# COMP-4540/8540
# Design and Analysis of Algorithms

## Winter 2024

## Assignment 4

**Due Date:** April 8 (before 11:59p.m.) ***This assignment is optional***

### Important instructions

1. Any algorithm you present in this course (for assignments or tests) without a proof of correctness *or* time complexity analysis will automatically receive a **0 mark**.
2. The time complexity of your algorithm must *not* be worse than that stated in the question.
3. You can receive a 0 mark for *poor presentation* even if your algorithm is correct.
   So, you are advised to do the following before presenting your algorithm. Give a clear description of the ***key idea*** underlying your algorithm. You may give some examples to illustrate the key idea.
4. Never submit a program to support the correctness of your algorithm. *It will be ignored* as it cannot prove the correctness of your algorithm. Instead, it shows that you lack confidence in your correctness proof.

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Decision Problem $\Pi$:

Given a directed graph $\vec{G} = (\vec{V}, \vec{E})$ and an integer $k$, determine if $\exists F \subseteq \vec{E}$, such that $|F| \leq k$ and every directed cycle in $\vec{G}$ contains at least one edge of $F$.

($i$) Prove that $\Pi \in$ `NP`.

($ii$) Prove that $\Pi$ is `NP-complete` by proving that `VC` $\propto \Pi$, where `VC` is the Vertex Cover decision problem (see the courseware).

**Hint:**

(*i*) Removing $F$ from $\vec{G}$ results in an acyclic graph (a graph with no cycles).

(*ii*) A directed edge is represented by an ordered pair $(s, t)$, where $s$ is the tail and $t$ is the head.

An undirected edge is represented by an unordered pair $\{s, t\}$.

Let $(G = (V, E), k)$ be a problem instance of `VC`. Replace every vertex $w$ of $G$ with a directed edge $(w_s, w_t)$, and every edge $\{u, v\}$ of $G$ with a pair of directed edges $(u_1, v_1)$ and $(v_2, u_2)$, where $u_1, u_2 \in \{u_s, u_t\}$ and $v_1, v_2 \in \{v_s, v_t\}$.