CS3113 A3

1)a) $x_1 + 2x_2 - x_3 = 5$
$2x_1 - x_2 + 3x_3 = 0$
$x_1 + 3x_2 - 2x_3 = 7$

$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & 5 \\ 2 & -1 & 3 & 0 \\ 1 & 3 & -2 & 7 \end{array}\right] \quad R_2 = R_2 - 2R_1$$

$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & 5 \\ 0 & -5 & 5 & -10 \\ 1 & 3 & -2 & 7 \end{array}\right] \quad R_3 = R_3 - R_1$$

$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & 5 \\ 0 & -5 & 5 & -10 \\ 0 & 1 & -1 & 2 \end{array}\right] \quad R_3 = 5R_3 + R_2$$

$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & 5 \\ 0 & -5 & 5 & -10 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

This system of equations has infinitely many solutions, according to R_3.

$$0x_1 + 0x_2 + 0x_3 = 0$$

1) b) $\left[\begin{array}{ccc|c} 5 & -4 & 2 & 11 \\ -2 & 3 & 1 & 1 \\ 16 & -7 & 4 & 19 \end{array}\right]$ $R_2 = R_2 + \frac{2}{5} R_1$

$\left[\begin{array}{ccc|c} 5 & -4 & 2 & 11 \\ 0 & 7/5 & 9/5 & 27/5 \\ 16 & -7 & 4 & 19 \end{array}\right]$ $R_3 = R_3 - \frac{16}{5} R_1$

$\left[\begin{array}{ccc|c} 5 & -4 & 2 & 11 \\ 0 & 7/5 & 9/5 & 27/5 \\ 0 & 29/5 & -12/5 & -16.2 \end{array}\right]$ $R_3 = R_3 - \left(\frac{29}{5} \div \frac{7}{5} R_2\right)$

$\left[\begin{array}{ccc|c} 5 & -4 & 2 & 11 \\ 0 & 7/5 & 9/5 & 27/5 \\ 0 & 0 & -69/7 & -270/7 \end{array}\right]$

according to R_3, $0x_1 + 0x_2 - \frac{69}{7} x_3 = \frac{-270}{7}$

$x_3 = 3.91$

$R_2 \rightarrow \frac{7}{5} x_2 + \frac{9}{5} (3.91) = 27/5$

$x_2 = -1.17$

$R_1 \rightarrow 5x_1 - 4(-1.17) + 2(3.91) = 11$

$x_1 = -0.3$

$$\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -0.3 \\ -1.17 \\ 3.91 \end{bmatrix}$$