compoment :
            for x in range(8): # Iterating over the horizontal dimation
                dct_block[-1][-1].append([]) # adding new row inside the cell
                for y in range (8):# Iterating over the vertical dimation
                    #Calculating the basis
                    dct_block[-1][-1][-1].append(np.cos((2*x+1)*u*np.pi/16)*np.c
os((2*y+1)*v*np.pi/16))

            dct_block=np.array(dct_block)# 8*8*8*8 array that represent the bases functi
ons
            result=[]
            for i in range(8):# Multiplying the image by the corresponding base function
                result.append([])
                for j in range(8):
                    result[-1].append(sum(sum(np.array( image)*np.array(dct_block[:, :, i, j
]))))
            return np.array(result)
```

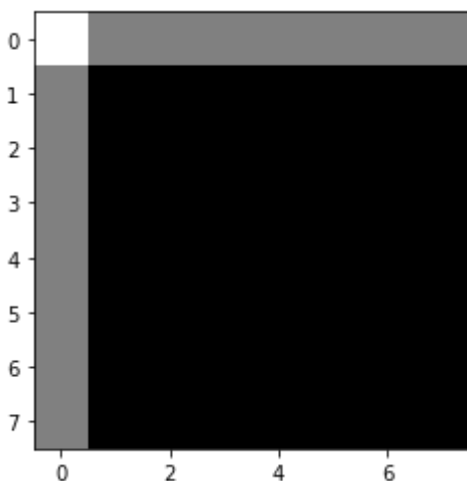
Testing both DCT and IDCT

In [6]:

```
image=Img[0:8,0:8]
plt.imshow(image, cmap=cm.gray)
```

Out[6]:

<matplotlib.image.AxesImage at 0x7f8809705730>

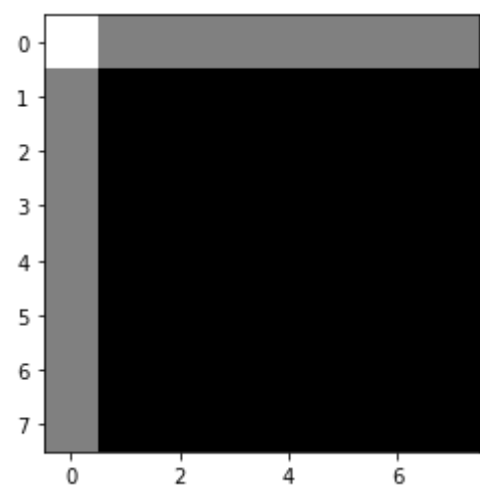


In [7]:

```
plt.imshow(idct2_8(dct2_8(image)), cmap=cm.gray)
retrived_image=idct2_8(dct2_8(image))
sum(sum(abs(image - retrived_image)))
```

Out[7]:

3.424815986363683e-12



In []:

Quantization Table

a. Low compression								b. High compression							
1	1	1	1	1	2	2	4	1	2	4	8	16	32	64	128
1	1	1	1	1	2	2	4	2	4	4	8	16	32	64	128
1	1	1	1	2	2	2	4	4	4	8	16	32	64	128	128
1	1	1	1	2	2	4	8	8	8	16	32	64	128	128	256
1	1	2	2	2	2	4	8	16	16	32	64	128	128	256	256
2	2	2	2	2	4	8	8	32	32	64	128	128	256	256	256
2	2	2	4	4	8	8	16	64	64	128	128	256	256	256	256
4	4	4	4	8	8	16	16	128	128	128	256	256	256	256	256

FIGURE 27-13
JPEG quantization tables. These are two example quantization tables that might be used during compression. Each value in the DCT spectrum is divided by the corresponding value in the quantization table, and the result rounded to the nearest integer.

In [79]:

```
def quantization_table(type_, degree=1):
    if type_.upper() == "HIGH" or type_.upper() == "H":
        quantable_high = np.ones((8,8))*256
        for i in range(8):
            for j in range(8):
                if ((i==0 or j ==0) or (i==1 and j ==1)):
                    quantable_high[i,j]=(2**i)*(2**j)
                elif (i+j<=8):
                    quantable_high[i,j]=(2**(i))*(2**(j))/2
                elif (i+j==9):
                    quantable_high[i,j]=(2**(i))*(2**(j))/4
            return quantable_high.astype(int)
    elif type_.upper() == "LOW" or type_.upper() == "L":
        quantable_low = np.ones((8,8))
        quantable_low[4,2:6]=quantable_low[2:6,4]=2
        quantable_low[5:7,0:3]=quantable_low[0:3,5:7]=2
        quantable_low[3,5]=quantable_low[5,3]=2
        quantable_low[0:3,7]=quantable_low[7,0:3]=4
        quantable_low[3:5,6]=quantable_low[6,3:5]=4
        quantable_low[5,5]=4
        quantable_low[3:6,7]=quantable_low[7,3:6]=8
        quantable_low[5:7,6]=quantable_low[6,5:7]=8
        quantable_low[6:8,7]=quantable_low[7,6:8]=16
        return quantable_low.astype(int)
    elif type_.upper() == "VERYHIGH" or type_.upper() == "VH":
        quantable_high = np.ones((8,8))*256
        for i in range(8):
            for j in range(8):
                if ((i==0 or j ==0) or (i==1 and j ==1)):
                    quantable_high[i,j]=(2**i)*(2**j)
                elif (i+j<=8):
                    quantable_high[i,j]=(2**(i))*(2**(j))/2
                elif (i+j==9):
                    quantable_high[i,j]=(2**(i))*(2**(j))/4
            return quantable_high.astype(int)*degree

    elif type_.upper() == "Moderate" or type_.upper() == "M":
        quantable_low = np.ones((8,8))
        quantable_low[4,2:6]=quantable_low[2:6,4]=2
        quantable_low[5:7,0:3]=quantable_low[0:3,5:7]=2
        quantable_low[3,5]=quantable_low[5,3]=2
        quantable_low[0:3,7]=quantable_low[7,0:3]=4
        quantable_low[3:5,6]=quantable_low[6,3:5]=4
        quantable_low[5,5]=4
        quantable_low[3:6,7]=quantable_low[7,3:6]=8
        quantable_low[5:7,6]=quantable_low[6,5:7]=8
        quantable_low[6:8,7]=quantable_low[7,6:8]=16
        return quantable_low.astype(int)*degree
```

In [80]:

```
Matrix(quantization_table("l"))
```

Out[80]:

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 2 & 2 & 4 \\ 1 & 1 & 1 & 1 & 1 & 2 & 2 & 4 \\ 1 & 1 & 1 & 1 & 2 & 2 & 2 & 4 \\ 1 & 1 & 1 & 1 & 2 & 2 & 4 & 8 \\ 1 & 1 & 2 & 2 & 2 & 2 & 4 & 8 \\ 2 & 2 & 2 & 2 & 2 & 4 & 8 & 8 \\ 2 & 2 & 2 & 4 & 4 & 8 & 8 & 16 \\ 4 & 4 & 4 & 8 & 8 & 8 & 16 & 16 \end{bmatrix}$$

In [81]:

```
Matrix(quantization_table("h"))
```

Out[81]:

$$\begin{bmatrix} 1 & 2 & 4 & 8 & 16 & 32 & 64 & 128 \\ 2 & 4 & 4 & 8 & 16 & 32 & 64 & 128 \\ 4 & 4 & 8 & 16 & 32 & 64 & 128 & 128 \\ 8 & 8 & 16 & 32 & 64 & 128 & 128 & 256 \\ 16 & 16 & 32 & 64 & 128 & 128 & 256 & 256 \\ 32 & 32 & 64 & 128 & 128 & 256 & 256 & 256 \\ 64 & 64 & 128 & 128 & 256 & 256 & 256 & 256 \\ 128 & 128 & 128 & 256 & 256 & 256 & 256 & 256 \end{bmatrix}$$

In [82]:

```
Matrix(quantization_table("M",2))
```

Out[82]:

$$\begin{bmatrix} 2 & 2 & 2 & 2 & 2 & 4 & 4 & 8 \\ 2 & 2 & 2 & 2 & 2 & 4 & 4 & 8 \\ 2 & 2 & 2 & 2 & 4 & 4 & 4 & 8 \\ 2 & 2 & 2 & 2 & 4 & 4 & 8 & 16 \\ 2 & 2 & 4 & 4 & 4 & 4 & 8 & 16 \\ 4 & 4 & 4 & 4 & 4 & 8 & 16 & 16 \\ 4 & 4 & 4 & 8 & 8 & 16 & 16 & 32 \\ 8 & 8 & 8 & 16 & 16 & 16 & 32 & 32 \end{bmatrix}$$

In [83]:

```
Matrix(quantization_table("VH",3))
```

Out[83]:

3	6	12	24	48	96	192	384
6	12	12	24	48	96	192	384
12	12	24	48	96	192	384	384
24	24	48	96	192	384	384	768
48	48	96	192	384	384	768	768
96	96	192	384	384	768	768	768
192	192	384	384	768	768	768	768
384	384	384	768	768	768	768	768

2D to 1D Transformer

In [57]:

```
def D2_D1(block):
    one_D= []
    for i in range(0,8):
        if i%2 != 0:
            for j in range(0,i+1):
                one_D.append(block[j,i-j])
        else:
            for j in range(0,i+1):
                one_D.append(block[i-j,j])

    for i in range(8,0,-1):
        if i%2 != 0:
            for j in range(1,i):
                one_D.append(block[j+(8-i),i-j+(8-i)])
        else:
            for j in range(1,i):
                one_D.append(block[i-j+(8-i),j+(8-i)])
    return np.array(one_D)
#Matrix(one_D)
```

Testing D2_D1

In [12]:

```
test_D2_D1=np.zeros((8,8))
s=1
for i in range(0,8):
    for j in range(0,8):
        test_D2_D1[i,j]=s;
        s=s+1
Matrix(test_D2_D1)
```

Out[12]:

1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0
9.0	10.0	11.0	12.0	13.0	14.0	15.0	16.0
17.0	18.0	19.0	20.0	21.0	22.0	23.0	24.0
25.0	26.0	27.0	28.0	29.0	30.0	31.0	32.0
33.0	34.0	35.0	36.0	37.0	38.0	39.0	40.0
41.0	42.0	43.0	44.0	45.0	46.0	47.0	48.0
49.0	50.0	51.0	52.0	53.0	54.0	55.0	56.0
57.0	58.0	59.0	60.0	61.0	62.0	63.0	64.0

In [13]:

```
D2_D1(test_D2_D1)
```

Out[13]:

```
array([ 1.,  2.,  9., 17., 10.,  3.,  4., 11., 18., 25., 33., 26., 1
9.,
       12.,  5.,  6., 13., 20., 27., 34., 41., 49., 42., 35., 28., 2
1.,
       14.,  7.,  8., 15., 22., 29., 36., 43., 50., 57., 58., 51., 4
4.,
       37., 30., 23., 16., 24., 31., 38., 45., 52., 59., 60., 53., 4
6.,
       39., 32., 40., 47., 54., 61., 62., 55., 48., 56., 63., 64.] )
```

Test is correct

1D to 2D Transformer

In [14]:

```
def D1_D2(one_D):
    two_D= np.zeros((8,8))
    counter = 0
    for i in range(0,8):
        if i%2 != 0:
            for j in range(0,i+1):
                two_D[j,i-j]=one_D[counter]
                counter=counter+1
        else:
            for j in range(0,i+1):
                two_D[i-j,j]=one_D[counter]
                counter=counter+1

    for i in range(8,0,-1):
        if i%2 != 0:
            for j in range(1,i):
                two_D[j+(8-i),i-j+(8-i)]=one_D[counter]
                counter=counter+1
            else:
                for j in range(1,i):
                    two_D[i-j+(8-i),j+(8-i)]=one_D[counter]
                    counter=counter+1

    return np.array(two_D)
# Matrix(one_D)
```

Testing D1_D2

In [15]:

```
Matrix(D1_D2(D2_D1(test_D2_D1)))
```

Out[15]:

1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0
9.0	10.0	11.0	12.0	13.0	14.0	15.0	16.0
17.0	18.0	19.0	20.0	21.0	22.0	23.0	24.0
25.0	26.0	27.0	28.0	29.0	30.0	31.0	32.0
33.0	34.0	35.0	36.0	37.0	38.0	39.0	40.0
41.0	42.0	43.0	44.0	45.0	46.0	47.0	48.0
49.0	50.0	51.0	52.0	53.0	54.0	55.0	56.0
57.0	58.0	59.0	60.0	61.0	62.0	63.0	64.0

Test is correct

Run Length encoder to the zero streams

In [16]:

```
def run_encode_zeros(seq):
    out=np.array([])
    i=0
    while True:
        if i>= len(seq):
            return out
        out=np.append(out,seq[i])
        if (seq[i]==0):
            counter=0
            while(seq[i]==0 ):
                counter=counter+1
                i=i+1
            if i>= len(seq):
                out=np.append(out,counter)
                return out
            out=np.append(out,counter)
        else:
            i=i+1

    return out
```

Testing run_encode_last

In [17]:

```
run_encode_zeros(np.array([0,0,0,1,2,3,0,0,4,7,8,0,0,0,0,0,0]))
```

Out[17]:

```
array([0., 3., 1., 2., 3., 0., 2., 4., 7., 8., 0., 6.] )
```

Test is correct

Run Length Decoder to the zero streams

In [18]:

```
def run_decode_zeros(seq):
    out=np.array([])
    i=0
    j=0
    while True:
        if j>= len(seq):
            return out
        if seq[j]==0:
            out=np.append(out,np.zeros(int(seq[j+1])))
            i=i+int(seq[j+1])
            j=j+2
        else:
            out=np.append(out,seq[j])
            i=i+1
            j=j+1
    return out
```

Testing run_decode_last

In [19]:

```
run_decode_zeros(run_encode_zeros(np.array([0,0,0,1,2,3,0,0,4,7,8,0,0,0,0,0,0])))
```

Out[19]:

```
array([0., 0., 0., 1., 2., 3., 0., 0., 4., 7., 8., 0., 0., 0., 0., 0., 0.])
```

Test is correct

Entropy Encoder and Decoder

In this section I use the dahuffman 0.4.1 python library to encode and decode a list numbers

Documentation of the library : <https://pypi.org/project/dahuffman/> (<https://pypi.org/project/dahuffman/>)

In [20]:

```
pip install dahuffman # install the library if not yet installed
```

Requirement already satisfied: dahuffman in /home/mahmoud/anaconda3/lib/python3.8/site-packages (0.4.1)

Note: you may need to restart the kernel to use updated packages.

In [21]:

```
testimage= dct2_8(image)//quantization_table("h") # applying quantable to get zeros in the end  
testimage
```

Out[21]:

```
array([[118.,  0.,  0.,  0.,  0.,  0.,  0.,  0.],  
       [ 0., -1., -1., -1.,  0., -1., -1., -1.],  
       [ 0., -1.,  0.,  0.,  0., -1., -1., -1.],  
       [ 0., -1., -1., -1.,  0., -1.,  0.,  0.],  
       [ 0.,  0., -1., -1.,  0., -1.,  0.,  0.],  
       [ 0.,  0., -1., -1.,  0.,  0., -1., -1.],  
       [ 0., -1., -1.,  0.,  0., -1., -1.,  0.],  
       [ 0., -1., -1.,  0.,  0., -1.,  0.,  0.]])
```


In [22]:

```
seq=run_encode_zeros(D2_D1(testimage))
seq
```

Out[22]:

```
array([[118.,  0.,  3., -1.,  0.,  2., -1., -1.,  0.,  2.,
        -1.,
         0.,  1., -1.,  0.,  4., -1.,  0.,  4., -1., -1.,
        0.,
         1., -1.,  0.,  2., -1., -1.,  0.,  1., -1., -1.,
        -1.,
         0.,  1., -1., -1., -1.,  0.,  1., -1., -1., -1.,
        -1.,
         0.,  1., -1.,  0.,  2., -1.,  0.,  6., -1., -1.,
        0.,
         1., -1., -1., -1.,  0.,  3.]])
```

In [23]:

```
from dahuffman import HuffmanCodec #importing https://pypi.org/project/dahuffman/the library
codec = HuffmanCodec.from_data(seq) #generating the Huffman dictionary using the seq
codec.print_code_table()
```

Bits	Code	Value	Symbol
1	0	0	-1.0
2	10	2	0.0
5	11000	24	118.0
6	110010	50	_EOF
6	110011	51	6.0
4	1101	13	2.0
5	11100	28	3.0
5	11101	29	4.0
4	1111	15	1.0

In [24]:

```
table=codec.get_code_table() # getting the coding table from the library
dic={x:np.binary_repr(y[1], width=y[0]) for x,y in zip(table.keys(),table.values
())} # constructing encoding dictionary
dic_inverse={np.binary_repr(y[1], width=y[0]):x for y,x in zip(table.values(),ta
ble.keys())}# constructing decoding dictionary
dic,dic_inverse
```

Out[24]:

```
({-1.0: '0',
 0.0: '10',
 118.0: '11000',
 _EOF: '110010',
 6.0: '110011',
 2.0: '1101',
 3.0: '11100',
 4.0: '11101',
 1.0: '1111'},
{'0': -1.0,
 '10': 0.0,
 '11000': 118.0,
 '110010': _EOF,
 '110011': 6.0,
 '1101': 2.0,
 '11100': 3.0,
 '11101': 4.0,
 '1111': 1.0})
```

In [25]:

```
encoded=np.array([dic[x] for x in seq]) # encoding the sequence using the encodi
ng dictionary
encoded
```

Out[25]:

```
array(['11000', '10', '11100', '0', '10', '1101', '0', '0', '10', '1
101',
      '0', '10', '1111', '0', '10', '11101', '0', '10', '11101',
      '0',
      '0', '10', '1111', '0', '10', '1101', '0', '0', '10', '1111',
      '0',
      '0', '0', '10', '1111', '0', '0', '0', '10', '1111', '0',
      '0', '0',
      '0', '10', '1111', '0', '10', '1101', '0', '10', '110011',
      '0',
      '0', '10', '1111', '0', '0', '0', '10', '11100'], dtype='<U
6')
```

In [26]:

```
decoded=D1_D2(run_decode_zeros(np.array([dic_inverse[x] for x in encoded])))  
decoded==testimage # comparing the decoded block and the original block
```

Out[26]:

```
array([[ True,  True,  True,  True,  True,  True,  True,  True],  
       [ True,  True,  True,  True,  True,  True,  True,  True],  
       [ True,  True,  True,  True,  True,  True,  True,  True],  
       [ True,  True,  True,  True,  True,  True,  True,  True],  
       [ True,  True,  True,  True,  True,  True,  True,  True],  
       [ True,  True,  True,  True,  True,  True,  True,  True],  
       [ True,  True,  True,  True,  True,  True,  True,  True],  
       [ True,  True,  True,  True,  True,  True,  True,  True]])
```

Test of Entropy encoder and Decoder is successful for the 8x8 block. (With augmented run length and 1D,2D)

Putting all together

In [85]:

```
def JPEP_encode(image_location, quantization_type, degree=1):  
    Img= imageio.imread(image_location, as_gray=True) #reading the image  
    Img=Img-128 # centering the image values to be -128:127  
    x,y=Img.shape # finding image dimensions  
    quan_table=quantization_table(quantization_type, degree)  
    runlength_coded_streams=[] #initialize an empty list for the runlength coded  
    streams  
    for i in range(0, x- x%8, 8):  
        for j in range(0, y-y%8, 8):  
            runlength_coded_streams.append(run_encode_zeros(D2_D1(np.around(dct2  
_8(Img[i:i+8, j:j+8])/quan_table))))  
  
    codec = HuffmanCodec.from_data([item for sublist in runlength_coded_streams  
for item in sublist ]) #generating the Huffman dictionary using the seq  
    table=codec.get_code_table() # getting the coding table from the library  
    dic={x:np.binary_repr(y[1], width=y[0]) for x,y in zip(table.keys(), table.va  
lues())} # constructing encoding dictionary  
    dic_inverse={np.binary_repr(y[1], width=y[0]):x for y,x in zip(table.values  
( ), table.keys())} # constructing decoding dictionary  
    encoded_streams=np.array([dic[word] for stream in runlength_coded_streams fo  
r word in stream ])  
    final_encoded_stream=""  
    for symbole in encoded_streams:  
        final_encoded_stream = final_encoded_stream+symbole  
  
    return final_encoded_stream, dic_inverse, x- x%8, y-y%8
```

In [86]:

```
def JPEG_decode(encoded_streams,dic_inverse,x,y,quantization_type,degree=1):
    entropy_decoded=np.array([])
    j=0
    while j<len(encoded_streams):
        code=""
        while(code not in dic_inverse.keys() ):
            code=code+encoded_streams[j]
            j=j+1
        entropy_decoded=np.append(entropy_decoded,dic_inverse[code])
    entropy_decoded=run_decode_zeros(entropy_decoded)
    quan_table=quantization_table(quantization_type,degree)
    decoded_image=np.zeros((x,y))
    k=0
    for i in range(0,x,8):
        for j in range(0,y,8):
            decoded_image[i:i+8,j:j+8]=idct2_8(quan_table*D1_D2(entropy_decoded[
k:k+64]))
            k=k+64
    return decoded_image
```

Working on an example

In [170]:

```
Img= imageio.imread('test.png',as_gray=True) #original image
encoded_stream_low,dic_inverse_low,x,y=JPEG_encode('test.png',"l")
encoded_stream_moderate,dic_inverse_moderate,x,y=JPEG_encode('test.png',"M",3)
encoded_stream_high,dic_inverse_high,x,y=JPEG_encode('test.png',"H")
encoded_stream_very_high,dic_inverse_very_high,x,y=JPEG_encode('test.png',"VH",4
)
```

In [171]:

```
print("The Compression ratio for the low quantization table is ",(Img.shape[0]*I
mg.shape[1]*8)/len(encoded_stream_low),": 1" )
print("The Compression ratio for the Moderate (degree 3) quantization table is "
,(Img.shape[0]*Img.shape[1]*8)/len(encoded_stream_moderate),": 1" )
print("The Compression ratio for the High quantization table is ",(Img.shape[0]*
Img.shape[1]*8)/len(encoded_stream_high),": 1" )
print("The Compression ratio for the Very High (degree 4) quantization table is
", (Img.shape[0]*Img.shape[1]*8)/len(encoded_stream_very_high),": 1" )
```

The Compression ratio for the low quantization table is 6.505270702801567 : 1

The Compression ratio for the Moderate (degree 3) quantization table is 9.19797309103617 : 1

The Compression ratio for the High quantization table is 13.120638085742772 : 1

The Compression ratio for the Very High (degree 4) quantization table is 20.237591788089656 : 1

In [172]:

```
decoded_low=JPEP_decode(encoded_stream_low,dic_inverse_low,x,y,"l")
decoded_moderate=JPEP_decode(encoded_stream_moderate,dic_inverse_moderate,x,y,
"M",degree=3)
decoded_high=JPEP_decode(encoded_stream_high,dic_inverse_high,x,y,"H")
decoded_very_high=JPEP_decode(encoded_stream_very_high,dic_inverse_very_high,x,y
,"VH",degree=4)
```

In [173]:

```
def rms_(image1,image2):
    x1,y1=image1.shape
    x2,y2=image2.shape
    return np.sqrt(sum(sum((image1[0:min(x1,x2),0:min(y1,y2)]-image2[0:min(x1,x2)
],0:min(y1,y2)])**2)))

print("The RMS for the low quantization table is ",rms_(Img,decoded_low) )
print("The RMS for the Moderate (degree 3) quantization table is ",rms_(Img,deco
ded_moderate) )
print("The RMS for the High quantization table is ",rms_(Img,decoded_high) )
print("The RMS for the Very High (degree 4) quantization table is ",rms_(Img,dec
oded_very_high))
```

```
The RMS for the low quantization table is 32634.75542272946
The RMS for the Moderate (degree 3) quantization table is 32736.347
75777205
The RMS for the High quantization table is 32717.645150318418
The RMS for the Very High (degree 4) quantization table is 32687.95
614523254
```

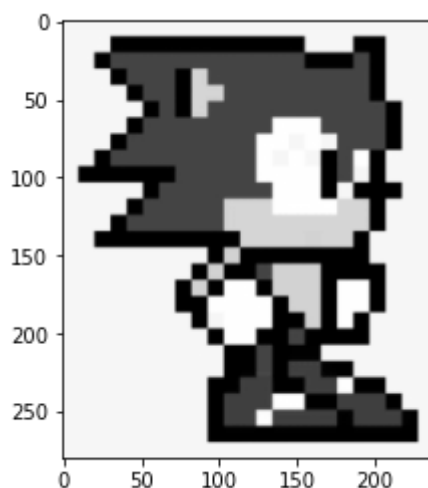
Because the image is smooth (have no high frequency content) the RMS is close to each other in each quantization table. That will make

In [174]:

```
#original image
plt.imshow(Img, cmap=cm.gray)
```

Out[174]:

<matplotlib.image.AxesImage at 0x7f8801d290a0>

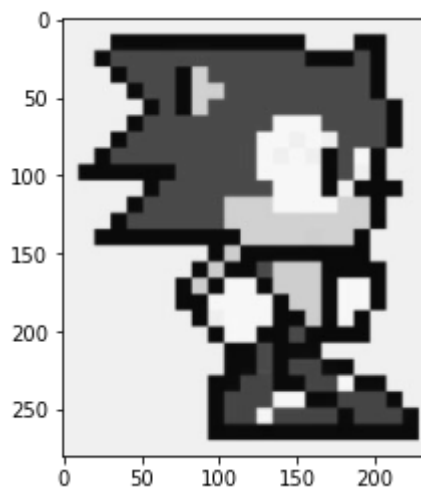


In [175]:

```
#low compressed image  
plt.imshow(decoded_low, cmap=cm.gray)
```

Out[175]:

<matplotlib.image.AxesImage at 0x7f880103b190>

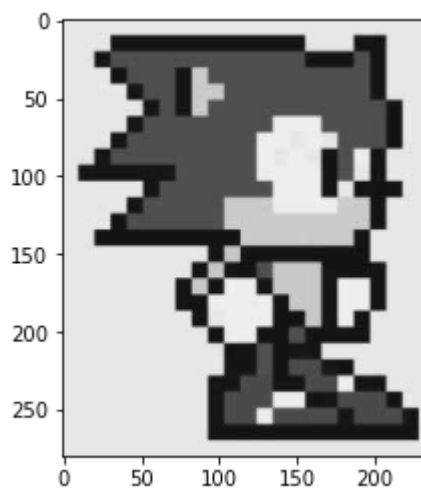


In [176]:

```
#moderate compressed image  
plt.imshow(decoded_moderate, cmap=cm.gray)
```

Out[176]:

<matplotlib.image.AxesImage at 0x7f87f96b2c70>

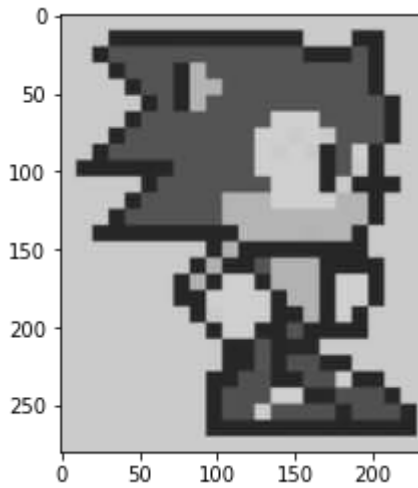


In [177]:

```
#high compressed image  
plt.imshow(decoded_high, cmap=cm.gray)
```

Out[177]:

<matplotlib.image.AxesImage at 0x7f8801d9e7c0>

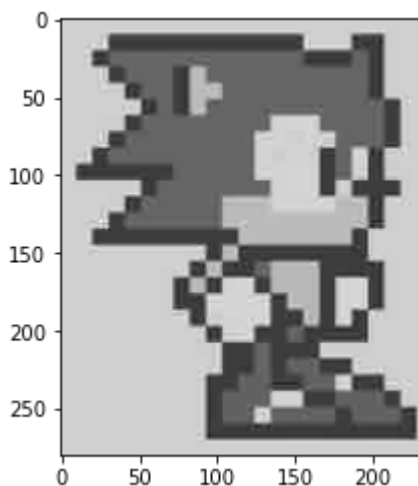


In [178]:

```
#very high compressed image  
plt.imshow(decoded_very_high, cmap=cm.gray)
```

Out[178]:

<matplotlib.image.AxesImage at 0x7f8801d7c6d0>



It's noticed that the decoded image is having less column (and/or potentially rows) than the original image. However, At most 7 rows (and/or columns) is to be cropped leaving no noticable effect on the image

In [128]:

```
#Testing for high resolution and high frequency image
Img= imageio.imread('high_freq.jpeg',as_gray=True) #original image
encoded_stream_low,dic_inverse_low,x,y=JPEP_encode('high_freq.jpeg',"l")
encoded_stream_moderate,dic_inverse_moderate,x,y=JPEP_encode('high_freq.jpeg',
"M",3)
encoded_stream_high,dic_inverse_high,x,y=JPEP_encode('high_freq.jpeg',"H")
encoded_stream_very_high,dic_inverse_very_high,x,y=JPEP_encode('high_freq.jpeg',
"VH",4)
decoded_low=JPEP_decode(encoded_stream_low,dic_inverse_low,x,y,"l")
decoded_moderate=JPEP_decode(encoded_stream_moderate,dic_inverse_moderate,x,y,
"M",degree=3)
decoded_high=JPEP_decode(encoded_stream_high,dic_inverse_high,x,y,"H")
decoded_very_high=JPEP_decode(encoded_stream_very_high,dic_inverse_very_high,x,y
,"VH",degree=4)
print("The Compression ratio for the low quantization table is ",(Img.shape[0]*I
mg.shape[1]*8)/len(encoded_stream_low),": 1" )
print("The Compression ratio for the Moderate (degree 3) quantization table is "
,(Img.shape[0]*Img.shape[1]*8)/len(encoded_stream_moderate),": 1" )
print("The Compression ratio for the High quantization table is ",(Img.shape[0]*
Img.shape[1]*8)/len(encoded_stream_high),": 1" )
print("The Compression ratio for the Very High (degree 4) quantization table is
", (Img.shape[0]*Img.shape[1]*8)/len(encoded_stream_very_high),": 1" )
print("The RMS for the low quantization table is ",rms_(Img,decoded_low) )
print("The RMS for the Moderate (degree 3) quantization table is ",rms_(Img,deco
ded_moderate) )
print("The RMS for the High quantization table is ",rms_(Img,decoded_high) )
print("The RMS for the Very High (degree 4) quantization table is ",rms_(Img,dec
oded_very_high))
```

The Compression ratio for the low quantization table is 6.409657407
924887 : 1

The Compression ratio for the Moderate (degree 3) quantization table
is 10.692414900483907 : 1

The Compression ratio for the High quantization table is 13.9579571
75791346 : 1

The Compression ratio for the Very High (degree 4) quantization tabl
e is 25.080916039040726 : 1

The RMS for the low quantization table is 191779.2226923756

The RMS for the Moderate (degree 3) quantization table is 191761.68
038718592

The RMS for the High quantization table is 191873.15798593996

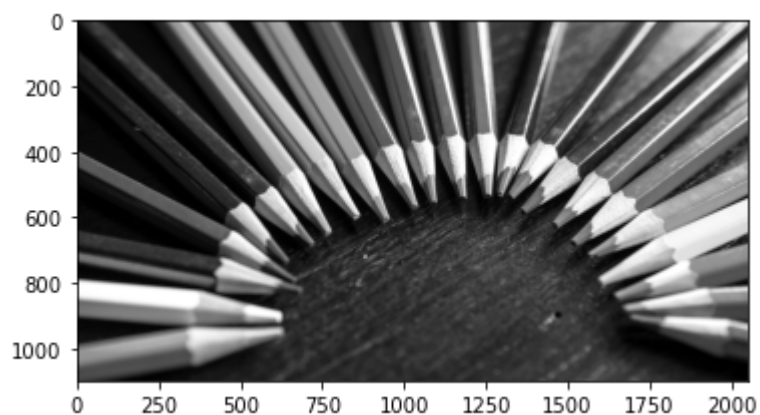
The RMS for the Very High (degree 4) quantization table is 191989.5
5496763604

In [130]:

```
#original image  
plt.imshow(Img, cmap=cm.gray)
```

Out[130]:

<matplotlib.image.AxesImage at 0x7f8802308ca0>

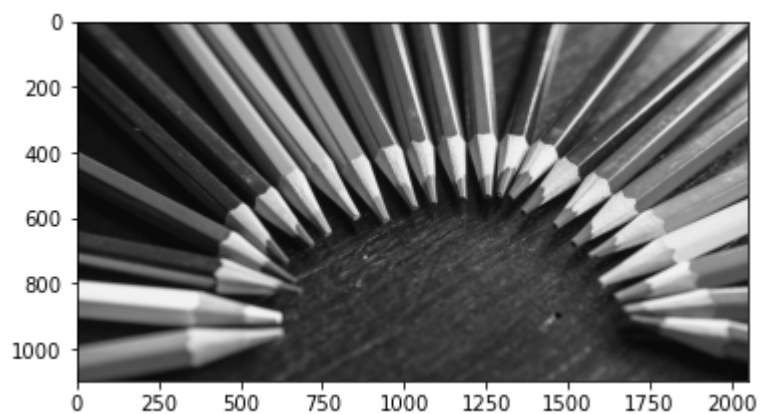


In [131]:

```
#original image  
plt.imshow(decoded_low, cmap=cm.gray)
```

Out[131]:

<matplotlib.image.AxesImage at 0x7f88021ad1f0>

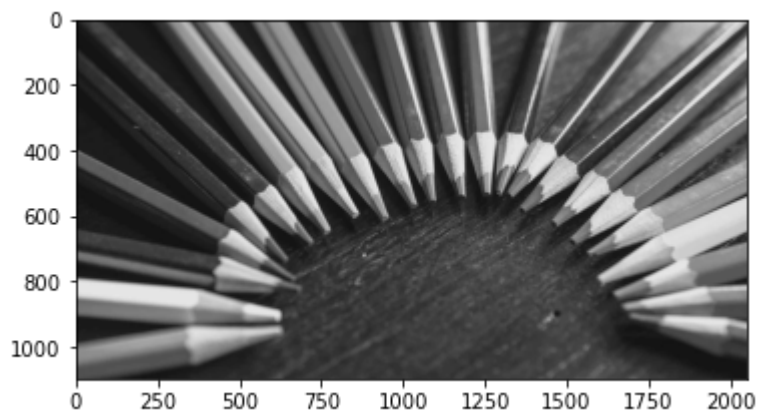


In [132]:

```
#moderate image  
plt.imshow(decoded_moderate, cmap=cm.gray)
```

Out[132]:

<matplotlib.image.AxesImage at 0x7f88022298b0>

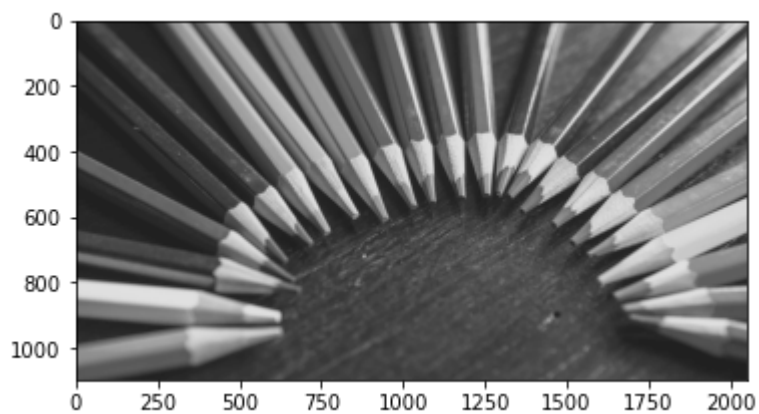


In [133]:

```
#high image  
plt.imshow(decoded_high, cmap=cm.gray)
```

Out[133]:

<matplotlib.image.AxesImage at 0x7f8801403190>

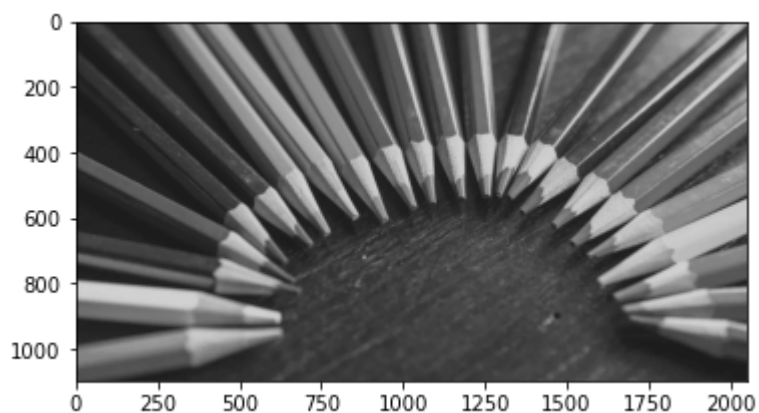


In [134]:

```
#very original image  
plt.imshow(decoded_very_high, cmap=cm.gray)
```

Out[134]:

<matplotlib.image.AxesImage at 0x7f8801438850>



Optimized implementation of The Algorithm

the code can be optimized by precalculating the DCT basis block and call it when needed. The not optimized algorithm calculate the DCT basis for each 8x8 block which make it severely inefficent.

In [143]:

```
def dct_block():
    dct_block=[] #initialize empty list for the basis function

    for u in range(8): # Iterating over horizontal frequencies
        dct_block.append([]) # adding new row
        for v in range(8): # Iterating over vertical frequencies
            dct_block[-1].append([]) # adding new column in the current row
            #This completes the allocation of the cell that will contain a basis
function

            #Calculating the basis function for that frequency compoment :
            for x in range(8): # Iterating over the horizontal dimention
                dct_block[-1][-1].append([]) # adding new row inside the cell
                for y in range (8):# Iterating over the vertical dimention
                    #Calculating the basis
                    dct_block[-1][-1][-1].append(np.cos((2*x+1)*u*np.pi/16)*np.c
os((2*y+1)*v*np.pi/16))

    return np.array(dct_block)# 8*8*8*8 array that represent the bases functions

def dct2_8_optimized( image,dct_block):

    result=np.zeros((8,8))
    for i in range(8):
        for j in range(8):
            if (i==0 or j == 0):# first row and column to be divided by 32 the r
est is by 16
                # Multiplying the image by the corresponding base function
                result[i,j]=(sum(sum(np.array( image)*np.array(dct_block[i,j])))/
32)
            else:
                result[i,j]=(sum(sum(np.array( image)*np.array(dct_block[i,j])))/
16)

    result[0][0]=result[0][0]/2 # / 2 is for normalization so that the first ter
m to be divied by 64

    return np.array(result)
def idct2_8_optimized( image,dct_block):
    result=np.zeros((8,8))
    for i in range(8):# Multiplying the image by the corresponding base function
        for j in range(8):
            result[i,j]=(sum(sum(np.array( image)*np.array(dct_block[:, :, i,j]))))
    return (result)
```

In [144]:

```
def JPEG_encode_optimized(image_location, quantization_type, degree=1):
    dct_block_=dct_block()

    Img= imageio.imread(image_location, as_gray=True) #reading the image
    Img=Img-128 # centering the image values to be -128:127
    x,y=Img.shape # finding image dimensions
    quan_table=quantization_table(quantization_type, degree)
    runlength_coded_streams=[] #initialize an empty list for the runlength coded
streams
    for i in range(0, x-x%8, 8):
        for j in range(0, y-y%8, 8):
            runlength_coded_streams.append(run_encode_zeros(D2_D1(np.around(dct2
_8_optimized(Img[i:i+8, j:j+8], dct_block_)/quan_table))))

    codec = HuffmanCodec.from_data([item for sublist in runlength_coded_streams
for item in sublist ]) #generating the Huffman dictionary using the seq
table=codec.get_code_table() # getting the coding table from the library
dic={x:np.binary_repr(y[1], width=y[0]) for x,y in zip(table.keys(), table.va
lues())} # constructing encoding dictionary
dic_inverse={np.binary_repr(y[1], width=y[0]):x for y,x in zip(table.values
()), table.keys()} # constructing decoding dictionary
encoded_streams=np.array([dic[word] for stream in runlength_coded_streams fo
r word in stream ])
final_encoded_stream=""
for symbole in encoded_streams:
    final_encoded_stream = final_encoded_stream+symbole

    return final_encoded_stream, dic_inverse, x-x%8, y-y%8
```

In [145]:

```
def JPEG_decode_optimized(encoded_streams, dic_inverse, x, y, quantization_type, degr
ee=1):
    dct_block_=dct_block()
    entropy_decoded=np.array([])
    j=0
    while j<len(encoded_streams):
        code=""
        while (code not in dic_inverse.keys() ):
            code=code+encoded_streams[j]
            j=j+1
        entropy_decoded=np.append(entropy_decoded, dic_inverse[code])
    entropy_decoded=run_decode_zeros(entropy_decoded)
    quan_table=quantization_table(quantization_type, degree)

    decoded_image=np.zeros((x,y))
    k=0
    for i in range(0, x, 8):
        for j in range(0, y, 8):
            decoded_image[i:i+8, j:j+8]=idct2_8_optimized(quan_table*D1_D2(entrop
y_decoded[k:k+64]), dct_block_)
            k=k+64
    return decoded_image
```

In [146]:

```
import timeit
start = timeit.default_timer()
encoded_stream_low,dic_inverse_low,x,y=JPEP_encode('emia.png',"l")
stop = timeit.default_timer()
print('Time of not optimized encoding: ', stop - start)
```

Time of not optimized encoding: 79.26895589600099

In [141]:

```
start = timeit.default_timer()
decoded_low=JPEP_decode(encoded_stream_low,dic_inverse_low,x,y,"l")
stop = timeit.default_timer()
print('Time of not optimized decoding: ', stop - start)
```

Time of not optimized decoding: 95.60079716199834

In [147]:

```
start = timeit.default_timer()
encoded_stream_low,dic_inverse_low,x,y=JPEP_encode_optimized('emia.png',"l")
stop = timeit.default_timer()
print('Time of optimized encoding: ', stop - start)
```

Time of optimized encoding: 7.27969544699954

In [148]:

```
start = timeit.default_timer()
decoded_low=JPEP_decode_optimized(encoded_stream_low,dic_inverse_low,x,y,"l")
stop = timeit.default_timer()
print('Time of optimized decoding: ', stop - start)
```

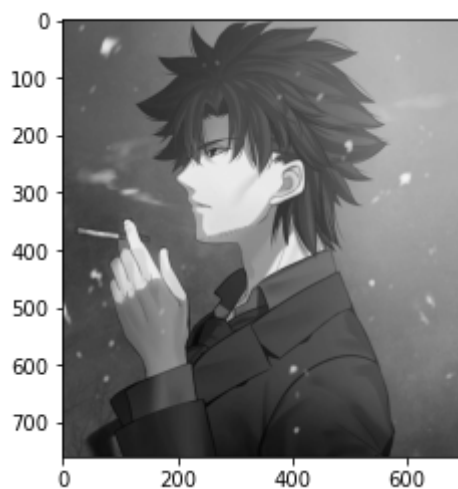
Time of optimized decoding: 18.312016687999858

In [149]:

```
plt.imshow(decoded_low, cmap=cm.gray)
```

Out[149]:

<matplotlib.image.AxesImage at 0x7f88016aabb0>



The optimization works 10 times faster for the encoding and 5 times faster for the decoding for the decoding to work with the same efficiency as the encoder an efficient implementation of the entropy encoder must be obtained (Huffman tree for example).

Image Quality

It looks like the RMS is not a good indicator of the image quality. So, another indicator is used which is Structural Similarity Index (SSIM).

In [156]:

```
!pip3 install scikit-image opencv-python imutils
```

Requirement already satisfied: scikit-image in /home/mahmoud/anaconda3/lib/python3.8/site-packages (0.16.2)
Requirement already satisfied: opencv-python in /home/mahmoud/anaconda3/lib/python3.8/site-packages (4.4.0.46)
Requirement already satisfied: imutils in /home/mahmoud/anaconda3/lib/python3.8/site-packages (0.5.3)
Requirement already satisfied: scipy>=0.19.0 in /home/mahmoud/anaconda3/lib/python3.8/site-packages (from scikit-image) (1.5.0)
Requirement already satisfied: networkx>=2.0 in /home/mahmoud/anaconda3/lib/python3.8/site-packages (from scikit-image) (2.4)
Requirement already satisfied: PyWavelets>=0.4.0 in /home/mahmoud/anaconda3/lib/python3.8/site-packages (from scikit-image) (1.1.1)
Requirement already satisfied: matplotlib!=3.0.0,>=2.0.0 in /home/mahmoud/anaconda3/lib/python3.8/site-packages (from scikit-image) (3.2.2)
Requirement already satisfied: imageio>=2.3.0 in /home/mahmoud/anaconda3/lib/python3.8/site-packages (from scikit-image) (2.9.0)
Requirement already satisfied: pillow>=4.3.0 in /home/mahmoud/anaconda3/lib/python3.8/site-packages (from scikit-image) (7.2.0)
Requirement already satisfied: numpy>=1.17.3 in /home/mahmoud/anaconda3/lib/python3.8/site-packages (from opencv-python) (1.18.5)
Requirement already satisfied: decorator>=4.3.0 in /home/mahmoud/anaconda3/lib/python3.8/site-packages (from networkx>=2.0->scikit-image) (4.4.2)
Requirement already satisfied: cycler>=0.10 in /home/mahmoud/anaconda3/lib/python3.8/site-packages (from matplotlib!=3.0.0,>=2.0.0->scikit-image) (0.10.0)
Requirement already satisfied: python-dateutil>=2.1 in /home/mahmoud/anaconda3/lib/python3.8/site-packages (from matplotlib!=3.0.0,>=2.0.0->scikit-image) (2.8.1)
Requirement already satisfied: pyparsing!=2.0.4,!=2.1.2,!=2.1.6,>=2.0.1 in /home/mahmoud/anaconda3/lib/python3.8/site-packages (from matplotlib!=3.0.0,>=2.0.0->scikit-image) (2.4.7)
Requirement already satisfied: kiwisolver>=1.0.1 in /home/mahmoud/anaconda3/lib/python3.8/site-packages (from matplotlib!=3.0.0,>=2.0.0->scikit-image) (1.2.0)
Requirement already satisfied: six in /home/mahmoud/anaconda3/lib/python3.8/site-packages (from cycler>=0.10->matplotlib!=3.0.0,>=2.0.0->scikit-image) (1.15.0)

In [157]:

```
#Testing for high resolution and high frequency image
Img= imageio.imread('high_freq.jpeg',as_gray=True) #original image
encoded_stream_low,dic_inverse_low,x,y=JPEP_encode_optimized('high_freq.jpeg',
"l")
encoded_stream_moderate,dic_inverse_moderate,x,y=JPEP_encode_optimized('high_freq.jpeg',"M",3)
encoded_stream_high,dic_inverse_high,x,y=JPEP_encode_optimized('high_freq.jpeg',"H")
encoded_stream_very_high,dic_inverse_very_high,x,y=JPEP_encode_optimized('high_freq.jpeg',"VH",4)
decoded_low=JPEP_decode_optimized(encoded_stream_low,dic_inverse_low,x,y,"l")
decoded_moderate=JPEP_decode_optimized(encoded_stream_moderate,dic_inverse_moderate,x,y,"M",degree=3)
decoded_high=JPEP_decode_optimized(encoded_stream_high,dic_inverse_high,x,y,"H")
decoded_very_high=JPEP_decode_optimized(encoded_stream_very_high,dic_inverse_very_high,x,y,"VH",degree=4)
print("The Compression ratio for the low quantization table is ",(Img.shape[0]*Img.shape[1]*8)/len(encoded_stream_low),": 1" )
print("The Compression ratio for the Moderate (degree 3) quantization table is ",(Img.shape[0]*Img.shape[1]*8)/len(encoded_stream_moderate),": 1" )
print("The Compression ratio for the High quantization table is ",(Img.shape[0]*Img.shape[1]*8)/len(encoded_stream_high),": 1" )
print("The Compression ratio for the Very High (degree 4) quantization table is ",(Img.shape[0]*Img.shape[1]*8)/len(encoded_stream_very_high),": 1" )
```

The Compression ratio for the low quantization table is 6.409657407924887 : 1

The Compression ratio for the Moderate (degree 3) quantization table is 10.692414900483907 : 1

The Compression ratio for the High quantization table is 13.957957175791346 : 1

The Compression ratio for the Very High (degree 4) quantization table is 25.080916039040726 : 1

In [168]:

```
from skimage.metrics import structural_similarity
import argparse
import imutils
import cv2
SSIM=[]

SSIM.append(list( structural_similarity(Img[0:x,0:y], decoded_low, full=True)))
SSIM[-1][1]= (SSIM[-1][1] * 255).astype("uint8")
SSIM.append(list( structural_similarity(Img[0:x,0:y], decoded_moderate, full=True
e))))
SSIM[-1][1]= (SSIM[-1][1] * 255).astype("uint8")
SSIM.append( list(structural_similarity(Img[0:x,0:y], decoded_high, full=True)))
SSIM[-1][1]= (SSIM[-1][1] * 255).astype("uint8")
SSIM.append(list( structural_similarity(Img[0:x,0:y], decoded_very_high, full=Tr
ue))))
SSIM[-1][1]= (SSIM[-1][1] * 255).astype("uint8")

print("The SSIM score for the low quantization table is ",SSIM[0][0])
print("The SSIM score for the Moderate (degree 3) quantization table is ",SSIM[1
][0] )
print("The SSIM score for the High quantization table is ",SSIM[2][0] )
print("The SSIM score for the Very High (degree 4) quantization table is ",SSIM[
3][0] )
```

<ipython-input-168-b41d1d289585>:7: UserWarning: Inputs have mismatched dtype. Setting data_range based on im1.dtype.

```
SSIM.append(list( structural_similarity(Img[0:x,0:y], decoded_low, full=True)))
```

<ipython-input-168-b41d1d289585>:9: UserWarning: Inputs have mismatched dtype. Setting data_range based on im1.dtype.

```
SSIM.append(list( structural_similarity(Img[0:x,0:y], decoded_moderate, full=True)))
```

<ipython-input-168-b41d1d289585>:11: UserWarning: Inputs have mismatched dtype. Setting data_range based on im1.dtype.

```
SSIM.append( list(structural_similarity(Img[0:x,0:y], decoded_high, full=True)))
```

<ipython-input-168-b41d1d289585>:13: UserWarning: Inputs have mismatched dtype. Setting data_range based on im1.dtype.

```
SSIM.append(list( structural_similarity(Img[0:x,0:y], decoded_very_high, full=True)))
```

The SSIM score for the low quantization table is -0.3285316281645728

The SSIM score for the Moderate (degree 3) quantization table is -0.27408778736771544

The SSIM score for the High quantization table is -0.277913717977796

The SSIM score for the Very High (degree 4) quantization table is -0.1935625987140633

The very highly compressed image do not meet the specifications but the other compressed versions meets it.