

5)

a) 2.718

$$\begin{array}{r} 2^1 \ 2^0 \\ 2 \ 1 \\ 1 \ 0 \end{array}$$

$$2 = (10)_2$$

$$0.718 =$$

$$0.718 \times 2 = 1.436 \quad 1$$

$$0.436 \times 2 = 0.872 \quad 0$$

$$0.872 \times 2 = 1.744 \quad 1$$

$$0.744 \times 2 = 1.488 \quad 1$$

$$0.488 \times 2 = 0.976 \quad 0$$

$$0.976 \times 2 = 1.952 \quad 1$$

$$0.952 \times 2 = 1.904 \quad 1$$

$$0.904 \times 2 = 1.808 \quad 1$$

$$0.808 \times 2 = 1.616 \quad 1$$

$$0.616 \times 2 = 1.232 \quad 1$$

$$0.232 \times 2 = 0.464 \quad 0$$

$$0.464 \times 2 = 0.928 \quad 0$$

$$0.928 \times 2 = 1.856 \quad 1$$

$$0.856 \times 2 = 1.712 \quad 1$$

$$\vdots$$

$$2.718 = (10.101101111\dots)$$

$$= 1.0101101111 \times 2^1$$

$$= 1.0110 \times 2^1$$

$$\text{sign} = 0$$

$$\text{bias} = 3$$

$$\text{exponent} = 3 + 1 = 4$$

$$\text{mantissa} = 0110$$

0100 0110 \Rightarrow 8-bit representation (format)

b) (10100111) to decimal

sign | 010 | 0111
 exponent | mantissa

$$\text{exponent} = 2 - 3 = -1$$

(010)₂ - bias

$$\text{mantissa} = 1.0111$$

$$1.0111 \times 2^{-1} = 0.10111 = 2^{-1} + 2^{-3} + 2^{-4} + 2^{-5}$$

$$= 0.71875$$

c)

$$\epsilon_{\text{mach}} = 2^{-b} \rightarrow 2^{-4} = \frac{1}{16}$$

$$\text{upper bound of the relative error} = \frac{1}{2} \epsilon_{\text{mach}}$$

$$= \frac{1}{2} \times \frac{1}{16}$$

$$= \frac{1}{32}$$

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