University of New Brunswick
Faculty of Computer Science
CS2333: Computability and Formal Languages
Homework Assignment 6, ***Due Time, Date*** *5:00 PM, March 14, 2022*

Student Name: _____ Matriculation Number: _____

Instructor: Rongxing Lu
The marking scheme is shown in the left margin and [100] constitutes full marks.

[**75**] 1. For each of the following languages, draw the diagram for a nondeterministic PDA that accepts exactly those strings that are in the language.

[25] (a) $L_1 = \{a^i b^j c^k \mid i, j, k \geq 0, \text{ and } i = j \text{ or } j = k\}$.

[25] (b) $L_2 = \{a^i b^j c^k \mid i, j, k \geq 0, \text{ and } i + j = k\}$.

[25] (c) $L_3 = \{a^{2n} b^{3n} \mid n \geq 0\}$.

[**25**] 2. Show that the language

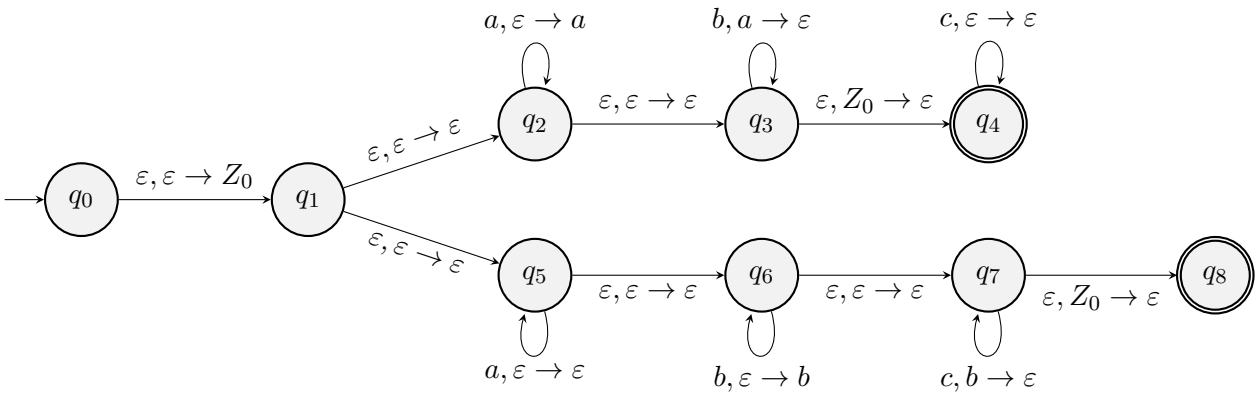
$$L = \{a^n b^j : n = j^2\}$$

is not context free.

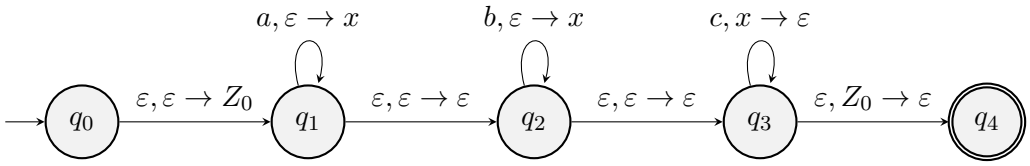
Solutions.

1. For each of the following languages, draw the diagram for a nondeterministic PDA that accepts exactly those strings that are in the language.

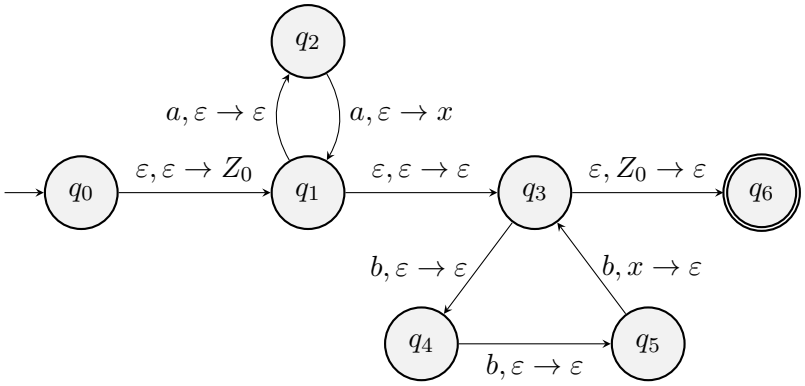
   (a) $L_1 = \{a^i b^j c^k \mid i, j, k \geq 0, \text{ and } i = j \text{ or } j = k\}$.



   (b) $L_2 = \{a^i b^j c^k \mid i, j, k \geq 0, \text{ and } i + j = k\}$.



   (c) $L_3 = \{a^{2n} b^{3n} \mid n \geq 0\}$.



2. Show that the language

$$L = \{a^n b^j : n = j^2\}$$

   is not context free.

Proof by Contradiction. (We only need to find one counter example.)

Assume that L is context free.

Let p be the pumping length given by the pumping lemma. Choose s to be the string $a^{p^2}b^p$. Because s is a member of L and s has length more than p, the pumping lemma guarantees that s can be split into five pieces, $s = uvxyz$, where for any $i \geq 0$ the string uv^ixy^iz is in L.

For clarity, consider $p = 3$ as an example, then

$$s = a^{p^2}b^p = aaaaaaaaabbb$$

According to the pumping lemma, three conditions should be satisfied, i) $uv^ixy^iz \in L$ for every $i \geq 0$; ii) $|vy| > 0$; and iii) $|vxy| \leq p$. Therefore, based on ii) and iii), we first consider cases of y in the splitting of $s = uvxyz$. Clear, we will have several cases for $|vy| = 2$ and $|vxy| = p = 3$. We just take one case for a discussion.

$$s = a^{p^2}b^p = \underbrace{aaaaaaa}_{u}\underbrace{a}_{v}\underbrace{a}_{x}\underbrace{b}_{y}\underbrace{bb}_{z}$$

Consider uv^ixy^iz as uv^2xy^2z when $i = 2$, we have

$$uv^2xy^2z = \underbrace{aaaaaaa}_{u}\underbrace{aa}_{v^2}\underbrace{a}_{x}\underbrace{bb}_{y^2}\underbrace{bb}_{z} \notin L$$

Clearly, uv^2xy^2z is not in the form of $a^{p^2}b^p$, and thus the condition i) is not satisfied.

(For other cases, i.e., vxy are all in a-part $s = a^{p^2}b^p = \underbrace{aaaaa}_{u}\underbrace{a}_{v}\underbrace{a}_{x}\underbrace{a}_{y}\underbrace{abbb}_{z}$, or all in b-part $s = a^{p^2}b^p = \underbrace{aaaaaaaaa}_{u}\underbrace{b}_{v}\underbrace{b}_{x}\underbrace{b}_{y}\underbrace{\varepsilon}_{z}$, we can easily see uv^2xy^2z is also not in the form of $a^{p^2}b^p$.)

Therefore, L is not context free.