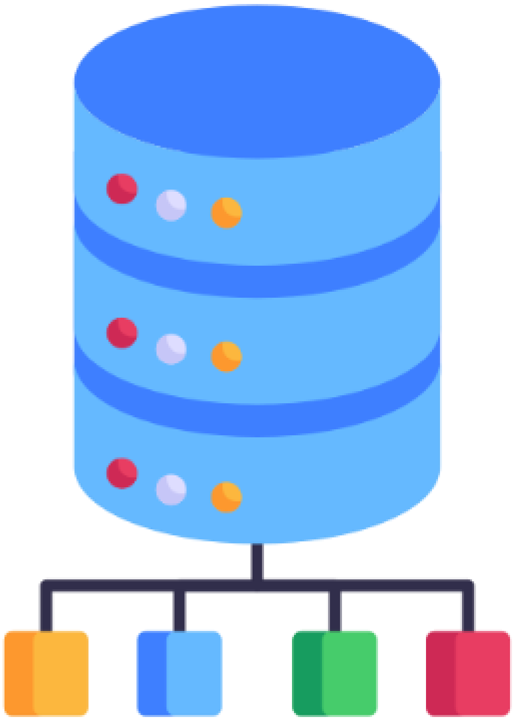
Exploratory Data Analysis (EDA)

Outline

- Features and Feature Types
- Dataset Types, Properties and Sources
- Data Exploration – Univariate
- Data Exploration - Bivariate
- Data Exploration - Multivariate

Features and Feature Types



Steps for Doing Machine Learning

1. Acquire and load the data
2. Explore the data with Pandas and visualization
3. Clean and transform the data as necessary
   - E.g., Scikit-Learn requires numeric data
4. Split the data for training and testing
5. Create the machine learning model
6. Train and test the model
7. Tune the model and evaluate its accuracy
8. Use the model to make predictions on live data that the model hasn't seen before

Feature

- A feature or a variable is any characteristic, number, or quantity that can be measured or counted
- E.g.,
  - Age (21, 35, 62, ...)
  - Gender (male, female)
  - Income ($25000, $35000, $50000, ...)
  - House price ($450000, $980000, ...)
  - Country of birth (Qatar, Australia, Saudi, ...)
  - Eye colour (blue, brown, green, ...)
  - Vehicle make (Toyota, Kia, ...)

Feature Types

Type	Subtype	Examples
Categorical (Qualitative)	**Nominal**	Product type, name
	Ordinal	Size measured as small<medium<large
	Binary	Spam email (yes/no, true/false, 0/1)
	Date / Time	Job start date
Numerical (Quantitative)	**Discrete**	Number of students in a class
	Continuous	Height, weight

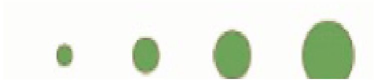
Understanding the type of variables is crucial for selecting appropriate statistical methods, visualization techniques, and ML algorithms

Categorical Features

- **Categorical data are strings that represent qualitative data**
  - **Often selected from a group of categories, also called labels**
- **Nominal**, e.g., country of birth, gender, eye color, etc.

  – No inherent order or ranking
  – Operators applicable: **=, ≠**
  – 1:1 transformation permissible, e.g. ID: 974 ⇒ Qatar

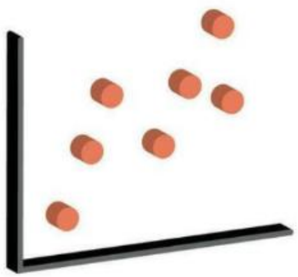
- **Ordinal**, e.g. grade (A, B, C, D, F), degree (bachelor, master, PhD), height (tall, medium, short), etc.



  – Represent categories that can be meaningfully ordered
  – Operator applicable: **=, ≠, <, >, ≥, ≤**
  – Order-preserving transformation permitted,
  - e.g. height (tall, medium, short) to (1, 2, 3)

Numerical Features

- **Discrete**



  - Whole numbers (counts) typically integers
  - E.g., The number of cars in a parking lot, the number of students in a class, or the count of items in a basket.

- **Continuous**

  - Measurable numeric variable that may contain any value within a range
  - Typically represented decimal numbers and fractions
  - E.g., Height, weight, temperature, or distance

Features and Data Objects

- **Data object:** (also known as record, sample, or entity) individual object/event
  - Characterized by its recorded values on a fixed set of features

- **Features:** (also known as attribute, variable, field, or characteristic) a specific property or characteristic of the data object
  - **Raw Features:**
    - **Collected or measured** value of an attribute according to an appropriate measurement scale
  - **Derived Features**
    - Constructed from data in one or more raw features

Features

Objects

Tid	Refund	Marital Status	Taxable Income	Cheat
1	Yes	Single	125K	No
2	No	Married	100K	No
3	No	Single	70K	No
4	Yes	Married	120K	No
5	No	Divorced	95K	Yes
6	No	Married	60K	No
7	Yes	Divorced	220K	No
8	No	Single	85K	Yes
9	No	Married	75K	No
10	No	Single	90K	Yes

Derived Features

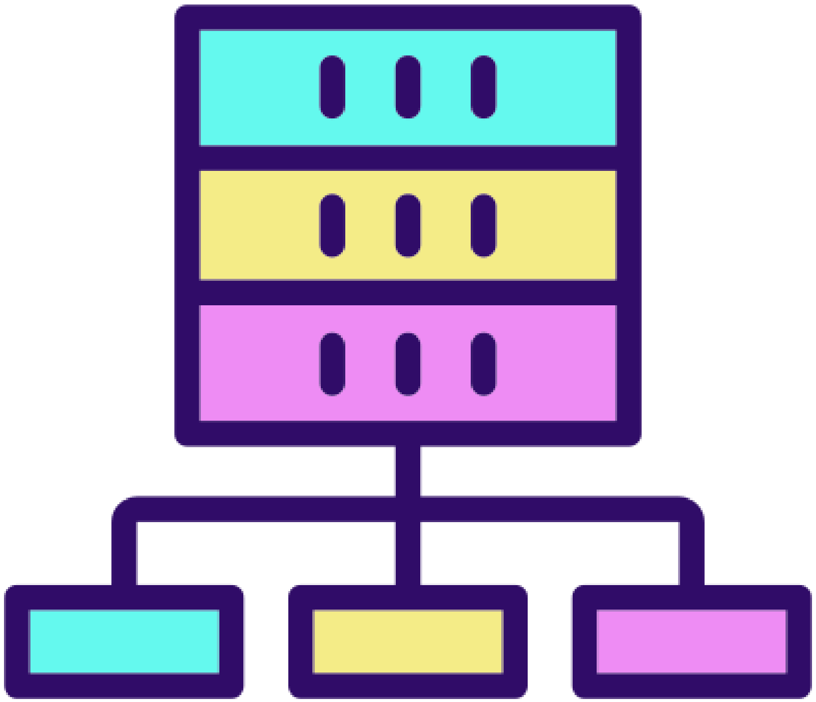
- **Aggregates:** defined over a group or period, e.g., count, sum, average, minimum, or maximum of the values
- **Flags:** indicate presence or absence of some characteristic within a dataset, e.g., a flag indicating whether or not a bank account has ever been overdrawn
- **Ratios:** capture relationship between two or more raw data values, e.g., a ratio between a loan applicant's salary and the amount for which they are requesting
- **Mappings:** convert continuous features into categorical features, e.g., map the salary values to low, medium, and high
- **Others:** no restrictions to the ways in which we can combine data to make derived features, e.g., use satellite photos to count the number of cars in the parking lots and use this as a proxy measure of activity within a competitor's stores!

Goals for Derived Features

- To **improve** the accuracy and performance of machine learning models by transforming the raw data into a more meaningful representation that can better capture the underlying relationships in the data

- To help to **reduce** the dimensionality of a dataset and make it easier to visualize and understand the relationships between variables

Dataset Types, Properties and Sources



Dataset Types

Age Group	Own Car	Income Band	Class
young	yes	low	risky
young	no	low	risky
middle aged	yes	middle	risky
middle aged	no	high	safe
middle aged	yes	low	risky
young	yes	high	risky
middle aged	no	low	safe
retired	yes	middle	safe
retired	no	middle	safe
retired	yes	high	safe

Relational Table

No.	studentID Numeric	Homework1 Numeric	Homework2 Numeric	Homework3 Numeric	Final Exam Numeric
1	1.0		94.0	34.0	42.0
2	2.0	35.0	94.0	85.0	45.0
3	3.0	31.0	46.0	22.0	48.0
4	4.0	46.0	90.0	60.0	50.0
5	5.0	52.0	94.0	49.0	50.0
6	6.0	58.0	94.0	30.0	51.0
7	7.0	47.0	90.0		52.0
8	8.0	37.0	94.0	25.0	52.0
9	9.0	35.0	94.0	45.0	54.0
10	10.0	57.0	94.0	100.0	54.0
11	11.0	51.0	94.0	5.0	54.0
12	12.0	45.0	94.0	33.0	55.0
13	13.0	44.0	0.0	35.0	55.0
14	14.0	52.0	95.0	56.0	56.0
15	15.0	35.0	94.0		57.0
16	16.0	57.0	97.0	57.0	57.0
17	17.0	45.0	90.0	71.0	57.0
18	18.0	39.0	94.0	54.0	57.0
19	19.0	31.0	94.0	63.0	57.0
20	20.0	45.0	94.0		59.0
21	21.0	35.0	90.0	84.0	59.0
22	22.0	37.0	90.0	40.0	61.0
23	23.0	83.0	97.0	26.0	61.0
24	24.0	68.0	97.0	55.0	62.0
25	25.0	50.0	95.0	56.0	62.0
26	26.0	77.0	93.0		63.0
27	27.0	84.0	48.0	18.0	63.0

Data Matrix

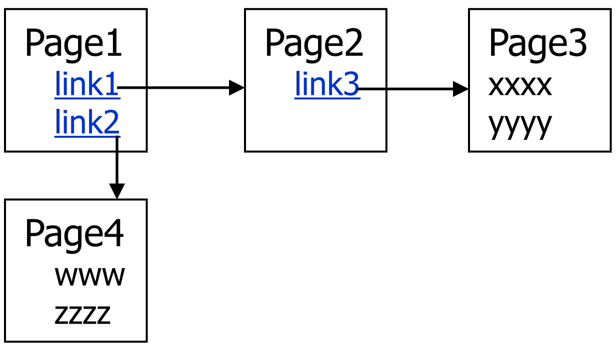
TID	Items
100	apple, milk, newspaper
200	apple, beef, milk, newspaper, potato
300	beef, potato
400	beef, noodles
500	beef, potato

Transaction Data

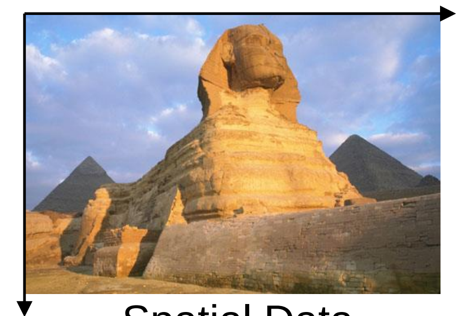
	team	coach	play	ball	score	game	win	lost	timeout	season
Document 1	3	0	5	0	2	6	0	2	0	2
Document 2	0	7	0	2	1	0	0	3	0	0
Document 3	0	1	0	0	1	2	2	0	3	0

Document-term Matrix

Types of data sets (cont.)



Web Structure



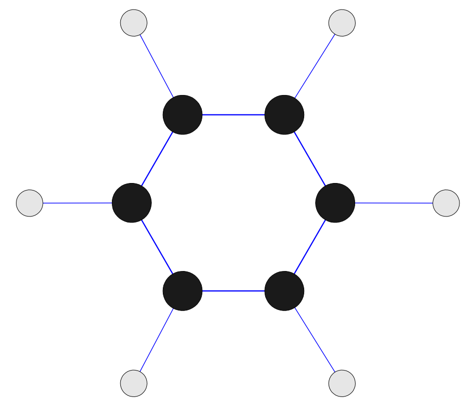
Spatial Data

GGTTCCGCCTTCAGCC
CCGCGCCCGCAGGG...

Data Sequence

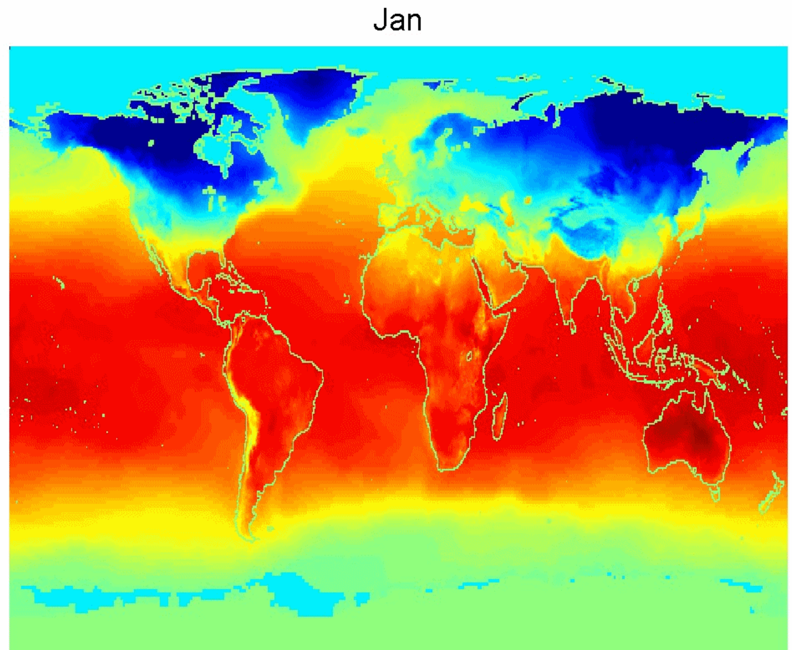
Types of data sets (cont.)

Chemical Data



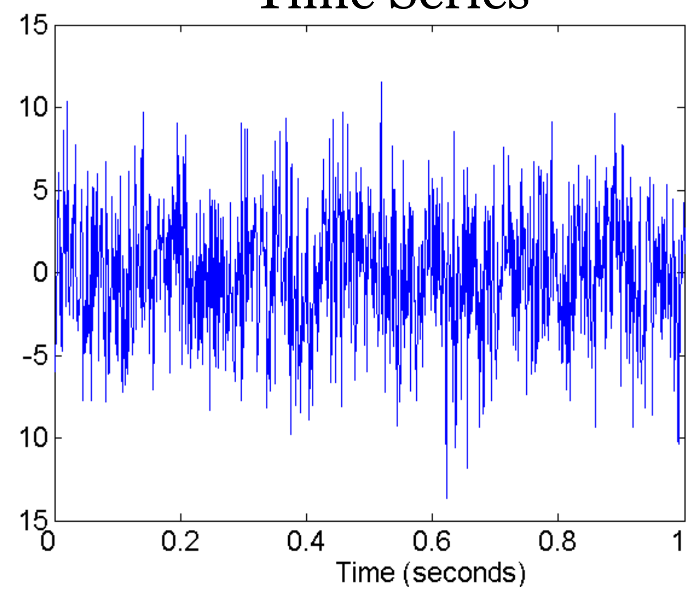
Benzene Molecule: C_6H_6

Spatio-Temporal Data



Average Monthly Temperature of land and ocean

Time Series



Data Matrix

- Data can often be represented or abstracted as an $\boldsymbol{n \times d}$ *data matrix*, with n rows and d columns, given as

$$\boldsymbol{D} = \left(\begin{array}{c|cccc} & X_1 & X_2 & \cdots & X_d \\ \hline \boldsymbol{x}_1 & x_{11} & x_{12} & \cdots & x_{1d} \\ \boldsymbol{x}_2 & x_{21} & x_{22} & \cdots & x_{2d} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \boldsymbol{x}_n & x_{n1} & x_{n2} & \cdots & x_{nd} \end{array}\right)$$

- **Rows:** Also called *instances*, *examples*, *records*, *transactions*, *objects*, *points*, *feature-vectors*, etc. Given as a d-tuple

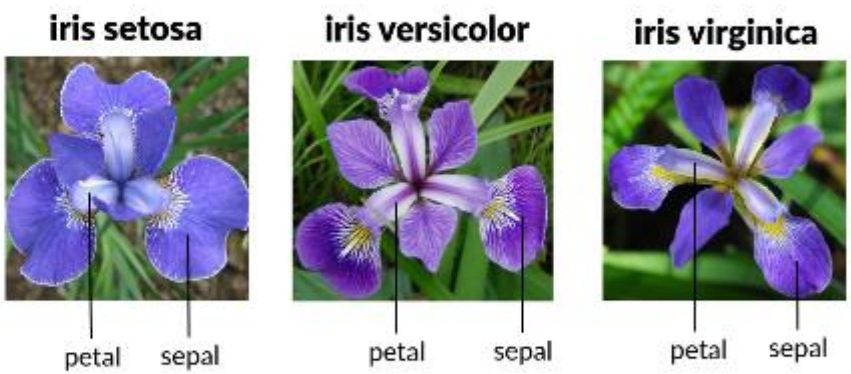
$$\boldsymbol{x}_i = (x_{i1}, x_{i2}, \ldots, x_{id})$$

- **Columns:** Also called *attributes*, *properties*, *features*, *dimensions*, *variables*, *fields*, etc. Given as an n-tuple

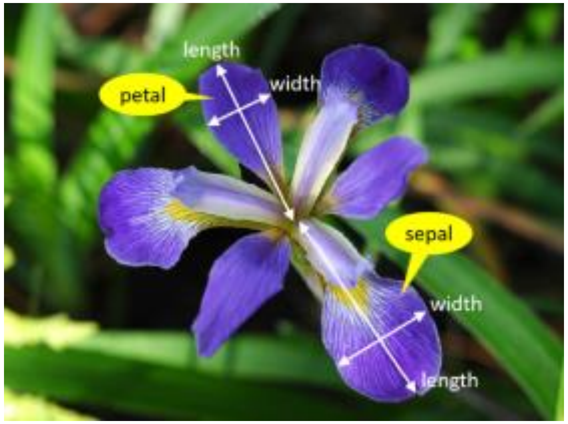
$$X_j = (x_{1j}, x_{2j}, \ldots, x_{nj})$$

Iris Dataset Extract

Data to quantify
the morphologic variation
of *Iris* flowers
Wikipedia



	Sepal length X_1	Sepal width X_2	Petal length X_3	Petal width X_4	Class X_5
$\mathbf{x}_1$	5.9	3.0	4.2	1.5	Iris-versicolor
$\mathbf{x}_2$	6.9	3.1	4.9	1.5	Iris-versicolor
$\mathbf{x}_3$	6.6	2.9	4.6	1.3	Iris-versicolor
$\mathbf{x}_4$	4.6	3.2	1.4	0.2	Iris-setosa
$\mathbf{x}_5$	6.0	2.2	4.0	1.0	Iris-versicolor
$\mathbf{x}_6$	4.7	3.2	1.3	0.2	Iris-setosa
$\mathbf{x}_7$	6.5	3.0	5.8	2.2	Iris-virginica
$\mathbf{x}_8$	5.8	2.7	5.1	1.9	Iris-virginica
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$\mathbf{x}_{149}$	7.7	3.8	6.7	2.2	Iris-virginica
$\mathbf{x}_{150}$	5.1	3.4	1.5	0.2	Iris-setosa



Dataset Properties

Size:	Measured in terms of the total number of records or total number of bytes, e.g. Small (MB), medium (GB) and large (TB)
Dimensionality:	Number of attributes
Sparsity:	• Values are skewed to some extreme or sub-ranges • Asymmetric values (some are more important than others)
Resolution:	• Right level of data details • Related to the intended purpose

Data Sources

- **Public data**
  - Data hubs https://www.kaggle.com/datasets https://www.openml.org , GitHub
  - Open data such as https://www.data.gov.qa/ & https://data.gov/
  - Data conferences
  - Many others…
- **Enterprise/Organisational data warehouse**
  - An organisational database for decision making
  - A central data repository separate from operational systems
  - Equipped with data analysis and reporting tools
- **Your own generated/collected data**

Data Exploration - Univariate

`03.eda\5-univariate-eda.ipynb`

Exploratory Data Analysis (EDA)

- Exploratory Data Analysis (EDA): exploring data through summary statistics and visual charts, and graphs.

  Purpose:
  - Better understanding of the characteristics of data
  - Spot anomalies (e.g., missing data, outliers)
  - Better decision regarding data pre-processing tasks
- The three main types of EDA:
  - **Univariate EDA** explore a single feature at a time to understand the data distribution and identify any outliers.
  - **Bivariate EDA** looking at two features at a time to understand the relationship and identify any patterns that might exist
  - **Multivariate EDA** looking at three or more features at a time to understand the relationships and identify any patterns

Summary Statistics - Central Tendency

- Mean and Median for continuous attributes:

  - **Mean**

  $$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

  - **Median** (Middle value if odd number of values, or average of the middle two values otherwise)

  Median is a better indication of "average" when data distribution is skewed, or outliers are present

  - Trimmed Mean and Median (after trimming top and bottom p%)

Summary Statistics - Central Tendency

- **Mode** for categorical attributes:
  - Frequency counts of values that a feature takes
  - Proportion: Frequency count for a value divided by the total sample size
  - **Mode**: the most frequently occurred value

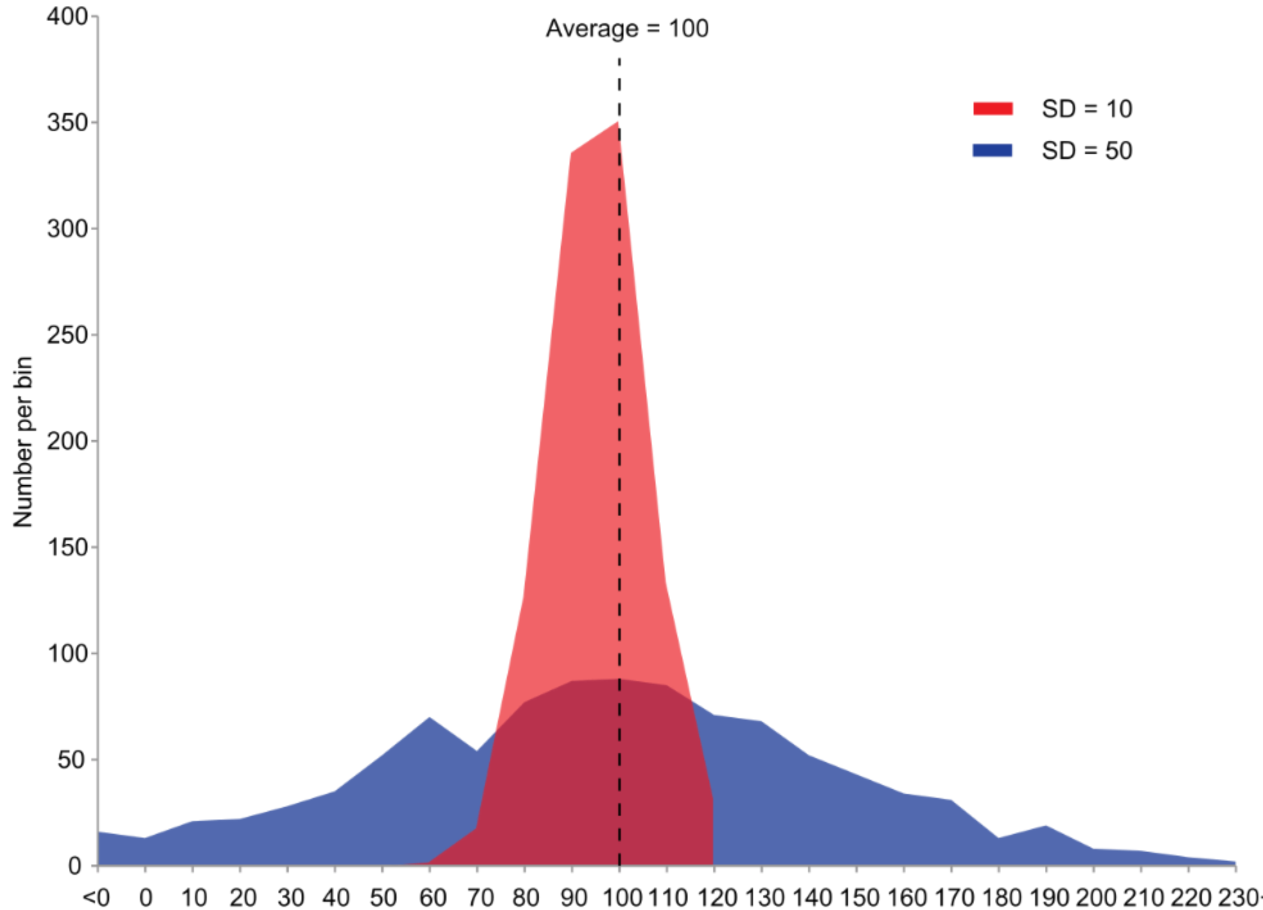
Summary Statistics - Measures of Spread

- Measure how "spread out" the values are
- Range $range(x) = \max(x) - \min(x)$
- Variance (σ^2) $\sigma^2 = \frac{1}{m-1}\sum_{i=1}^{m}(x_i - \bar{x})^2$
- Standard Deviation (σ) $\sigma = \sqrt{\frac{1}{m-1}\sum_{i=1}^{m}(x_i - \bar{x})^2}$
- Percentiles of continuous attributes:
  - Given an attribute *x* and an integer *p* ($0 \le p \le 100$), the percentile x_p is a value of x such that *p*% observed values of *x* are less than x_p
  - Q_1 (25th percentile), Q_3 (75th percentile). Q_3 means 75% of the data values are less than Q_3
  - Inter-quartile range: $IQR = Q_3 - Q_1$

Measures of Spread, *cont'd*

- Measure how the values are stretched or squeezed
  - aka **Measures of dispersion**

Two datasets with the <u>same mean</u> but **different dispersion**. The blue dataset is much more **dispersed** than the red dataset.



https://en.wikipedia.org/wiki/Statistical_dispersion

Summary Statistics using Pandas

```
df.describe()

df[['DepTime', 'DepDelay', 'ArrTime',
'ArrDelay']].agg(['mean', 'min', 'max'])


price_mean = df['price'].mean()

price_median = df['price'].median()

price_std = df['price'].std()

price_var = df['price'].var()

price_quantiles =

     df['price'].quantile([0.25,0.5,0.75])
```

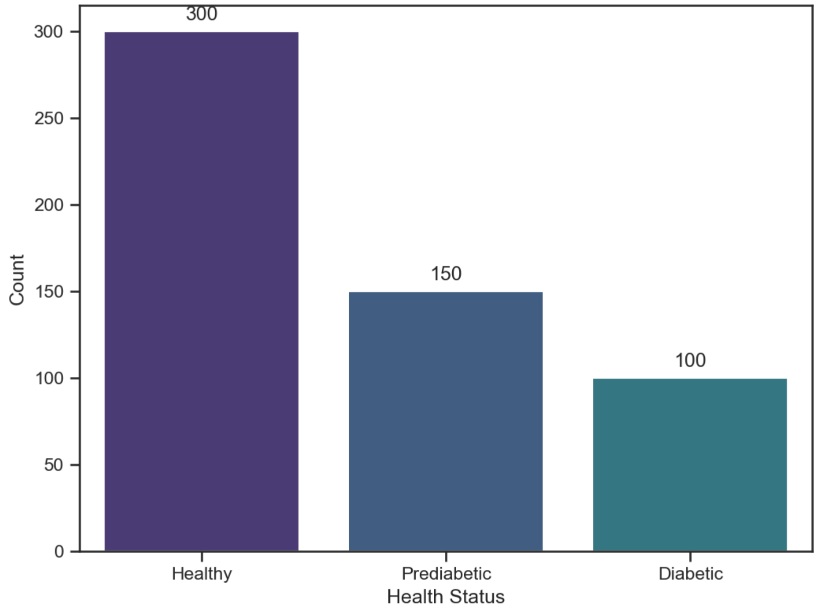
Motto: Visualize Before Analyzing!

- Data visualization gives us a more holistic sense
- Allows understanding patterns, distributions, and relationships among different features
- *Anscombe's quartet* datasets having the same mean, standard deviation, and regression line, but which are qualitatively different.
  - It illustrates the importance of looking at a set of data graphically and not only relying on basic statistic properties.

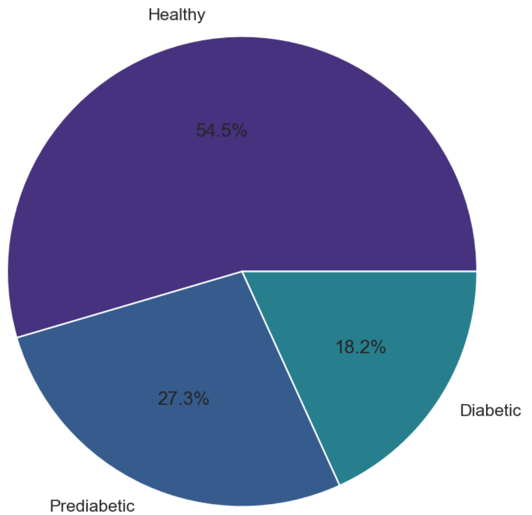
`03.eda\4.why-visualization-anscombe.ipynb`

Data visualization for categorical data

- **Bar Chart**: categories on one axis and the corresponding frequencies or proportions on the other axis



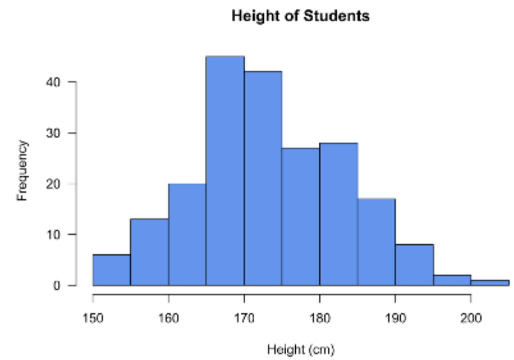
- **Pie Chart**: shows relative size of each category within the whole

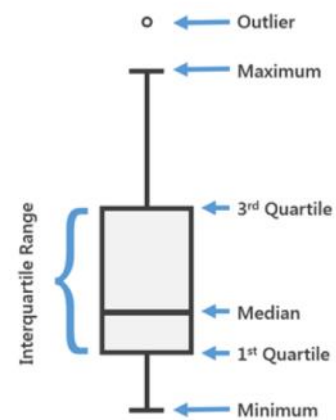


`03.eda\2-categorical-variables.ipynb`

Data visualization for numerical data

- **Histogram**: represent the distribution of a continuous numerical variable
- **Boxplot**: Depicts the spread and central tendency of the data, including median, quartiles, and potential outliers
- **Line plot**: visualize trends in data over time



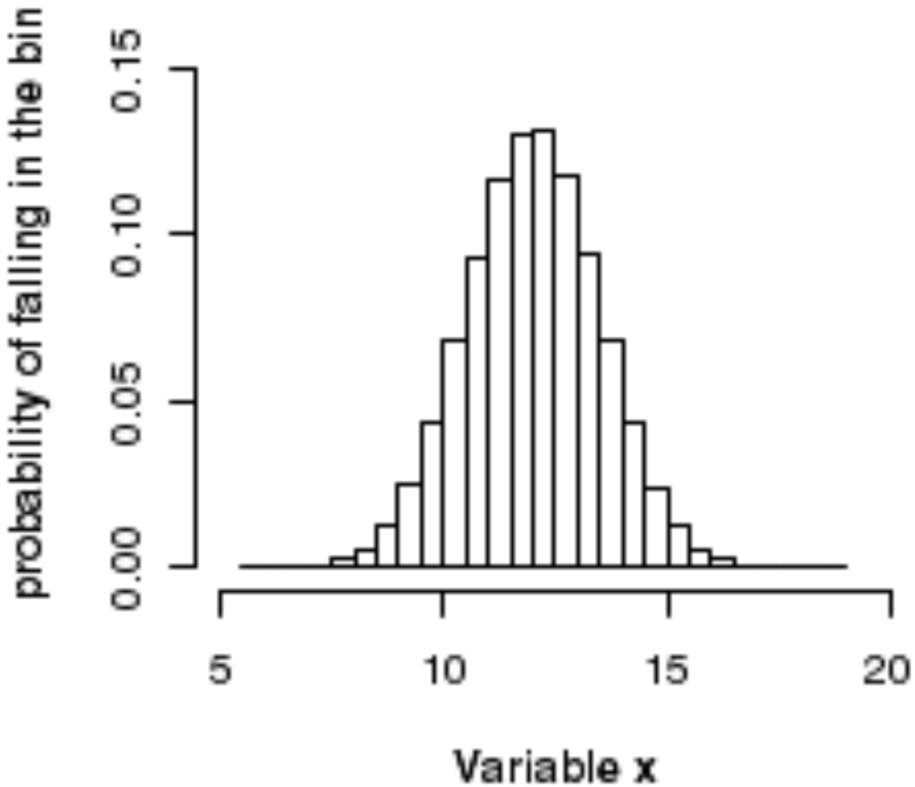


`03.eda\1-numerical-variables.ipynb`

Histogram to probability distribution

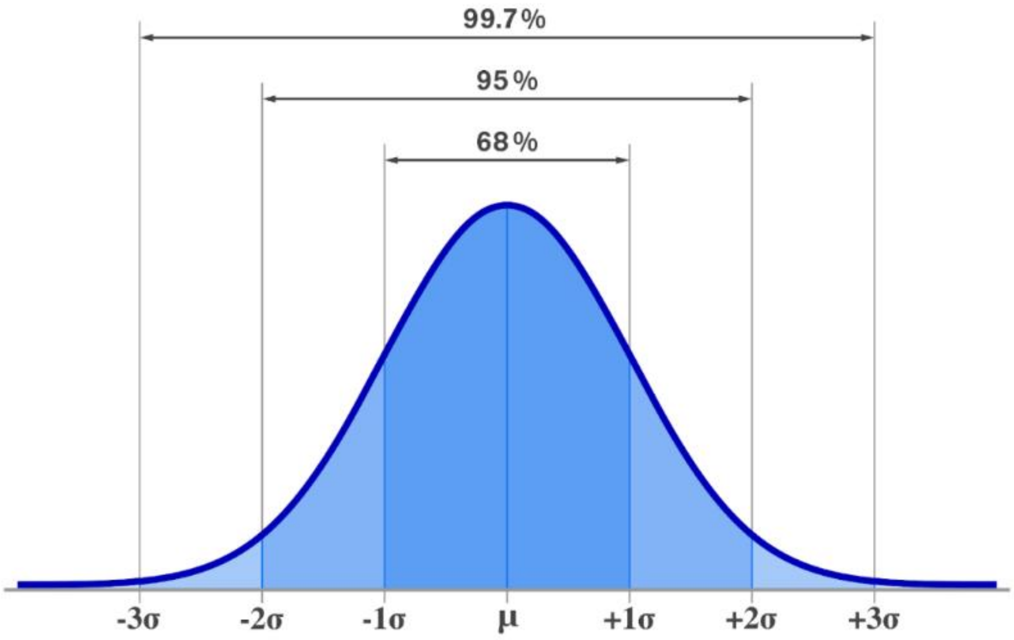
- Divide the count for each interval by the total number of observations in the dataset multiplied by the width of the interval

$$\text{Probability distribution} = \frac{\text{Count for each interval}}{\text{Count of observations in the dataset} \times \text{Width of the interval}}$$



Normal Distribution

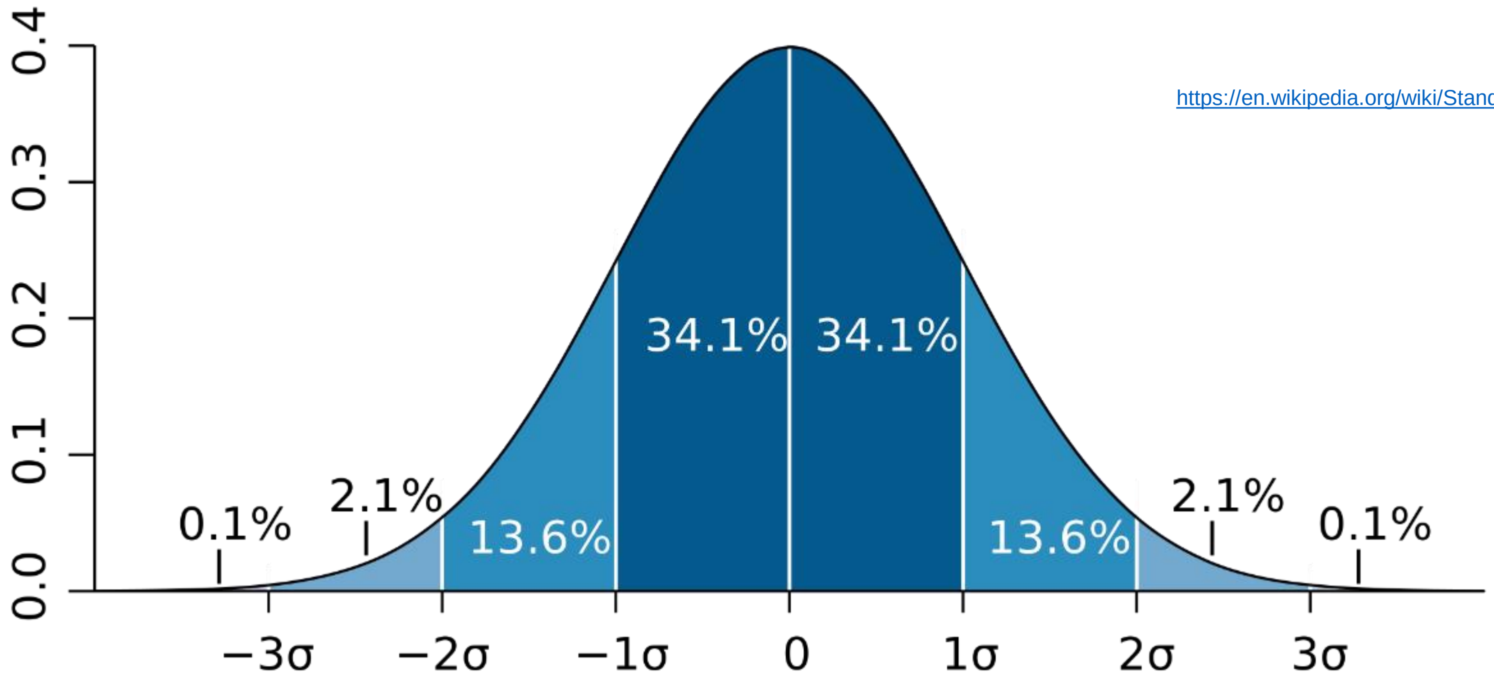
- Many phenomena follow a normal distribution
  - Height, blood pressure, exam scores, etc.



- Symmetric:
  - Most of the observations occur around the central peak
  - Probabilities for values further away from the center decrease equally in both directions
  - Extreme values in both tails of the distribution are similarly unlikely

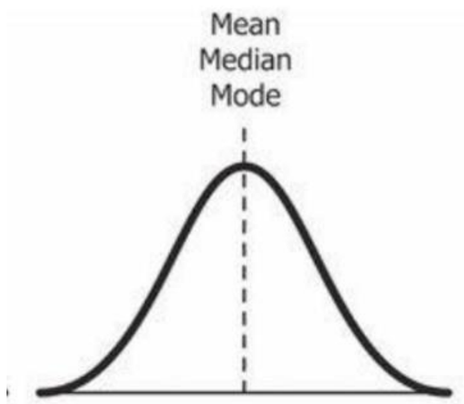
Spread Properties of Normal Distribution Curve

- The 68−95−99.7 rule:
  - From μ–σ to μ+σ: contains about 68% of the values
  - From μ–2σ to μ+2σ: contains about 95% of it
  - From μ–3σ to μ+3σ: contains about 99.7% of it

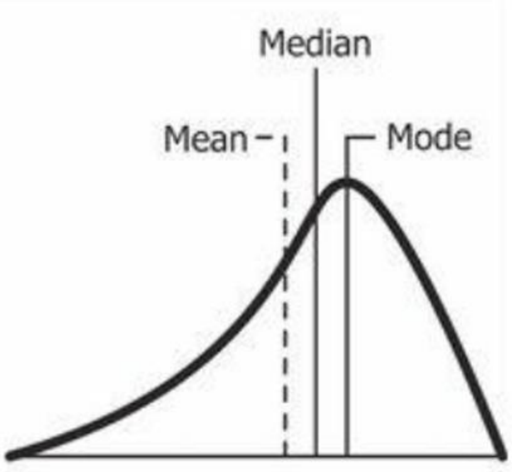


https://en.wikipedia.org/wiki/Standard_deviation

Dark blue is one standard deviation on either side of the mean, or 68% of the values

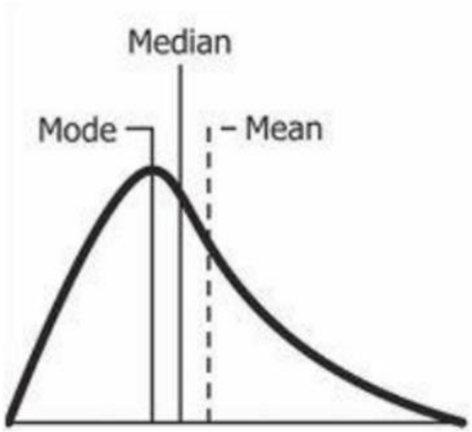


- In normal **symmetric distribution**, the mean, median and mode are the same



- A distribution is skewed if one of its tails is longer than the other

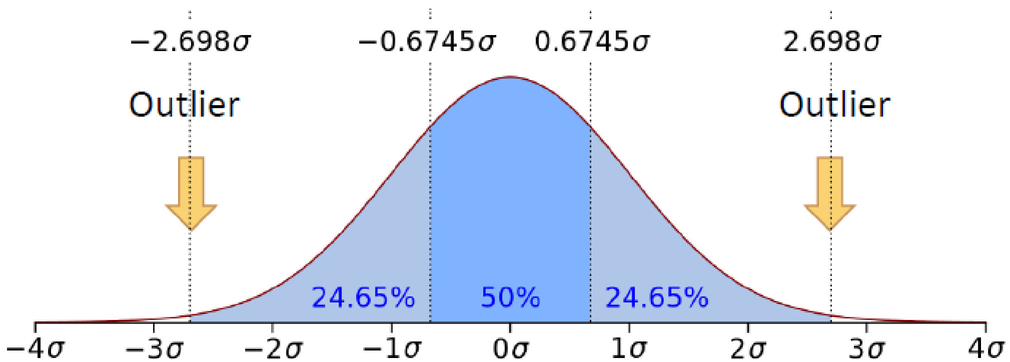
- A **left-skewed** (negative-skewed) distribution has a long left tail



- A **right-skewed** (positive-skewed) distribution has a long right tail

Detecting Outliers

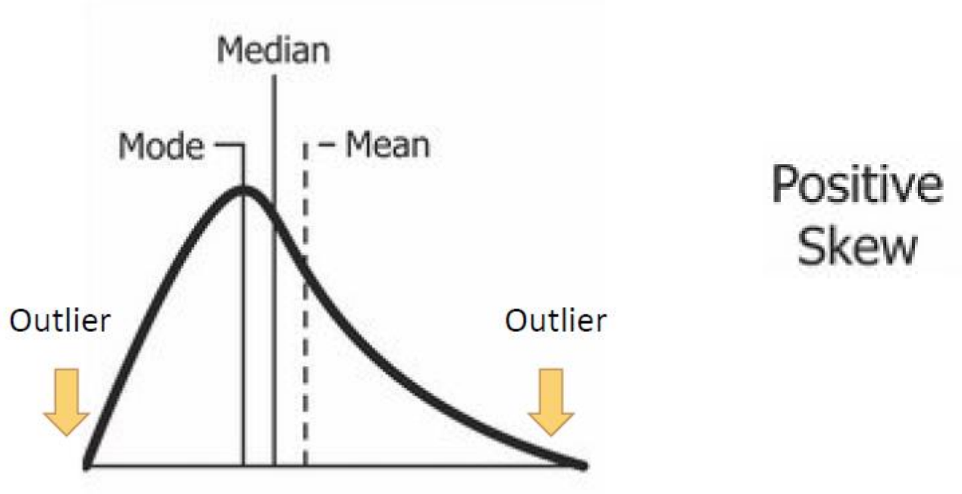
- An outlier is a data point which is **significantly different** from the remaining data



- ≈99% of the observations of a normally distributed variable lie within the mean ± 3 x standard deviations
- Values outside mean **± 3 x standard deviations** are considered outliers

`03.eda\5-univariate-eda.ipynb`

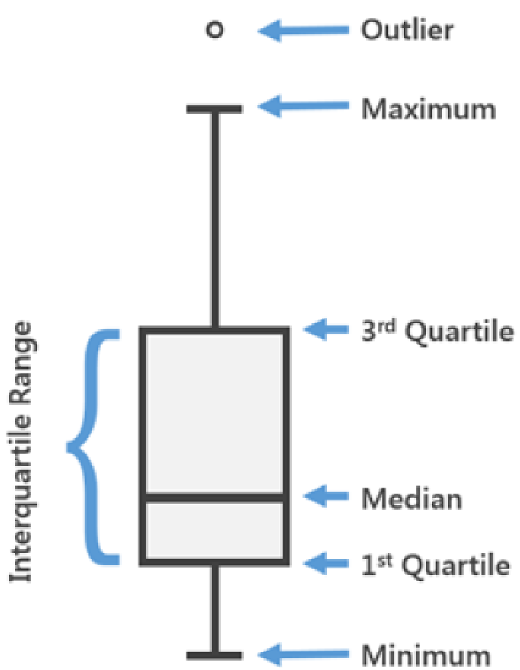
Outliers for skewed distributions

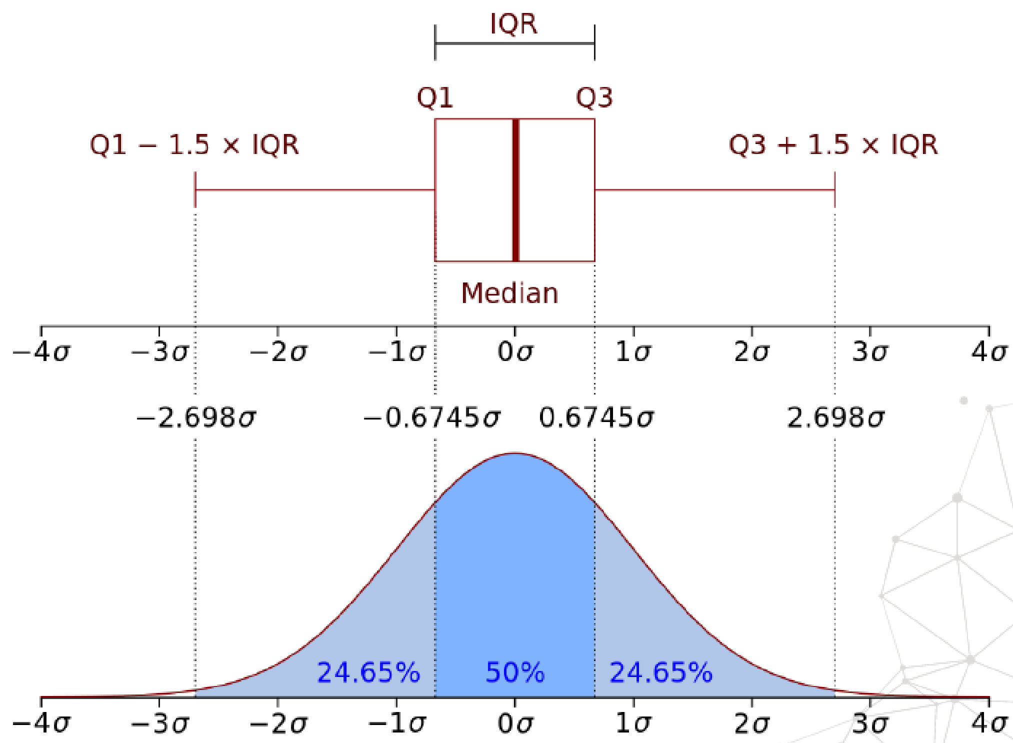


Calculate the quantiles, and then the inter-quantile range (IQR), as follows:

- `IQR = 75`th `Quantile - 25`th `Quantile`
- `Upper limit = 75`th `Quantile + IQR × 1.5`
- `Lower limit = 25`th `Quantile - IQR × 1.5`
- Values outside the limits are considered **outliers** that can be dropped or replaced by the mean or median

Visualizing outliers using Boxplots





Images taken from pro.arcgis.com and wiki.commons

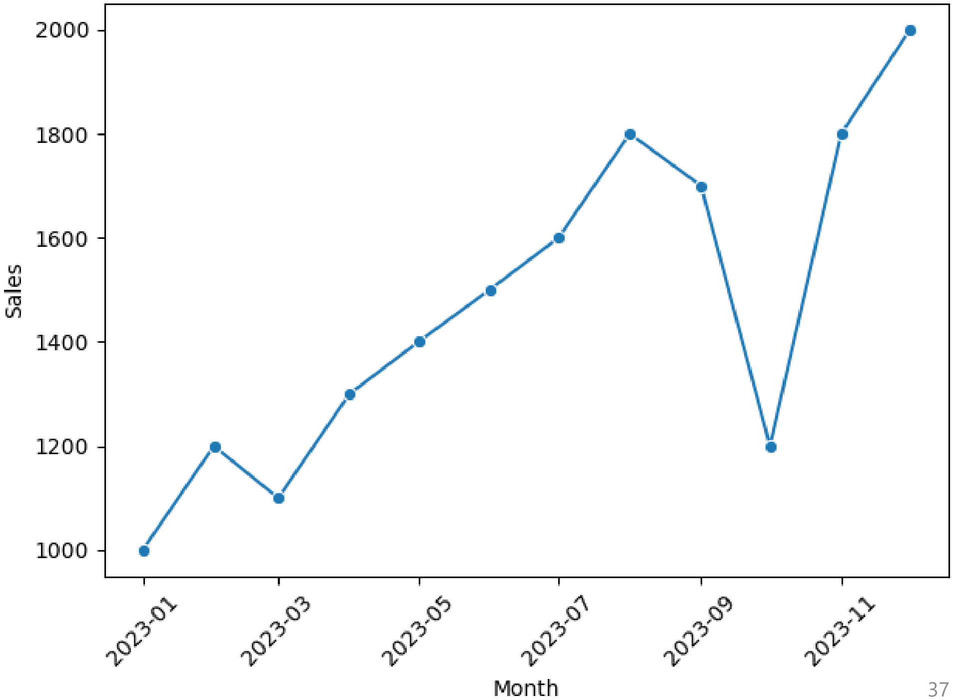
Boxplot: data is represented with a box

- The ends of the box are at the 1st and 3rd quartiles, i.e., the height of the box is IQR
- The median is marked by a line within the box
- Whiskers: two lines outside the box extended to `± IQR × 1.5`

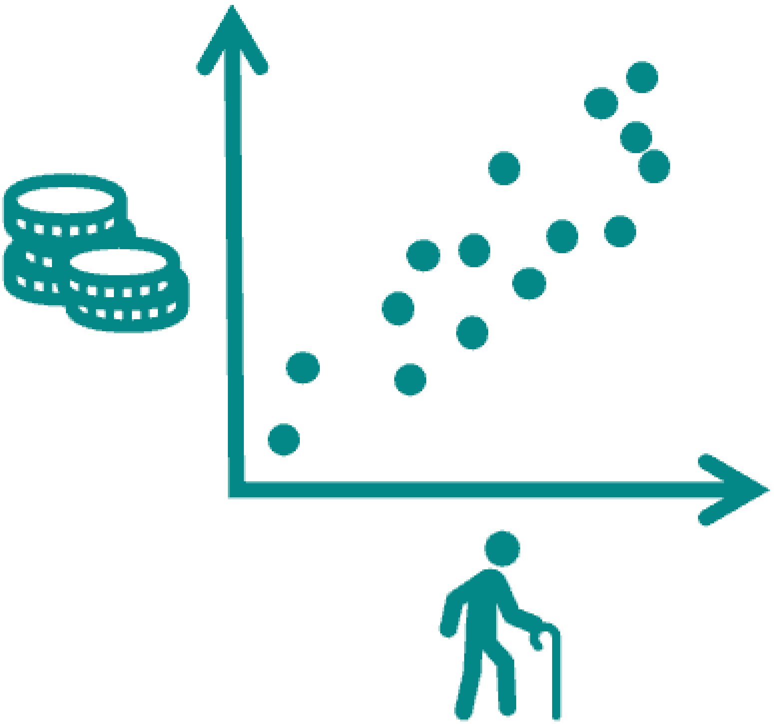
Line Plot - analyzing a Single Variable over Time

- Line Plot display data points connected by straight lines
- It is particularly effective for visualizing trends in data over time
  - E.g., Tracking stock prices over months to identify trends or patterns
- They help identify seasonal patterns such as increasing, decreasing, or cyclical trends
  - E.g., Peak air travel around June-August

e.g., LinePlot visualizes the monthly sales.
We can observe any fluctuations or any seasonal patterns in sales over the course of the year
e.g. upward trend with growth in sales except a drop in the month of October.



Data Exploration - Bivariate



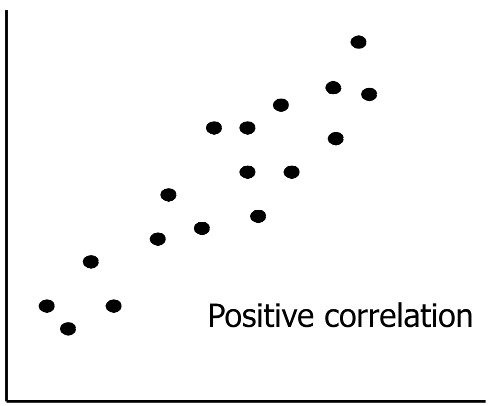
`03.eda\`**`6-bivariate-eda.ipynb`**

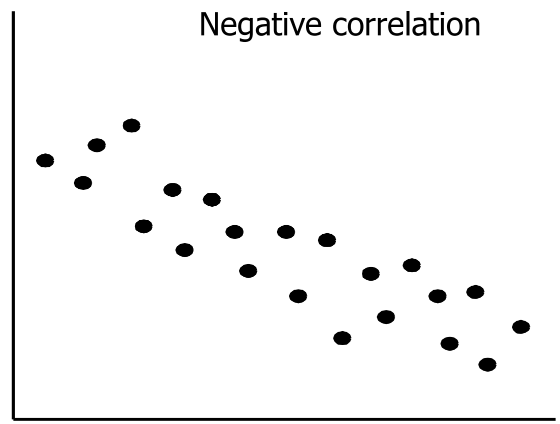
Types of Bivariate Analysis

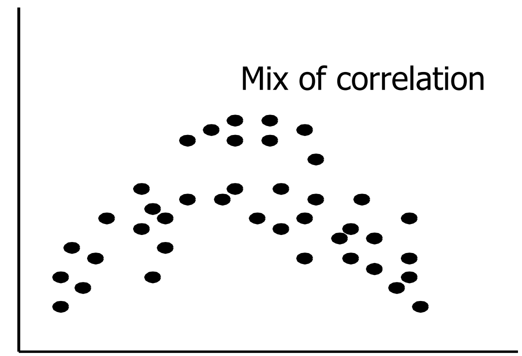
- 📈 **Numerical vs. Numerical**: examines the relationship between two numerical variables.
  - E.g.,: In a real estate dataset, we analyze the correlation between the house size in m^2 of a house and its price. Are larger houses more expensive?
- 📊 **Categorical vs. Numerical**: explore how a categorical variable affects a numerical one.
  - E.g.,: Studying how the type of car (SUV, sedan, etc.) impacts fuel efficiency (miles per gallon) in an automotive dataset.
  - E.g., Connection between the level of education and income in demographic studies
- 📚 **Categorical vs. Categorical**: focuses on the association between two categorical variables.
  - E.g.,: In an e-commerce dataset, we assess if there's a connection between a customer's gender and their preferred payment method

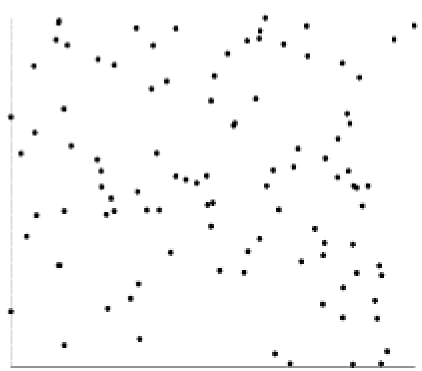
Scatter Plot

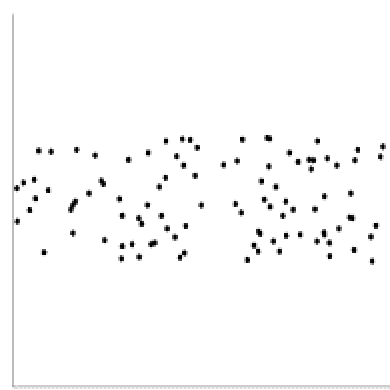
- **Scatter Plot** visualize the relationship between two continuous variables by plotting one variable along the x-axis and the other variable along the y-axis
- Useful to determine whether there is a correlation between the variables (positive, negative, or none), the strength of the correlation, and the presence of any outliers









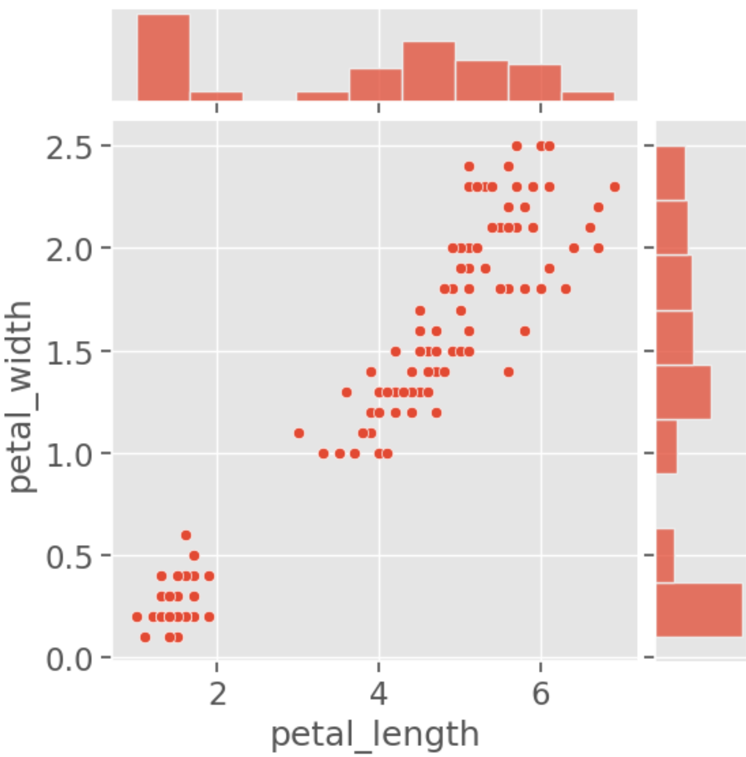


Not Correlated Data

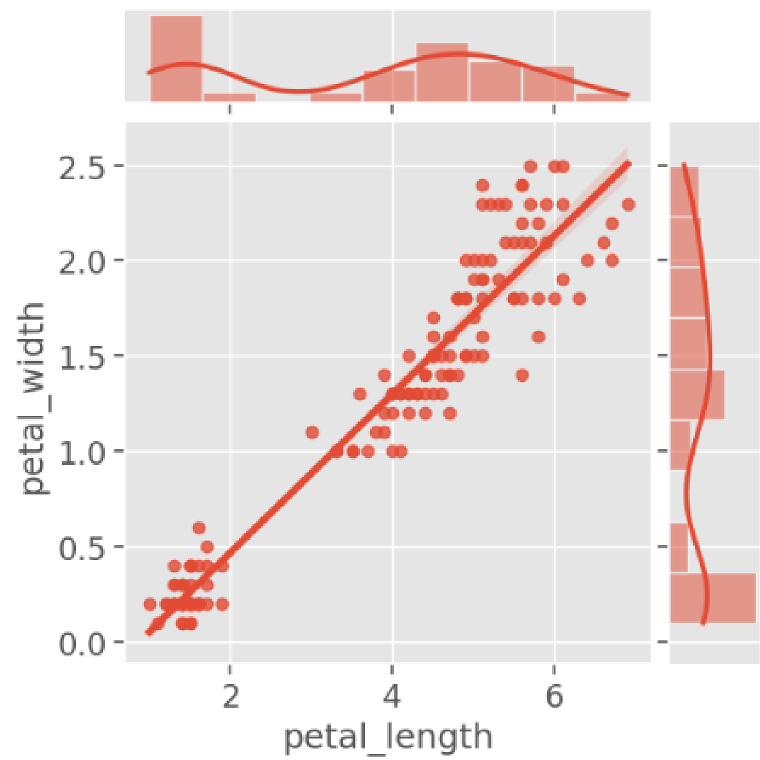
Joint Plot - Visualizing Pairs of Continuous Features

- **Joint Plot** combines multiple plots, such as scatter plot and histogram, to visualize the correlation between the variables, including their individual distributions

```python
sns.jointplot(x='petal_length', y='petal_width', data=iris, kind='scatter')
pass
```




```python
sns.jointplot(x='petal_length', y='petal_width', data=iris, kind='reg')
pass
```



Iris Characteristics: Strong linear relationship between petal length and width

Pearson's correlation

- Pearson correlation is a statistical measure that **quantifies the linear relationship between two continuous variables**. It provides insights into how closely related two variables are and the direction of their relationship (positive or negative)
  - The Pearson correlation coefficient, denoted by r, ranges from -1 to 1
  - $r=1$ indicates a perfect positive linear relationship
  - $r=-1$ indicates a perfect negative linear relationship
  - $r=0$ indicates no linear relationship
  - A positive r value suggests that as one variable increases, the other tends to increase as well
  - A negative r value indicates that as one variable increases, the other tends to decrease
- In Python, you can calculate the Pearson correlation coefficient using the corr() function from pandas or pearsonr() function from the scipy.stats module

Pearson correlation coefficient

$$r = \frac{\sum(X_i - \bar{X})(Y_i - \bar{Y})}{\sqrt{\sum(X_i - \bar{X})^2 \sum(Y_i - \bar{Y})^2}}$$

Where:

- X_i and Y_i are individual data points.
- $\bar{X}$ and $\bar{Y}$ are the means of X and Y respectively.

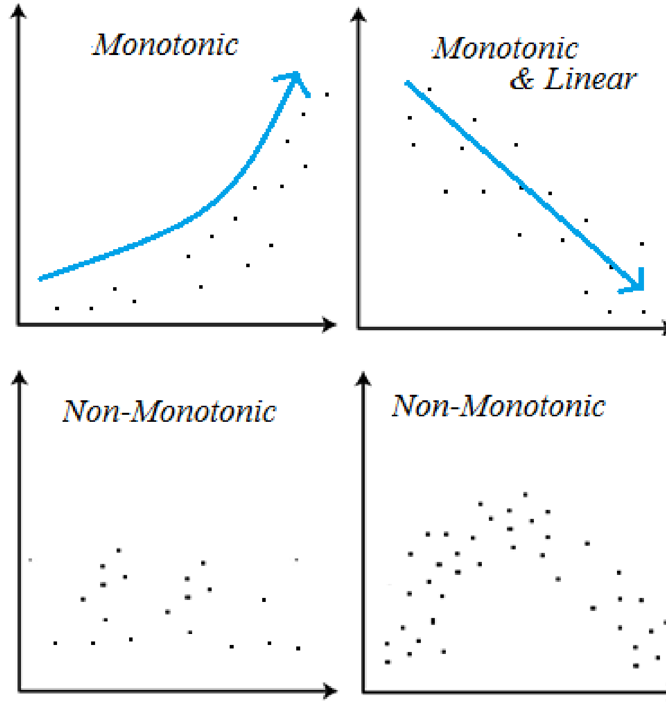
- The numerator calculates the covariance between X and Y, which measures how they vary together from their means
- The denominator normalizes the covariance by the standard deviations of X and Y, ensuring that the correlation coefficient is scaled appropriately

Pearson's correlation assumptions

- Pearson's correlation coefficient is a parametric measure of the linear relationship between two continuous variables. It makes certain assumptions about the data, including:
    - **Linearity**: there is a linear relationship between the two variables. If the relationship between the variables is not linear, Pearson's correlation may not accurately reflect the relationship.
    - **Normality**: the data is normally distributed. This means that the distribution of the residuals (the difference between the values) should follow a normal distribution.
    - **Independence**: the observations are independent of one another. This means that the value of one observation does not influence the value of another observation.
- If these assumptions are not met, Pearson's correlation may not accurately reflect the relationship between the variables. In these cases, non-parametric methods, such as Spearman's rank correlation, may be more appropriate

Spearman's Rank Correlation

- Non-parametric: does not assume a specific distribution of the data
- Measure of the monotonic relations: the variables tend to move in the same direction, but not necessarily at a constant
- Measure the degree of correlation between two variables
- Calculated based on the ranks of the data points instead of the actual values.



Students	Maths	Science
A	35	24
B	20	35
C	49	39
D	44	48
E	30	45

Students	Maths	Rank	Science	Rank	d	d square
A	35	3	24	5	2	4
B	20	5	35	4	1	1
C	49	1	39	3	2	4
D	44	2	48	1	1	1
E	30	4	45	2	2	4
						14

$$\rho = 1 - \frac{6 \sum d_i^2}{n(n^2 - 1)}$$

ρ = Spearman's rank correlation coefficient

d_i = difference between the two ranks of each observation

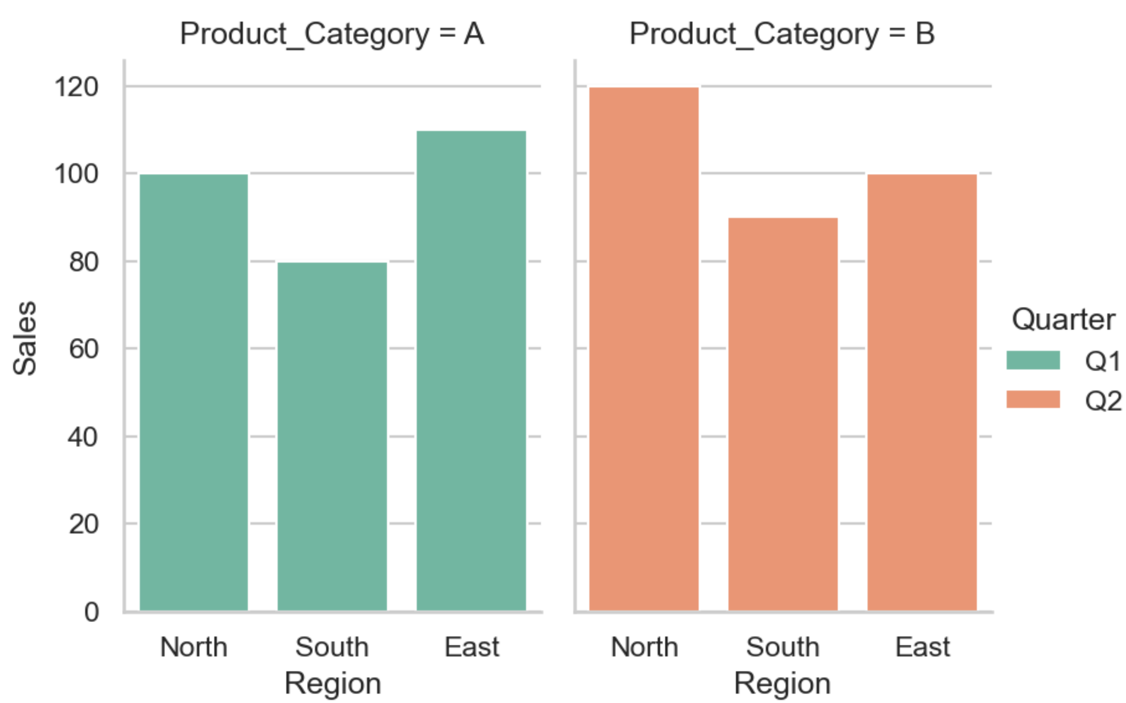
n = number of observations

1 - (6 * 14) / 5(25 - 1) = 0.3

The Spearman’s Rank Correlation for the given data is 0.3. The value is near 0, which means that there is a weak correlation between the two ranks.

A collection of Bar Plots - Visualizing Pairs of Categorical Features

- A collection of bar plots allows comparing multiple categorical features for exploring and analyzing relationships and trends within datasets
  - E.g., visualize how the sales of each product category vary across different regions and quarters

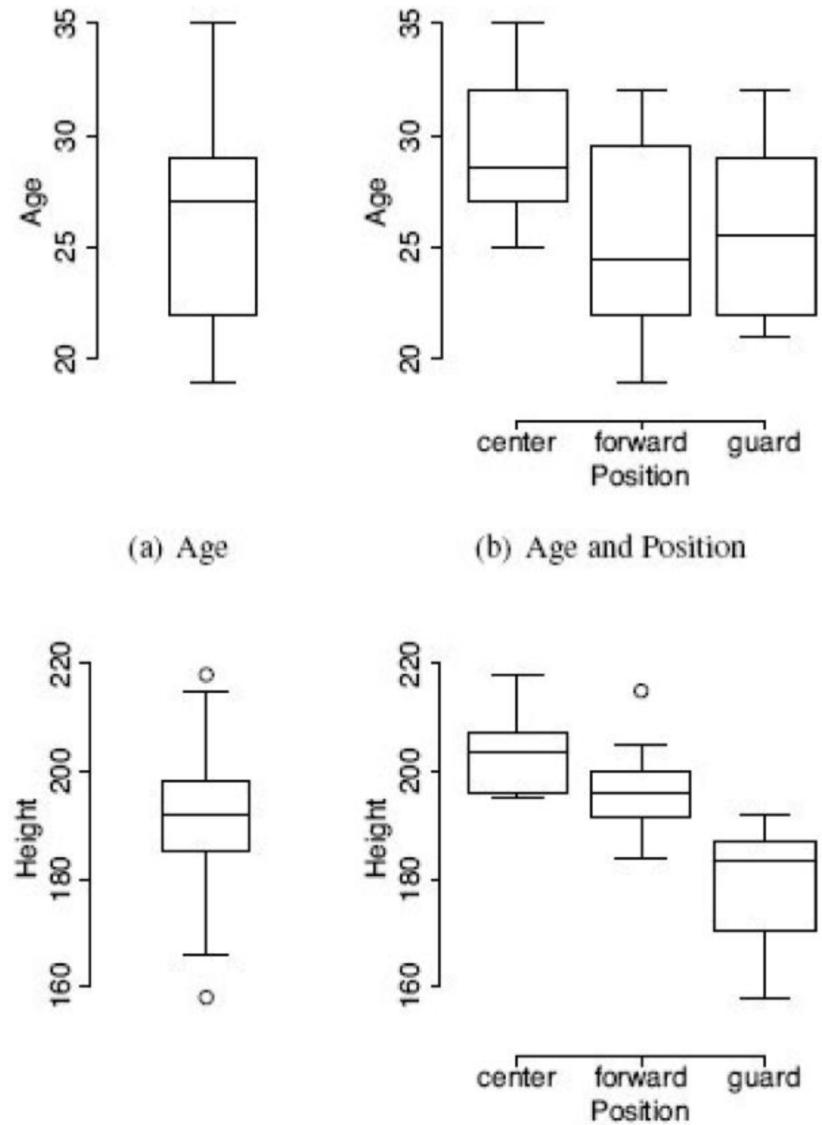


Stacked Bar Plot - Visualizing Pairs of Categorical Features

- **Stacked Bar Plot** is used to represent the **distribution** of a categorical variable, showcasing the composition of each category as a stack of subcategories.
  - Each bar in the plot represents the total value of the categorical variable, and the segments within the bar correspond to different subcategories.

Grouped Box Plots

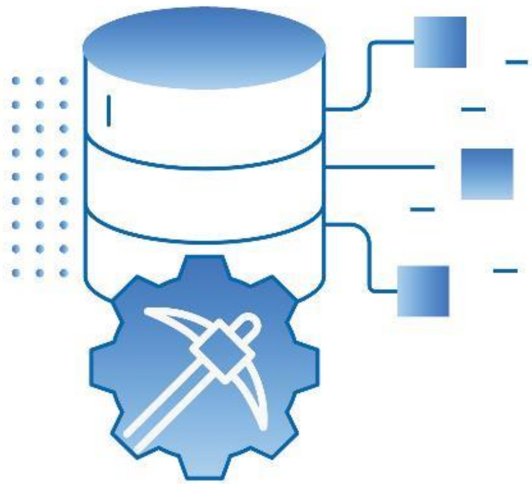
- **Grouped Box Plots**, allows comparing the distributions of the continuous variable across **multiple categories simultaneously**.
  - This visualization is particularly useful for identifying patterns, differences, and relationships between the categorical and continuous variables



(a) Age (b) Age and Position

(c) Height (d) Height and Position

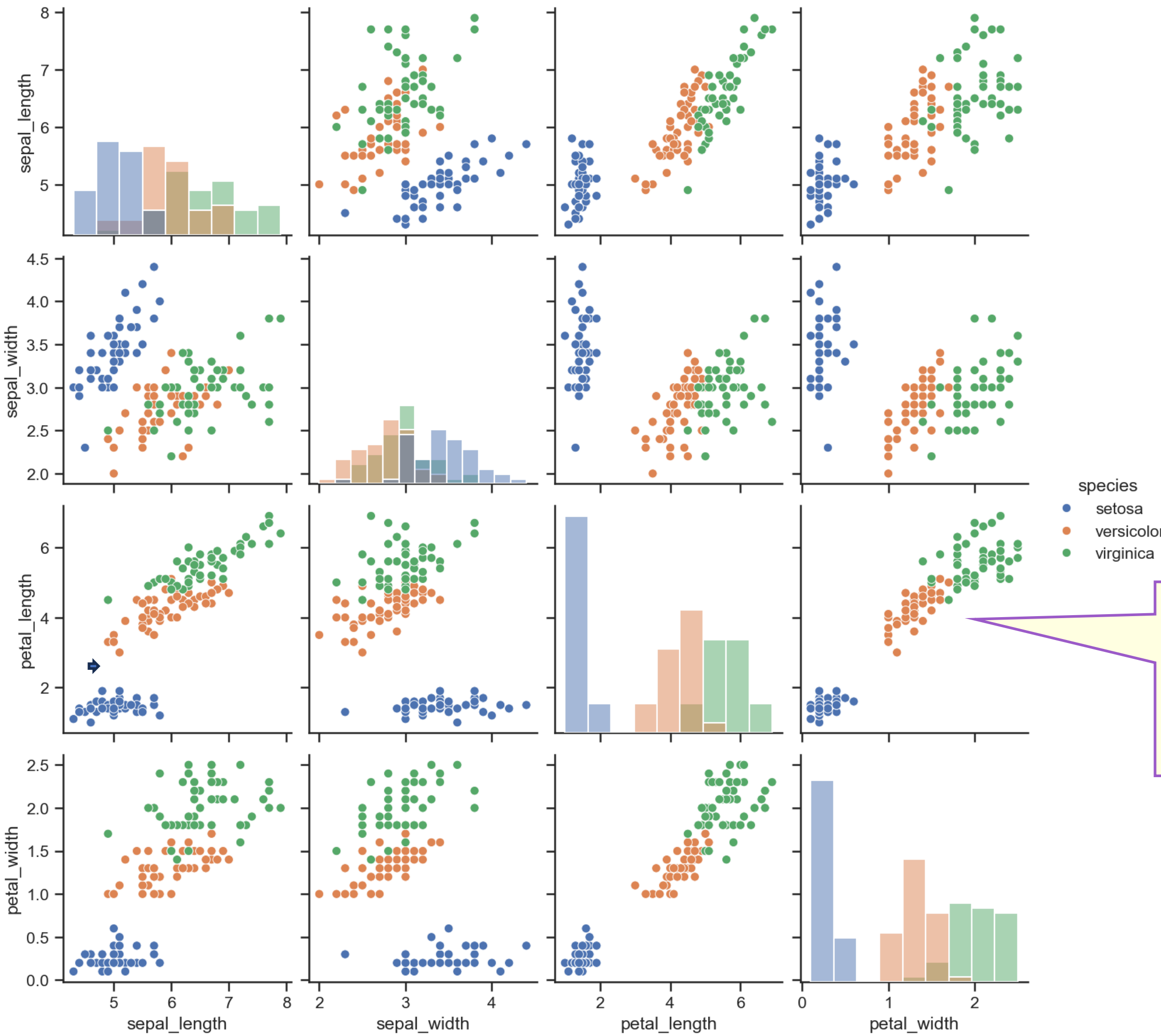
Data Exploration - Multivariate



`03.eda\7-multivariate-eda.ipynb`

Scatter Plot matrix

- **A Scatter Plot matrix**, also known as a **Pairs Plot**, is used to examine the relationships between multiple variables simultaneously.
  - It consists of a grid of scatter plots where each variable is plotted against every other variable in the dataset



Petal dimensions discriminate species more strongly than sepal dimensions

Strong linear relationship between petal length and width

Correlation Matrix & Heatmap

- A correlation matrix is a table that shows the correlation coefficients between many variables
- A heatmap plots data as a color-encoded matrix

```
cormat = iris.corr(method="pearson")
round(cormat,2)
```

]:

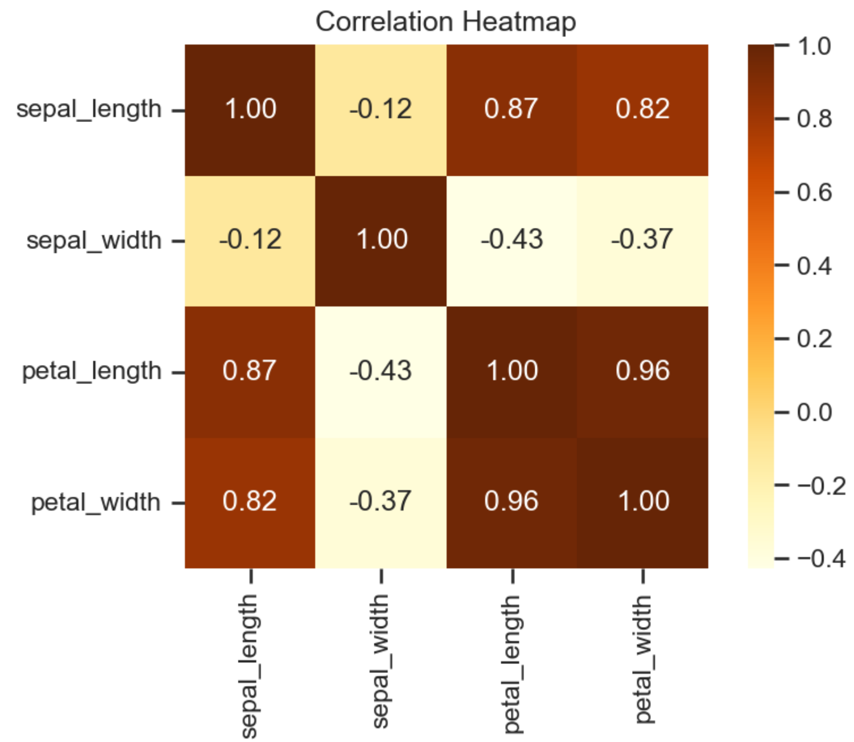
	sepal_length	sepal_width	petal_length	petal_width
sepal_length	1.00	-0.12	0.87	0.82
sepal_width	-0.12	1.00	-0.43	-0.37
petal_length	0.87	-0.43	1.00	0.96
petal_width	0.82	-0.37	0.96	1.00

In [65]:
```
cormat = iris.corr(method="spearman")
round(cormat,2)
```

Out[65]:

	sepal_length	sepal_width	petal_length	petal_width
sepal_length	1.00	-0.17	0.88	0.83
sepal_width	-0.17	1.00	-0.31	-0.29
petal_length	0.88	-0.31	1.00	0.94
petal_width	0.83	-0.29	0.94	1.00

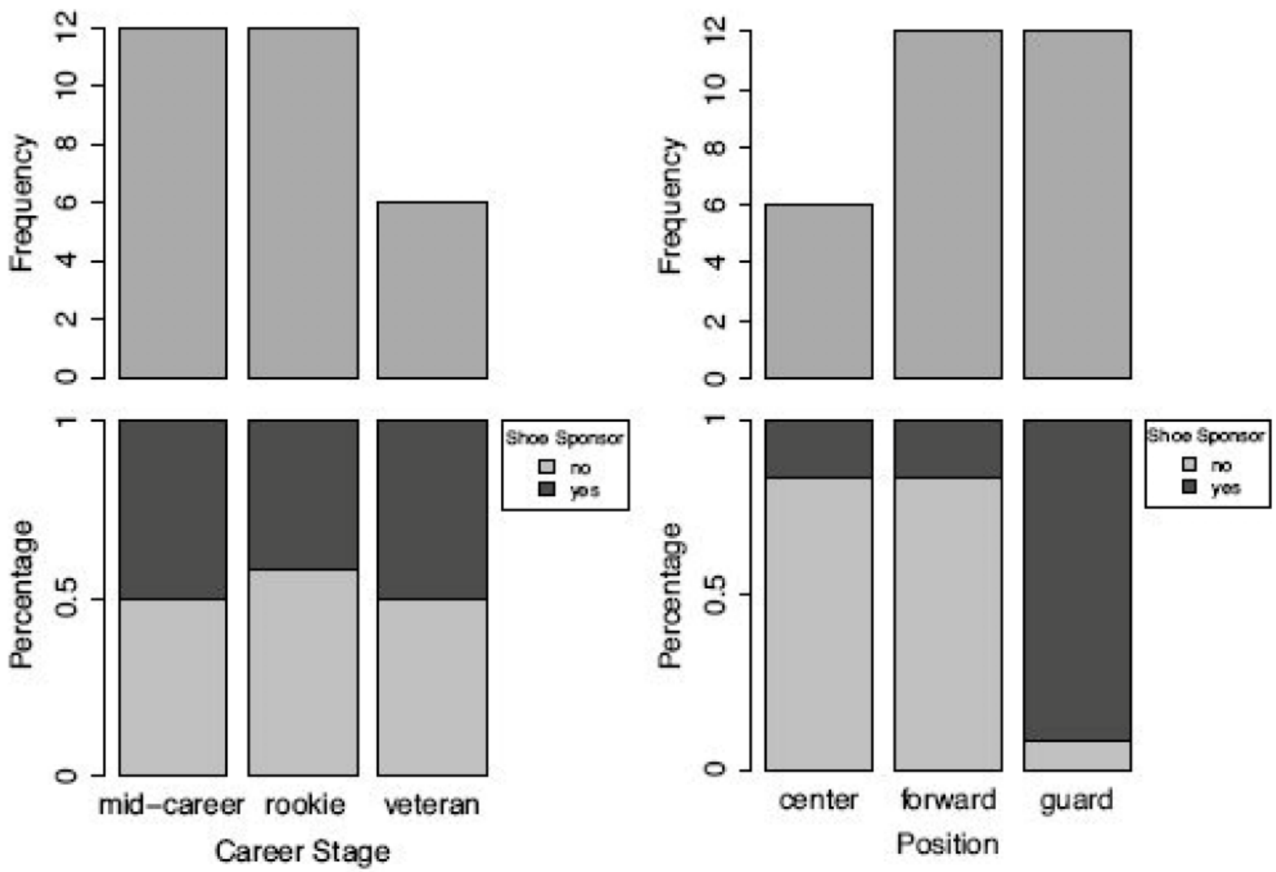
```
sns.heatmap(cormat)
```



We can observe a strong positive correlation between petal length and width

Analyzing the Relationship Between Two Variables

- Visualizing Pairs of Categorical Features
  - **Stacked bar plots**

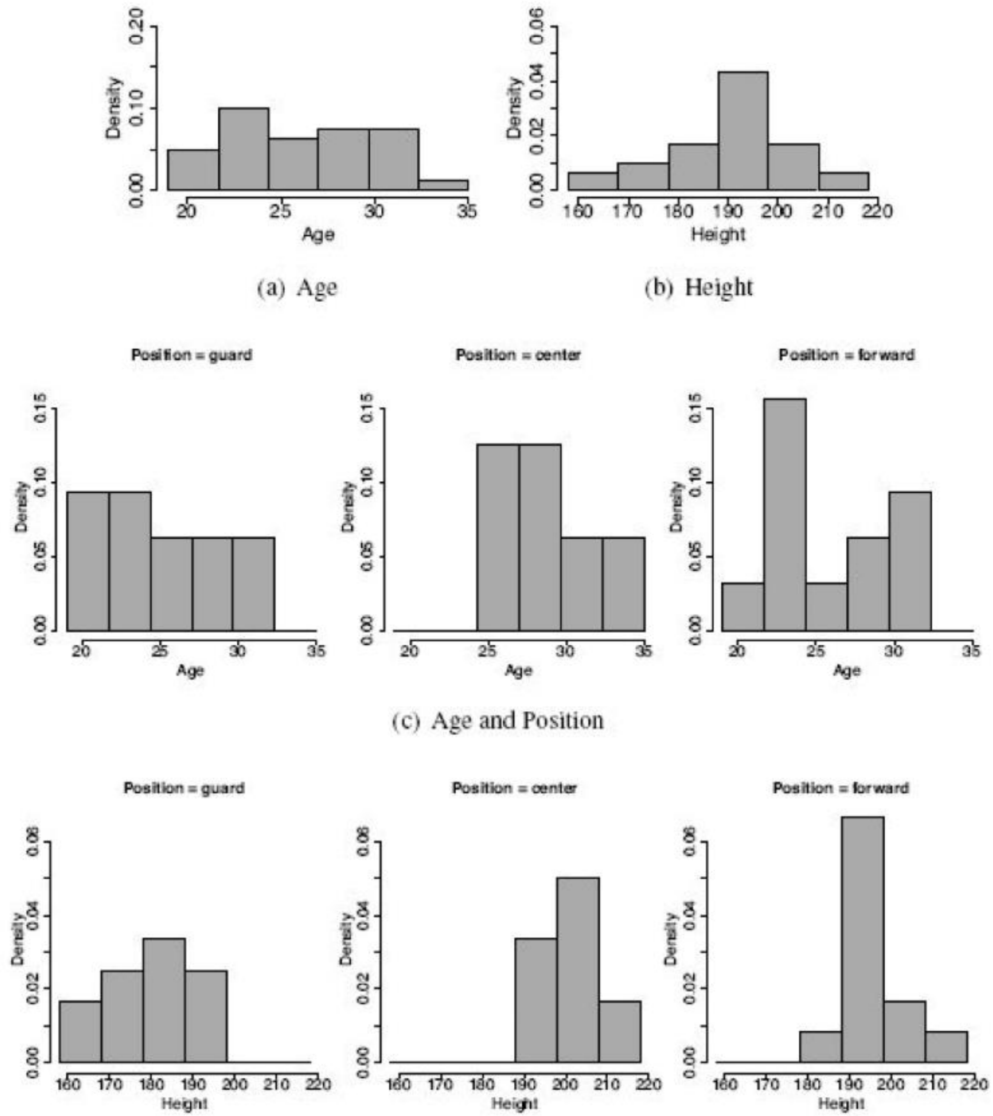


(a) Career Stage and Shoe Sponsor

(b) Position and Shoe Sponsor

Analyzing the Relationship Between Two Variables

- Visualizing a Categorical Feature and a Continuous Feature
    - **Collection of bar plots**



(a) Age (b) Height

(c) Age and Position

(d) Height and Position

Multivariate Data Analysis

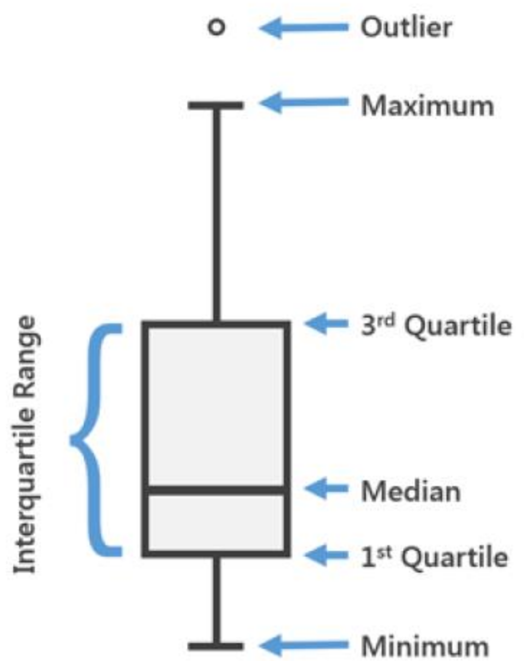
- Measures relationship between pairs of continues features
  - Covariance $\sigma_{xy} = \text{covariance}(x,y) = \frac{1}{m-1}\sum_{i=1}^{m}(x_i - \bar{x})(y_i - \bar{y})$
    - a measure of the linear relations
    - measures the extent to which the variables change together.
  - The covariance matrix is a d x d (square) symmetric matrix

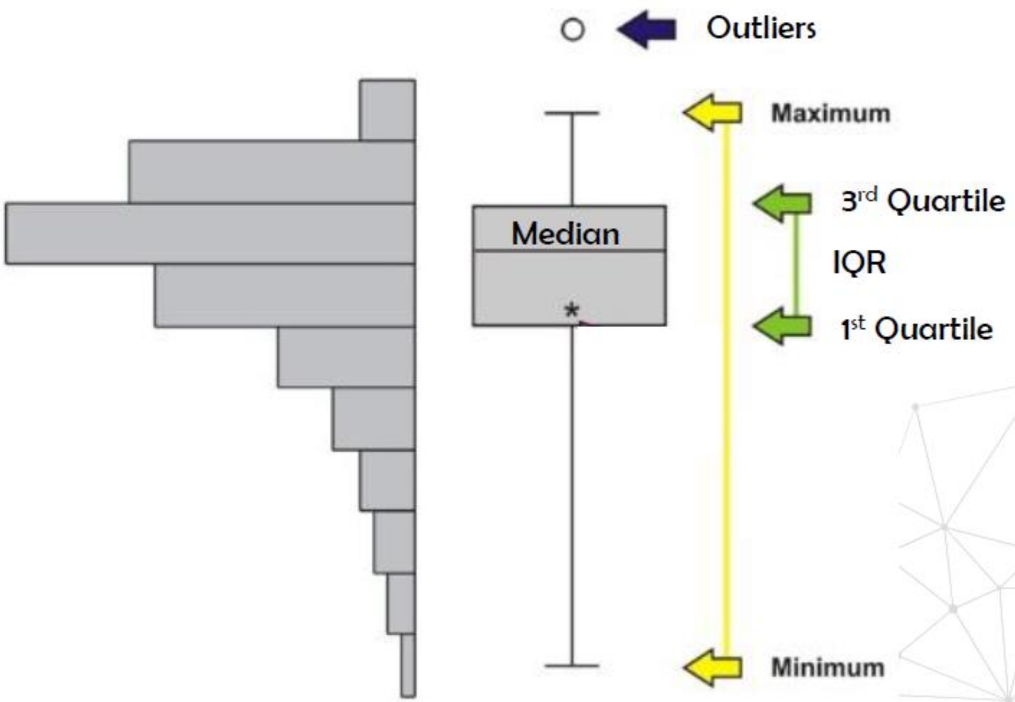
$$\begin{pmatrix} \sigma_1^2 & \sigma_{12} & \cdots & \sigma_{1d} \\ \sigma_{21} & \sigma_2^2 & \cdots & \sigma_{2d} \\ \cdots & \cdots & \cdots & \cdots \\ \sigma_{d1} & \sigma_{d2} & \cdots & \sigma_d^2 \end{pmatrix}$$

  - Correlation (Pearson's Correlation Coefficient) $\rho_{x,y} = \text{correlation}(x,y) = \frac{\text{covariance}(x,y)}{\sigma_x \sigma_y}$
    - a measure of the linear relations
    - Between −1 and +1
    - If >0 or < 0, positively/negatively correlated (x's values increase/decrease as y's).
    - The closer to +1 or -1, the stronger correlation.
    - If = 0: independent.
  - The correlation matrix is a d x d (square) symmetric matrix

$$\begin{pmatrix} \rho_1^2 & \rho_{12} & \cdots & \rho_{1d} \\ \rho_{21} & \rho_2^2 & \cdots & \rho_{2d} \\ \cdots & \cdots & \cdots & \cdots \\ \rho_{d1} & \rho_{d2} & \cdots & \rho_d^2 \end{pmatrix}$$

Visualizing outliers -Boxplots





Images taken from pro.arcgis.com and wiki.commons

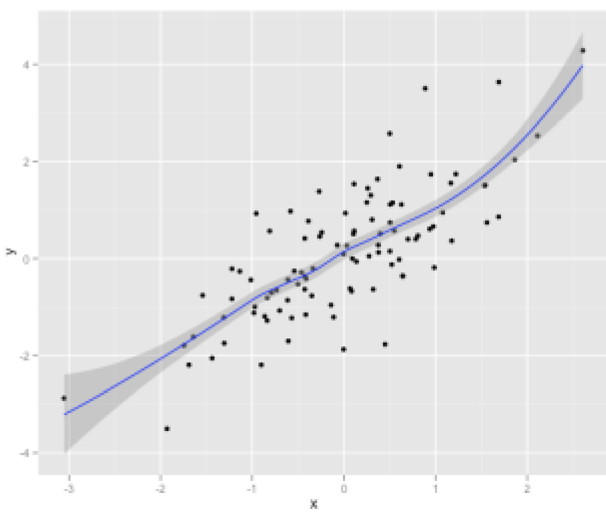
Data Exploration - Visualization

Summary statistics give us some sense of the data:

- Mean vs. Median.
- Standard deviation
- Quartiles, Min/Max
- Correlations between variables.

```
Summary(data)
       x                    y
 Min.    :-3.05439   Min.    :-3.50179
 1st Qu.:-0.61055   1st Qu.:-0.75968
 Median : 0.04666   Median : 0.07340
 Mean    :-0.01105   Mean    : 0.09383
 3rd Qu.: 0.56067   3rd Qu.: 0.88114
 Max.    : 2.60614   Max.    : 4.28693
```

Why Visualize?



Visualization gives us a more holistic sense

Data Visualization – Why?

Anscombe's Quartet

4 data sets, characterized by the following. Are they the same, or are they different?

Property	Values
Mean of x in each case	9
Exact variance of x in each case	11
Exact mean of y in each case	7.5
Variance of Y in each case	4.13
Correlations between x and y in each case	0.816
Linear regression line in each case	Y = 3.00 + 0.500x

i

X	y
10.00	8.04
8.00	6.95
13.00	7.58
9.00	8.81
11.00	8.33
14.00	9.96
6.00	7.24
4.00	4.26
12.00	10.84
7.00	4.82
5.00	5.68

ii

x	y
10.00	9.14
8.00	8.14
13.00	8.74
9.00	8.77
11.00	9.26
14.00	8.10
6.00	6.13
4.00	3.10
12.00	9.13
7.00	7.26
5.00	4.74

iii

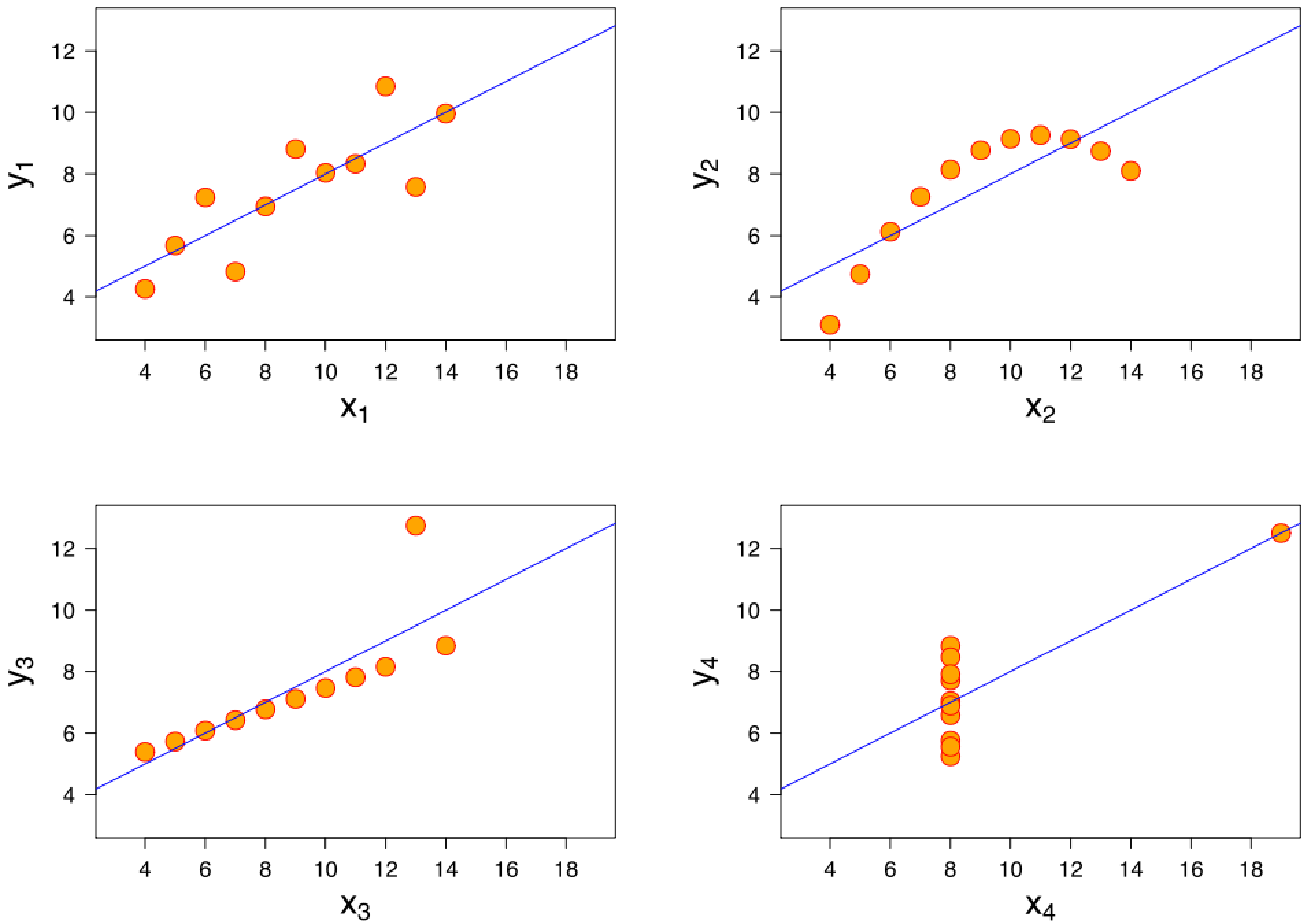
x	y
10.00	7.46
8.00	6.77
13.00	12.74
9.00	7.11
11.00	7.81
14.00	8.84
6.00	6.08
4.00	5.39
12.00	8.15
7.00	6.42
5.00	5.73

iv

x	y
8.00	6.58
8.00	5.76
8.00	7.71
8.00	8.84
8.00	8.47
8.00	7.04
8.00	5.25
19.00	12.50
8.00	5.56
8.00	7.91
8.00	6.89

Motto: Visualize Before Analyzing!

Visualization gives us a more holistic sense



Numeric Data Matrix

If all attributes are numeric, then the data matrix $\boldsymbol{D}$ is an $n \times d$ matrix, or equivalently a set of n row vectors $\boldsymbol{x}_i^T \in \mathbb{R}^d$ or a set of d column vectors $X_j \in \mathbb{R}^n$

$$\boldsymbol{D} = \begin{pmatrix} x_{11} & x_{12} & \cdots & x_{1d} \\ x_{21} & x_{22} & \cdots & x_{2d} \\ \vdots & \vdots & \ddots & \vdots \\ x_{n1} & x_{n2} & \cdots & x_{nd} \end{pmatrix} = \begin{pmatrix} - & \boldsymbol{x}_1^T & - \\ - & \boldsymbol{x}_2^T & - \\ & \vdots & \\ - & \boldsymbol{x}_n^T & - \end{pmatrix} = \begin{pmatrix} | & | & & | \\ X_1 & X_2 & \cdots & X_d \\ | & | & & | \end{pmatrix}$$

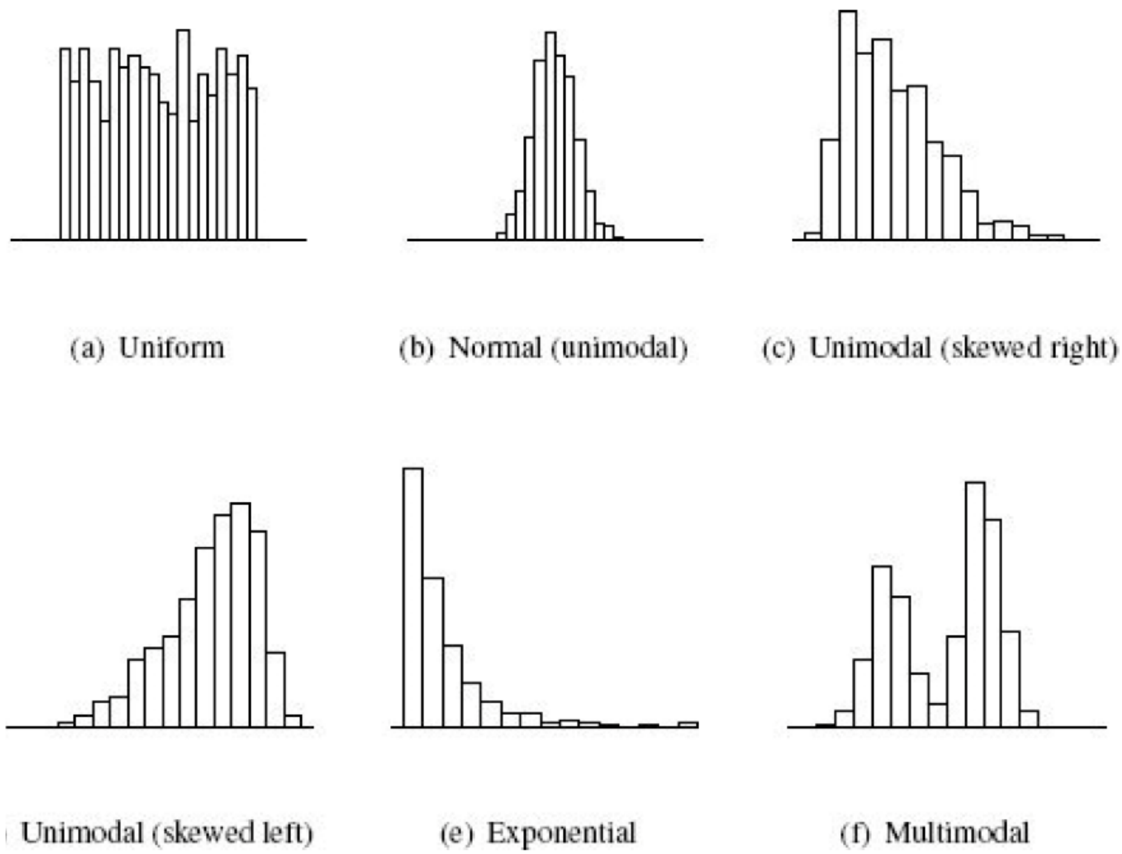
The *mean* of the data matrix $\boldsymbol{D}$ is the average of all the points: $mean(\boldsymbol{D}) = \boldsymbol{\mu} = \frac{1}{n}\sum_{i=1}^{n} \boldsymbol{x}_i$

The *centered data matrix* is obtained by subtracting the mean from all the points:

$$\boldsymbol{Z} = \boldsymbol{D} - \boldsymbol{1} \cdot \boldsymbol{\mu}^T = \begin{pmatrix} \boldsymbol{x}_1^T \\ \boldsymbol{x}_2^T \\ \vdots \\ \boldsymbol{x}_n^T \end{pmatrix} - \begin{pmatrix} \boldsymbol{\mu}^T \\ \boldsymbol{\mu}^T \\ \vdots \\ \boldsymbol{\mu}^T \end{pmatrix} = \begin{pmatrix} \boldsymbol{x}_1^T - \boldsymbol{\mu}^T \\ \boldsymbol{x}_2^T - \boldsymbol{\mu}^T \\ \vdots \\ \boldsymbol{x}_n^T - \boldsymbol{\mu}^T \end{pmatrix} = \begin{pmatrix} \boldsymbol{z}_1^T \\ \boldsymbol{z}_2^T \\ \vdots \\ \boldsymbol{z}_n^T \end{pmatrix} \tag{1}$$

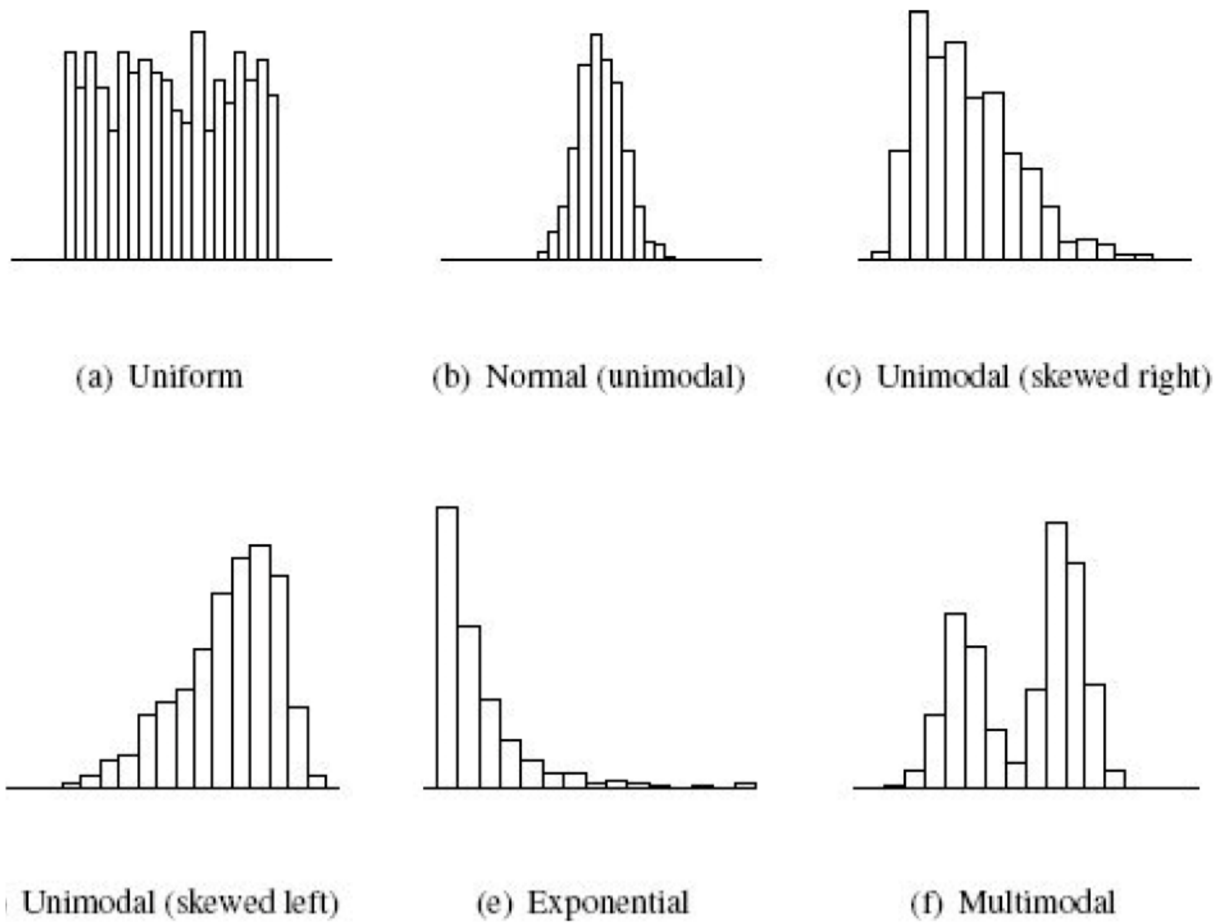
where $\boldsymbol{z}_i = \boldsymbol{x}_i - \boldsymbol{\mu}$ is a centered point, and $\boldsymbol{1} \in \mathbb{R}^n$ is the vector of ones.

Probablity Distributions



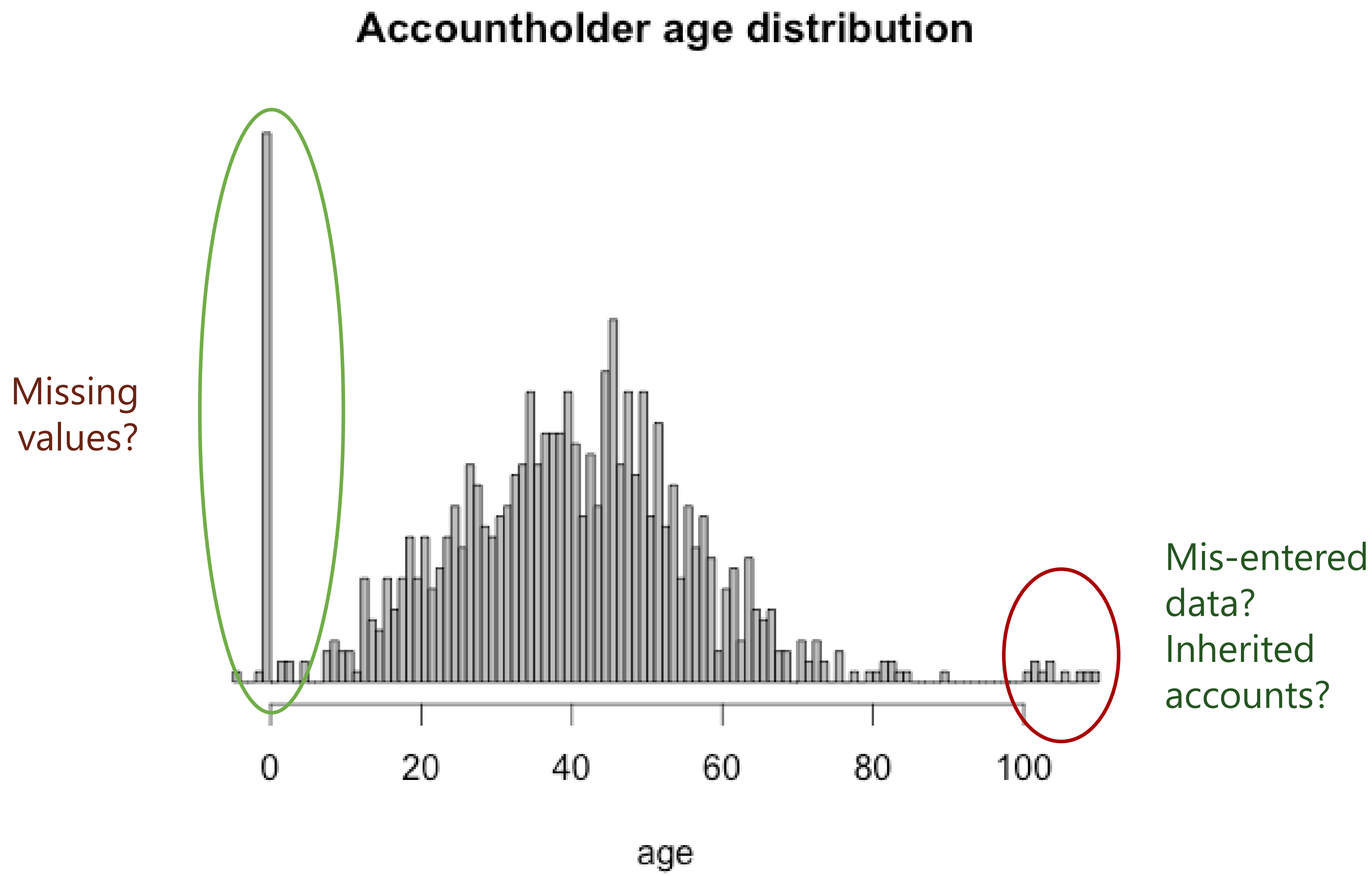
Histograms for six different sets of data, each of which exhibit well-known, common characteristics.

Probablity Distributions



Histograms for six different sets of data, each of which exhibit well-known, common characteristics.

Evidence of Dirty Data

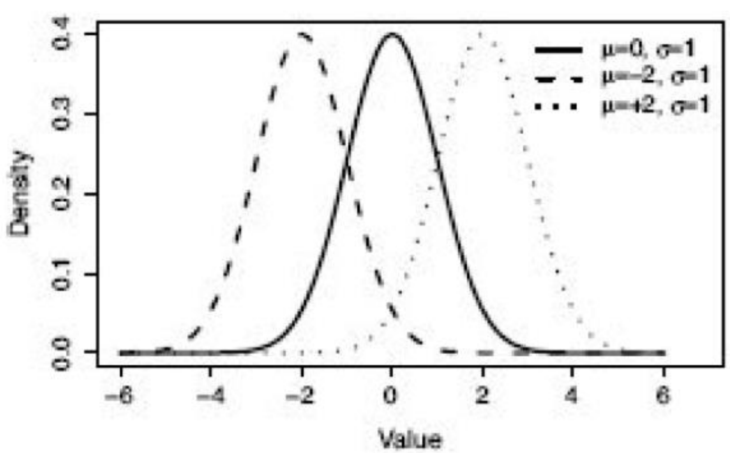


The Normal/Gaussian distribution

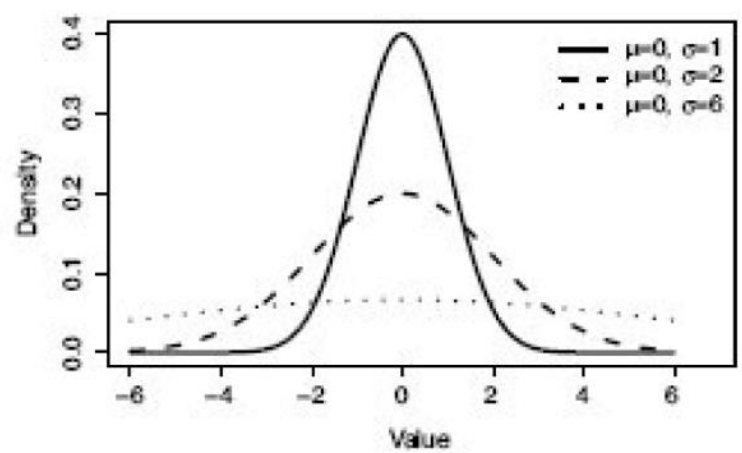
- Probability density functions, which define the characteristics of the distribution, the normal distribution is:

$$N(x, \mu, \sigma) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

where *x* is any value, and *μ* and *σ* are parameters that define the shape of the distribution.

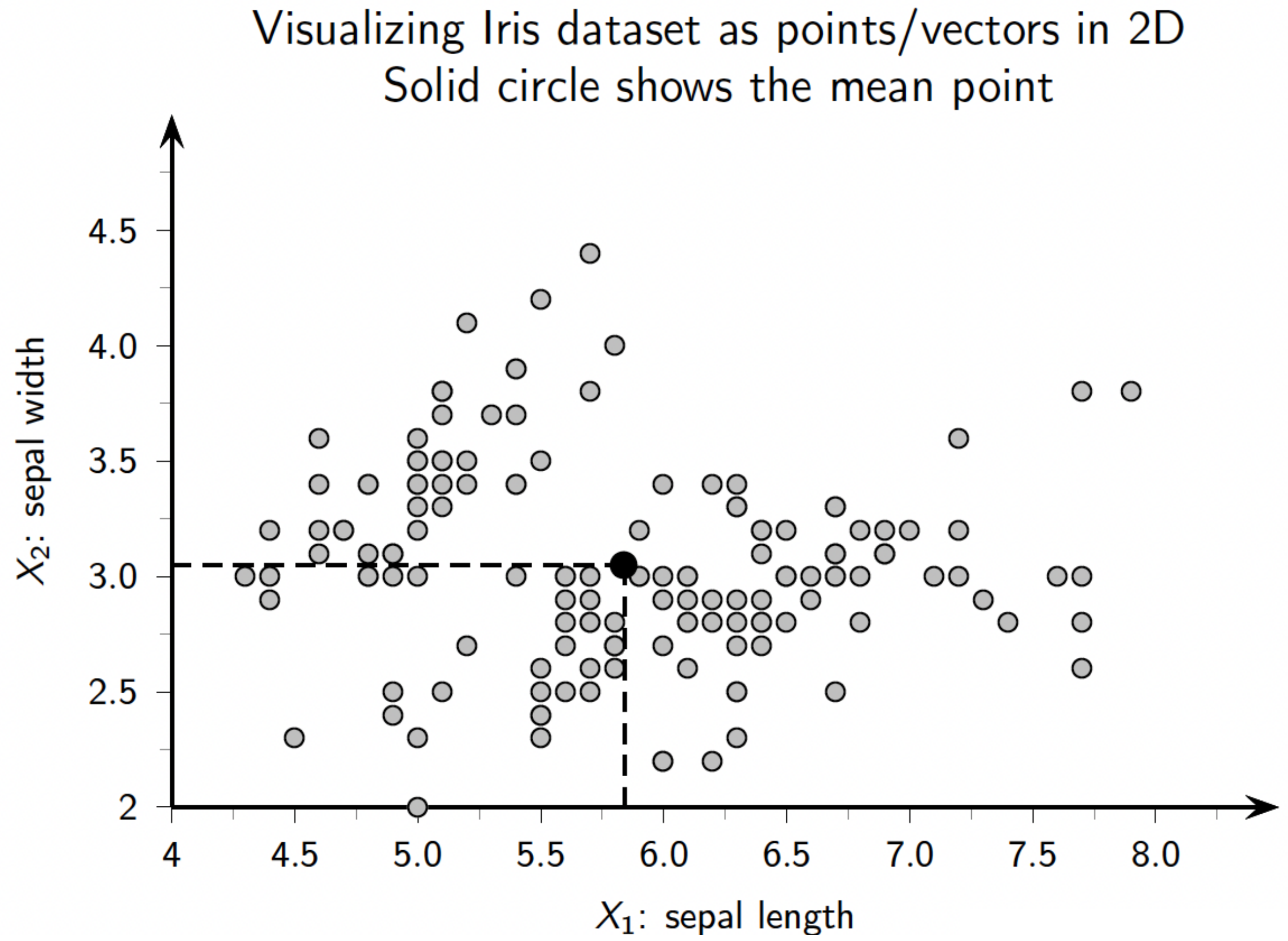


(a) Different means



(b) Different standard deviations

Scatterplot: 2D Iris Dataset
sepal length **versus** sepal width

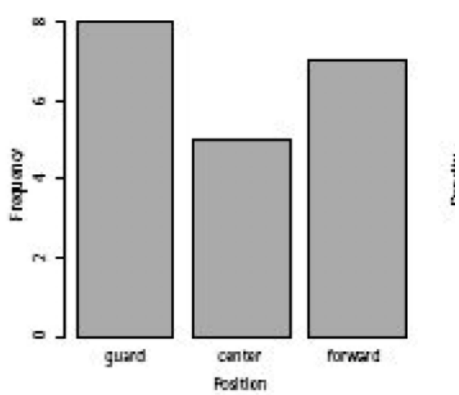


Data visualization for a single feature

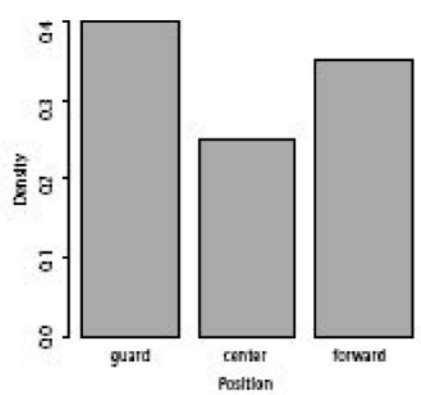
- Bar plot

A dataset showing the positions and monthly training expenses of a basketball team.

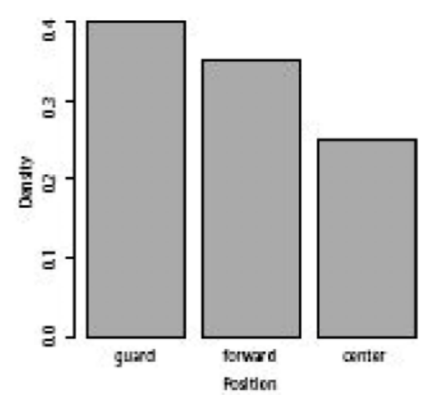
ID	Position	Training Expenses	ID	Position	Training Expenses
1	center	56.75	11	center	550.00
2	guard	1,800.11	12	center	223.89
3	guard	1,341.03	13	center	103.23
4	forward	749.50	14	forward	758.22
5	guard	1,150.00	15	forward	430.79
6	forward	928.30	16	forward	675.11
7	center	250.90	17	guard	1,657.20
8	guard	806.15	18	guard	1,405.18
9	guard	1,209.02	19	guard	760.51
10	forward	405.72	20	forward	985.41



Frequency bar plot for
for the POSITION feature



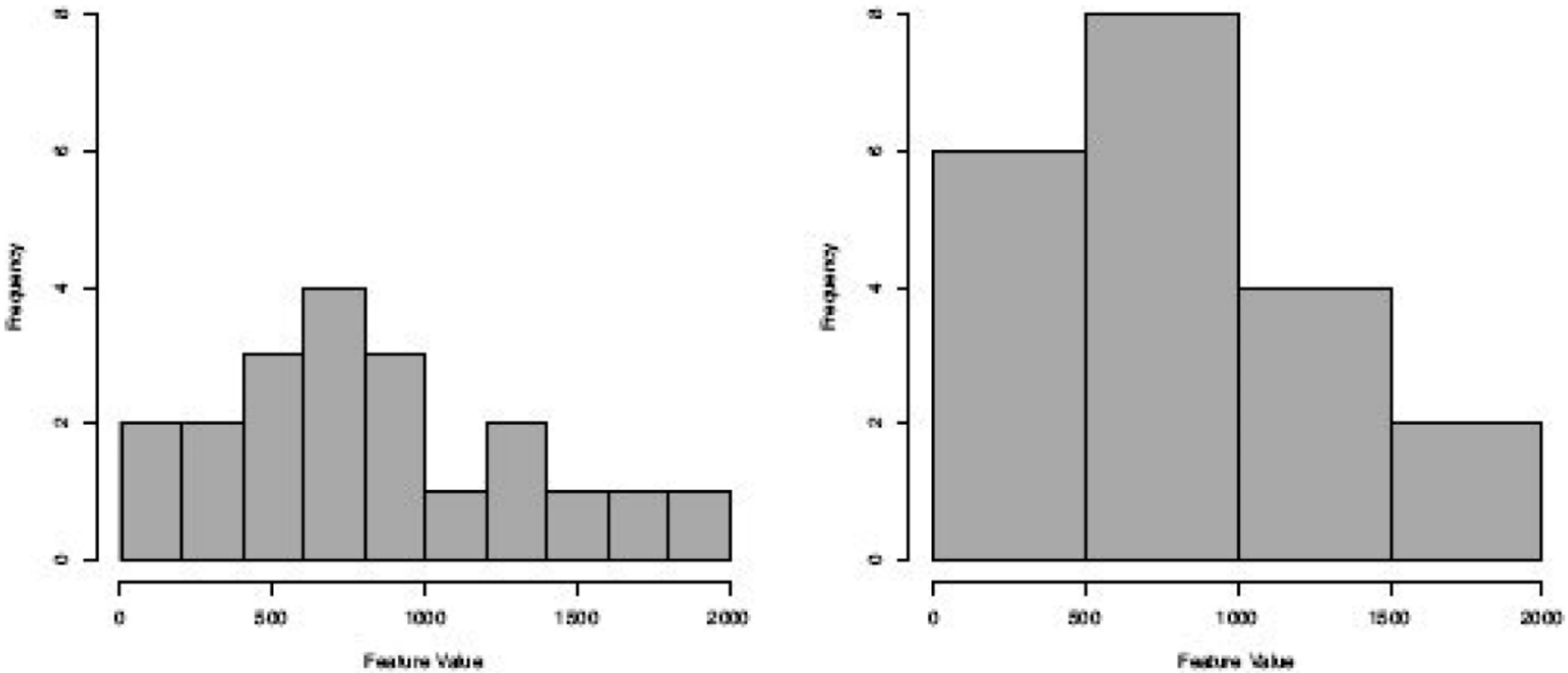
Density bar plot.
(Probability distribution)



Order density bar plot.

Data visualization for a single feature

- Histogram



Frequency histograms (200/500-unit intervals) for the continuous TRAINING EXPENSES feature