# CS-2333 A1

1)a) $\{x \in Z \mid x < 5\}$

b) $Z^+ = \{1, 2, 3, 4, 5, \ldots\}$

$x \in Z$ $\quad x \bmod 5 = 3$

$\Rightarrow x = K \cdot 5 + 3$

$\Rightarrow K \cdot 5 = x - 3$

$\Rightarrow 5 \mid x - 3$

$\{x \in Z^+ \mid 5 \mid x-3\}$

c) substrings of baab

substring length 0: $\varepsilon$

" " 1: b, a

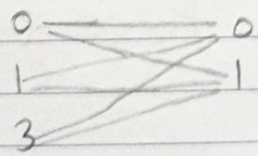
" " 2: ba, aa, ab

" " 3: baa, aab

" " 4: baab

$\{\varepsilon$, b, a, ba, aa, ab, baa, aab, baab$\}$

D) Cross product of $\{0, 1, 3\}$ and $\{0, 1\}$




$\{[0,0], [0,1]$
$[1,0], [1,1]$
$[3,0], [3,1]\}$

3)a) $R_1 = \{(x,y) : x, y \in \mathbb{Z}^{nonneg}$ and $3 \mid (x-y)\}$

$\mathbb{Z}^{nonneg} = \{0, 1, 2, 3, \ldots\}$

$3 \mid (x-y) \iff \exists k \in \mathbb{Z} \quad x - y = 3k \quad x = 3k + y \Rightarrow x \equiv y \mod 3$

$0 \leq y < 3 \Rightarrow x \equiv 0 \mod 3$

$y = 0 \qquad x \equiv 1 \mod 3$

$y = 1$

$y = 2 \qquad x \equiv 2 \mod 3$

Equivalence classes representative elements:

$[0] \leftarrow$

$[1] \leftarrow$

$[2] \leftarrow$

b) $(x, y) \mid x, y \in \{0, 1\}^{2022}$

$1^{st}$ symbol of $x$ similar to the first symbol of $y$

There will be $2^{2022}$ elements. these elements can be split in half. One half will start with 0. The other half will start with 1.

$[00000 \ldots\ldots] \quad [100000\ldots]$

b) When confirming reflexivity, prove $S1$ is related to $S1$. and $|S1| = |S1|$. ($S1$ has the same number of elements as itself. Same with $S2$.
To confirm symmetry, if $|S1| = |S2|$, then $|S2| = |S1|$
To confirm transitivity, if $|S1| = |S2|$ and $|S2| = |S3|$, then $|S1| = |S3|$. Because $S2$ has the same number of elements as $S1$ and $S3$.

5) $2 - \sqrt{2}$ irrational

given: $\sqrt{2}$ not rational

proof by contradiction

$Q(x) = x$ is rational

$p := \neg Q(2 - \sqrt{2})$

$\neg p = \neg\neg Q(2 - \sqrt{2})$

$\equiv Q(2 - \sqrt{2})$

Assuming $\neg p$ is true

$$2 - \sqrt{2} = \frac{x}{y}$$

$x, y \in \mathbb{Z}$

$y \neq 0$

$\gcd(x, y) = 1$

$$-\sqrt{2} = -2 + \frac{x}{y}$$

$$-\sqrt{2} = \frac{-2y + x}{y}$$

$$\sqrt{2} = -\frac{-2y + x}{y} = -\frac{A}{B}$$

$A, B \in \mathbb{Z}$

$B \neq 0$

$$\Rightarrow Q(\sqrt{2}) \wedge \neg Q(\sqrt{2}) = \text{"}c\text{"}$$

contradiction

$\therefore \neg p$ is false, $p$ is true

4) $L_1 = \{a\}^* = \{\varepsilon, a, aa, aaa, \ldots\}$

$L_2 = \{\varepsilon\}$

$L_3 = \{a\}$

$L_2 \cap L_3 = \emptyset$ , so $L_1(L_2 \cap L_3) = \emptyset$

With the given numbers

$L_1 L_2 = \{a\}^*$

$L_1 L_3 = \{a\}^+$

$L_1 L_2 \cap L_1 L_3 = \{a\}^+$ infinite

2) a) reflexive symmetric not transitive

Let $A = \{2, 4, 6\}$

R:

$\{(2,4), (4,6), (4,2), (6,4), (6,6), (4,4), (2,2)\}$

- Reflexive because we have $(2,2), (4,4), (6,6)$
- Not transitive because we don't have $(2,6)$ even though we have $(2,4), (4,6)$
- Symmetric because we have $(2,4), (4,2), (4,6), (6,4)$

b) Transitive symmetric not reflexive

Let $A = \{-2, -1\}$

R: $\{(-2,-1), (-1,-2), (-2,-2)\}$

- Symmetric because we have $(-2,-1), (-1,-2)$
- Not reflexive because we don't have $(-1,-1)$
- Transitive because we have $(-2,-1), (-1,-2) \in R$ and $(-2,-2) \in R$