5) a) $f(x) = x^3 + 4x^2 - 10 \qquad x_0 = 0 \qquad x_1 = 1$

$$x_{i+1} = x_i - \frac{f(x_i)(x_i - x_{i-1})}{f(x_i) - f(x_{i-1})}$$

$$x_2 = 1 - \frac{\left((1)^3 + 4(1)^2 - 10\right)(1-0)}{\left((1)^3 + 4(1)^2 - 10\right) - \left((0)^3 + 4(0)^2 - 10\right)} = 2$$

$$x_3 = 2 - \frac{\left((2)^3 + 4(2)^2 - 10\right)(2-1)}{\left((2)^3 + 4(2)^2 - 10\right) - \left((1)^3 + 4(1)^2 - 10\right)} = 1.26315$$

$$x_4 = 1.26315 - \frac{\left((1.26315)^3 + 4(1.26315)^2 - 10\right)(1.26315 - 2)}{\left((1.26315)^3 + 4(1.26315)^2 - 10\right) - \left((2)^3 + 4(2)^2 - 10\right)}$$

$$= 1.33882$$

$$x_5 = 1.33882 - \frac{((1.33882)^3 + 4(1.33882)^2 - 10)(1.33882 - 1.26315)}{((1.33882)^3 + 4(1.33882)^2 - 10) - ((1.26315)^3 + 4(1.26315)^2 - 10)}$$

$$= 1.36661$$

$$x_6 = 1.36661 - \frac{f(1.36661)(1.36661 - 1.33882)}{f(1.36661) - f(1.33882)}$$

$$= 1.36521$$

$$x_7 = 1.36521 - \frac{f(1.36521)(1.36521 - 1.36661)}{f(1.36521) - f(1.36661)}$$

$$= 1.36522$$

$$x_8 = 1.36522 - \frac{f(1.36522)(1.36522 - 1.36521)}{f(1.36522) - f(1.36521)}$$

$$= 1.36523$$

$$x_9 = 1.36523 - \frac{f(1.36523)(1.36523 - 1.36522)}{f(1.36523) - f(1.36522)}$$

$$= 1.36523$$

For this question, Newton's method was faster with 3 iteration vs 8 iteration for the secant method

tolerance: x_0 & x_1: $|1-0| < \epsilon$ X

x_2 & x_1: $|2-1| < \epsilon$ X

x_3 & x_2: $|1.26315 - 2| < \epsilon$ X

x_4 & x_3: $|1.33882 - 1.26315| < \epsilon$ X

x_5 & x_4: $|1.36661 - 1.33882| < \epsilon$ X

x_6 & x_5: $|1.36521 - 1.36661| < \epsilon$ X

x_7 & x_6: $|1.36522 - 1.36521| < \epsilon$ X
x_8 & x_7: $|1.36523 - 1.36522| < \epsilon$ X
x_9 & x_8: $|1.36523 - 1.36523| < \epsilon$ ✓

b) $x_0 = 0 \quad x_1 = 3$

$$x_2 = 3 - \frac{(f(3))(3-0)}{(f(3)) - f(0)} = 0.47619$$

tolerance: x_0 & x_1: $|3-0| < \epsilon$ X
x_2 & x_1: $|0.47619 - 3| < \epsilon$ X

$$x_3 = 0.47619 - \frac{f(0.47619)(0.47619-3)}{f(0.47619) - f(3)}$$
$$= 0.84202$$

tolerance: $|0.84202 - 0.47619| < \epsilon$ X

$$x_4 = 0.84202 - \frac{f(0.84202)(0.84202 - 0.47619)}{f(0.84202) - f(0.47619)}$$
$$= 1.83558$$

tolerance: $|1.83558 - 0.84202| < \epsilon$ X

$$x_5 = 1.83558 - \frac{f(1.83558)(1.83558 - 0.84202)}{f(1.83558) - f(0.84202)}$$
$$= 1.24405$$

tolerance: $|1.24405 - 1.83558| < \epsilon$ X

$$x_6 = 1.24405 - \frac{f(1.24405)(1.24405 - 1.83558)}{f(1.24405) - f(1.83558)}$$

$$= 1.34057$$

tolerance: $|1.34057 - 1.24405| < \epsilon$ ✗

$$x_7 = 1.34057 - \frac{f(1.34057)(1.34057 - 1.24405)}{f(1.34057) - f(1.24405)}$$
$$= 1.36677$$

tolerance: $|1.36677 - 1.34057| < \epsilon$ ✗

$$x_8 = 1.36677 - \frac{f(1.36677)(1.36677 - 1.34057)}{f(1.36677) - f(1.34057)}$$
$$= 1.36521$$

tolerance: $|1.36521 - 1.36677| < \epsilon$ ✗

$$x_9 = 1.36521 - \frac{f(1.36521)(1.36521 - 1.36677)}{f(1.36521) - f(1.36677)}$$
$$= 1.36522$$

tolerance: $|1.36522 - 1.36521| < \epsilon$ ✗

$$x_{10} = 1.36522 - \frac{f(1.36522)(1.36522 - 1.36521)}{f(1.36522) - f(1.36521)}$$
$$= 1.36523$$

tolerance: $|1.36523 - 1.36522| < \epsilon$ ✗

$$x_{11} = 1.36523 - \frac{f(1.36523)(1.36523 - 1.36522)}{f(1.36523) - f(1.36522)}$$
$$= 1.36523$$

tolerance: $|1.36523 - 1.36523| < \epsilon$ ✓

c) $x_0 = -2 \quad x_1 = 4$

$$x_2 = 4 - \frac{f(4)(4 - -2)}{f(4) - f(-2)} = -1.9$$

tolerance: $x_0 \& x_1 = |4 - -2| < \epsilon$ X

$x_2 \& x_1 = |-1.9 - 4| < \epsilon$ X

$$x_3 = -1.9 - \frac{f(-1.9)(-1.9 - 4)}{f(-1.9) - f(4)} = -1.78147$$

tolerance. $|-1.78147 - -1.9| < \epsilon$ X

$$x_4 = -1.78147 - \frac{f(-1.78147)(-1.78147 - -1.9)}{f(-1.78147) - f(-1.9)}$$

$= -2.43077$

tolerance: $|-2.43077 - -1.78147| < \epsilon$ X

$$x_5 = -2.43077 - \frac{f(-2.43077)(-2.43077 - -1.78147)}{f(-2.43077) - f(-1.78147)}$$

$= -2.64261$

tolerance: $|-2.64261 - -2.43077| < \epsilon$ X

$$x_6 = -2.64261 - \frac{f(-2.64261)(-2.64261 - -2.43077)}{f(-2.64261) - f(-2.43077)}$$

$= -3.17519$

$$x_7 = -3.17519 - \frac{f(-3.17519)(-3.17519 - -2.64261)}{f(-3.17519) - f(-2.64261)}$$

$= -2.40422$

tolerance: $|-2.40422 - -3.17519| < \epsilon$ X

$$x_8 = -2.40422 - \frac{f(-2.40422)(-2.40422 - -3.17519)}{f(-2.40422) - f(-3.17519)}$$

$= -1.74569$

tolerance: $|-1.74569 - -2.40422| < \epsilon$ X

$$x_9 = -1.74569 - \frac{f(-1.74569)(-1.74569 - -2.40422)}{f(-1.74569) - f(-2.40422)}$$

$= -2.62127$

tolerance: $|-2.62127 - -1.74569| < \epsilon$ X

$$x_{10} = -2.62127 - \frac{f(-2.62127)(-2.62127 - -1.74569)}{f(-2.62127) - f(-1.74569)}$$

$= -2.79839$

$$x_{11} = -2.79839 - \frac{f(-2.79839)(-2.79839 - -2.62127)}{f(-2.79839) - f(-2.62127)}$$

$= -1.15315$

tol: $|-1.15315 - -2.79839| < \epsilon$ X

$$x_{12} = -1.15315 - \frac{f(-1.15315)(-1.15315 - -2.79839)}{f(-1.15315) - f(-2.79839)}$$

$= -2.97104$

tol: $|-2.97104 - -1.15315| < \epsilon$ X

$$x_{13} = -2.97104 - \frac{f(-2.97104)(-2.97104 - -1.15315)}{f(-2.97104) - f(-1.15315)}$$

$= -3.28584$

tol: $|-3.28584 - -2.97104| < \epsilon$ X

$$x_{14} = -3.28584 - \frac{f(-3.28584)(-3.28584 - -2.97104)}{f(-3.28584) - f(-2.97104)}$$

$$= -2.76058$$

tol: $|-2.76058 - -3.28584| < \epsilon$ X

using the secant program, it starts to show convergence signs around 148^{th} iteration. It converges at a very slow rate

comments on 5a, b

a → takes a long time to converge
b → it takes longer than a because the starting bracket's range is bigger than a's