4)a) $x_1 + 3x_2 = -1$

$5x_1 + 4x_2 = 6$

$$\begin{bmatrix} 1 & 3 \\ 5 & 4 \end{bmatrix} \longrightarrow \begin{bmatrix} 5 & 4 \\ 1 & 3 \end{bmatrix}$$

Jacobi

$$x_1^{(k+1)} = 0.2(6 - 4x_2^{(k)}) \qquad \vec{x}^{(0)} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$x_2^{(k+1)} = 1/3(-1 - x_1^{(k)})$$

$$x_1^{(1)} = 0.2(6 - 4(0)) = 1.2$$

$$x_2^{(1)} = 1/3(-1 - (0)) = -1/3$$

$$x_1^{(2)} = 0.2(6 - 4(-1/3)) = 22/15$$

$$x_2^{(2)} = 1/3(-1 - (1.2)) = -11/15$$

Guass - Seidell

$$x_1^{(k+1)} = 0.2(6 - 4x_2^{(k)})$$

$$x_2^{(k+1)} = 1/3(-1 - x_1^{(k+1)})$$

$$x_1^{(1)} = 0.2(6 - 4(0)) = 1.2$$

$$x_2^{(1)} = 1/3(-1 - (1.2)) = -11/15$$

$$x_1^{(2)} = 0.2(6 - 4(-11/15)) = 134/75$$

$x_2^{(2)} = 1/3(-1-(134/75)) = -209/225$

b) $x_1 - 8x_2 - 2x_3 = 1$
$x_1 + x_2 + 5x_3 = 4$
$3x_1 - x_2 + x_3 = -2$

$$\begin{bmatrix} 1 & -8 & -2 \\ 1 & 1 & 5 \\ 3 & -1 & 1 \end{bmatrix} \longrightarrow \begin{bmatrix} 3 & -1 & 1 \\ 1 & -8 & -2 \\ 1 & 1 & 5 \end{bmatrix}$$

Jacobi

$x_1^{(k+1)} = 1/3(-2 + x_2^{(k)} - x_3^{(k)})$

$x_2^{(k+1)} = -1/8(1 - x_1^{(k)} + 2x_3^{(k)})$

$x_3^{(k+1)} = 1/5(4 - x_1^{(k)} - x_2^{(k)})$

$x_1^1 = 1/3(-2+(0)-(0)) = -2/3$

$x_2^1 = -1/8(1-(0)+2(0)) = -1/8$

$x_3^1 = 1/5(4-(0)-(0)) = 4/5$

$x_1^2 = 1/3(-2+(-1/8)-(4/5)) = -39/40$

$x_2^2 = -1/8(1-(-2/3)+2(4/5)) = -49/120$

$x_3^2 = 1/5(4-(-2/3)-(-1/8)) = 23/24$

Gauss - Seidel

$$x_1^{(k+1)} = 1/3(-2 + x_2^{(k)} - x_3^{(k)})$$

$$x_2^{(k+1)} = -1/8(1 - x_1^{(k+1)} + 2x_3^{(k)})$$

$$x_3^{(k+1)} = 1/5(4 - x_1^{(k+1)} - x_2^{(k+1)})$$

$x_1^1 = 1/3(-2 + (0) - (0)) = -2/3$

$x_2^1 = -1/8(1 - (-2/3) + 2(0)) = -5/24$

$x_3^1 = 1/5(4 - (-2/3) - (-5/24)) = 39/40$

$x_1^2 = 1/3(-2 + (-5/24) - (39/40)) = -191/180$

$x_2^2 = -1/8(1 - (-191/180) + 2(39/40)) = -361/720$

$x_3^3 = 1/5(4 - (-191/180) - (-361/720)) = 89/80$

4) c)

$$x_1 + 4x_2 = 5$$
$$x_2 + 2x_3 = 2$$
$$4x_1 + 3x_3 = 0$$

$$\begin{bmatrix} 1 & 4 & 0 \\ 0 & 1 & 2 \\ 4 & 0 & 3 \end{bmatrix} \longrightarrow \begin{bmatrix} 4 & 0 & 3 \\ 1 & 4 & 0 \\ 0 & 1 & 2 \end{bmatrix}$$

Jacobi

$$x_1^{(k+1)} = 1/4\,(0 - 3x_3^{(k)})$$

$$x_2^{(k+1)} = 1/4\,(5 - x_1^{(k)})$$

$$x_3^{(k+1)} = 1/2\,(2 - x_2^{(k)})$$

$$\vec{x}^{(0)} = [0 \quad 0 \quad 0]$$

$$x_1^1 = 1/4\,(0 - 3\,(0)) = 0$$

$$x_2^1 = 1/4\,(5 - (0)) = 5/4$$

$$x_3^1 = 1/2\,(2 - (0)) = = 1$$

$$x_1^2 = 1/4\,(0 - 3(1) = -3/4$$

$$x_2^2 = 1/4\,(5 - (0)) = 5/4$$

$$x_3^2 = 1/2\,(2 - (5/4)) = 3/8$$

Guass - Seidell

$$x_1^{(k+1)} = 1/4(0 - 3x_3^{(k)})$$

$$x_2^{(k+1)} = 1/4(5 - x_1^{(k+1)})$$

$$x_3^{(k+1)} = 1/2(2 - x_2^{(k+1)})$$

$$x_1^1 = 1/4(0 - 3(0)) = 0$$

$$x_2^1 = 1/4(5 - (0)) = 5/4$$

$$x_3^1 = 1/2(2 - (5/4)) = 3/8$$

$$x_1^2 = 1/4(0 - 3(3/8)) = -9/32$$

$$x_2^2 = 1/4(5 - (-9/32)) = 169/128$$

$$x_3^2 = 1/2(2 - (169/128)) = 87/256$$