

$$3)a) \begin{bmatrix} 4 & 2 & 0 \\ 4 & 4 & 2 \\ 2 & 2 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix}$$

$$\begin{aligned} R_2 &= R_2 - R_1 \\ R_3 &= R_3 - \frac{1}{2}R_1 \end{aligned} \begin{bmatrix} 4 & 2 & 0 \\ 0 & 2 & 2 \\ 0 & 1 & 3 \end{bmatrix}$$

$$P = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_3 = R_3 - \frac{1}{2}R_2 \begin{bmatrix} 4 & 2 & 0 \\ 0 & 2 & 2 \\ 0 & 0 & 2 \end{bmatrix}$$

$$PA = LU$$

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ \frac{1}{2} & \frac{1}{2} & 1 \end{bmatrix}; \quad U = \begin{bmatrix} 4 & 2 & 0 \\ 0 & 2 & 2 \\ 0 & 0 & 2 \end{bmatrix}$$

$$Lc = Pb$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ \frac{1}{2} & \frac{1}{2} & 1 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix}$$

$$c_1 = 2; \quad 1(2) + 1(c_2) = 4; \quad 0.5(2) + 0.5(2) + c_3 = 6$$

$$c_2 = 2; \quad c_3 = 4$$

$$Ux = c$$

$$c = \begin{bmatrix} 2 \\ 2 \\ 4 \end{bmatrix}$$

$$\begin{bmatrix} 4 & 2 & 0 \\ 0 & 2 & 2 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ 4 \end{bmatrix} \quad \begin{aligned} x_1 &= 1/2; \quad x_2 = -1 \\ x_3 &= 2 \end{aligned} \quad x = \begin{bmatrix} 1/2 \\ -1 \\ 2 \end{bmatrix}$$



$$3)b) \begin{bmatrix} -1 & 0 & 1 \\ 2 & 1 & 1 \\ -1 & 2 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -2 \\ 17 \\ 3 \end{bmatrix} \quad P = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 1 & 1 \\ -1 & 0 & 1 \\ -1 & 2 & 0 \end{bmatrix} \quad R_2 = R_2 + \frac{1}{2}R_1 \quad P = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 1 & 1 \\ 0 & 0.5 & 1.5 \\ 0 & 2.5 & 0.5 \end{bmatrix} \quad R_3 = R_3 + \frac{1}{2}R_1$$

$$\begin{bmatrix} 2 & 1 & 1 \\ 0 & 2.5 & 0.5 \\ 0 & 0.5 & 1.5 \end{bmatrix} \quad R_2 \leftrightarrow R_3 \quad P = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 1 & 1 \\ 0 & 2.5 & 0.5 \\ 0 & 0 & 1.4 \end{bmatrix}$$

$$PA = LU$$

$$L = \begin{bmatrix} 1 & 0 & 0 \\ -1/2 & 1 & 0 \\ -1/2 & 0.2 & 1 \end{bmatrix} \quad U = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 2.5 & 0.5 \\ 0 & 0 & 1.4 \end{bmatrix}$$

$$Lc = Pb$$

$$\begin{bmatrix} 1 & 0 & 0 \\ -1/2 & 1 & 0 \\ -1/2 & 0.2 & 1 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} -2 \\ 17 \\ 3 \end{bmatrix}$$

$$c_1 = 17; \quad -1/2(17) + c_2 = 3$$

$$c_2 = 11.5$$