

$$4) \quad f(x) = 5x^3 - 39x^2 + 22x - 60 \quad [7, 8] \quad x_0 = 7$$

$$f'(x) = 15x^2 - 78x + 22$$

$$x_1 = 7 - \frac{5(7)^3 - 39(7)^2 + 22(7) - 60}{15(7)^2 - 78(7) + 22} = 7.48341$$

$$x_2 = 7.48341 - \frac{5(7.48341)^3 - 39(7.48341)^2 + 22(7.48341) - 60}{15(7.48341)^2 - 78(7.48341) + 22}$$

$$x_3 = 7.42596 - \frac{5(7.42596)^3 - 39(7.42596)^2 + 22(7.42596) - 60}{15(7.42596)^2 - 78(7.42596) + 22}$$

$$= 7.42507$$

$$x_4 = 7.42507 - \frac{5(7.42507)^3 - 39(7.42507)^2 + 22(7.42507) - 60}{15(7.42507)^2 - 78(7.42507) + 22}$$

$$= 7.42507$$

Converged

[5, 10]

$x_0 = 5$

$$x_1 = 5 - \frac{5(5)^3 - 39(5)^2 + 22(5) - 60}{15(5)^2 - 78(5) + 22} = -47.8571$$

out of interval

Newton \rightarrow bisection

$$c = \frac{5+10}{2} = 7.5$$

$$F(5) = 5(5)^3 - 39(5)^2 + 22(5) - 60 = -300$$

$$F(5) < 0$$

$$F(10) = 5(10)^3 - 39(10)^2 + 22(10) - 60 = 1260$$

$$F(7.5) = 5(7.5)^3 - 39(7.5)^2 + 22(7.5) - 60 = 20.625$$

$$F(7.5) > 0$$

bracket $[5, 7.5]$

if we use $x_0 = 5$ for Newton's method again the result will be 47.85714

Therefore, bisection

$$c = 6.25$$

$$F(6.25) = 5(6.25)^3 - 39(6.25)^2 + 22(6.25) - 60 = -225.23$$

bracket: $[6.25, 7.5]$

Newton's method

$$x_1 = 6.25 - \frac{5(6.25)^3 - 39(6.25)^2 + 22(6.25) - 60}{15(6.25)^2 - 78(6.25) + 22} = 8.12013$$

8.12013 > 7.5 out of interval
Bisection

$$\frac{6.25 + 7.5}{2} = 6.875$$

$$F(7.5) = 5(7.5)^3 - 39(7.5)^2 + 22(7.5) - 60 = 20.625$$

$$F(6.875) = 5(6.875)^3 - 39(6.875)^2 + 22(6.875) - 60 = -127.35351$$

$$[6.875, 7.5]$$

Newton's

$$x_1 = 6.875 - \frac{5(6.875)^3 - 39(6.875)^2 + 22(6.875) - 60}{15(6.875)^2 - 78(6.875) + 22}$$

$$= 7.52898 \Rightarrow \text{out of interval}$$

bisection

$$c = 7.1875$$

$$f(7.1875) = 5(7.1875)^3 - 39(7.1875)^2 + 22(7.1875) - 60$$

$$= -60.08422 < 0 \quad -ve$$

$$[7.1875, 7.5]$$

Newton's

$$x_1 = 7.1875 - \frac{f(7.1875)}{f'(7.1875)} = 7.44179$$

$$x_2 = 7.44179 - \frac{f(7.44179)}{f'(7.44179)} = 7.42514$$

$$x_3 = 7.42514 - \frac{f(7.42514)}{f'(7.42514)} = 7.42507$$

$$x_4 = 7.42507 - \frac{f(7.42507)}{f'(7.42507)} = 7.42507$$

converged