1) a) Lagrange $\quad\overset{x_1 \; y_1}{(1, \ln 1)} \quad \overset{x_2 \; y_2}{(2, \ln 2)} \quad \overset{x_3 \; y_3}{(3, \ln 3)}$

$$P_2(x) = y_1 L_1(x) + y_2 L_2(x) + y_3 L_3(x)$$

$$L_1 = \frac{(x - x_2)(x - x_3)}{(x_1 - x_2)(x_1 - x_3)}$$

$$= \frac{(x - 2)(x - 3)}{(1 - 2)(1 - 3)} = \frac{(x - 2)(x - 3)}{2}$$

$$L_2 = \frac{(x - x_1)(x - x_3)}{(x_2 - x_1)(x_2 - x_3)}$$

$$= \frac{(x - 1)(x - 3)}{(2 - 1)(2 - 3)} = \frac{(x - 1)(x - 3)}{-1}$$

$$L_3 = \frac{(x - x_2)(x - x_1)}{(x_3 - x_2)(x_3 - x_1)}$$

$$= \frac{(x - 2)(x - 1)}{(3 - 2)(3 - 1)} = \frac{(x - 2)(x - 1)}{2}$$

$$P_2(x) = (\ln 1)\left(\frac{(x-2)(x-3)}{2}\right) + (\ln 2)(-(x-1)(x-3))$$

$$+ (\ln 3)\left(\frac{(x-2)(x-1)}{2}\right)$$

b) $P_2(1.5) = \ln(1)\left(\frac{(1.5-2)(1.5-3)}{2}\right) + \ln(2)\left(-(1.5-1)\right.$

$$\left.(1.5-3)\right) + \ln(3)\left(\frac{(1.5-2)(1.5-1)}{2}\right)$$

$$= 0.38253$$

$$P_2(2.4) = \ln(1)\left(\frac{(2.4-2)(2.4-3)}{2}\right) + \ln(2)$$

$$\left(-(2.4-1)(2.4-3)\right) + \ln(3)$$

$$\left(\frac{(2.4-2)(2.4-1)}{2}\right)$$

$$= 0.88985$$

$P_2(1.5)$ absolute error $= |F(1.5) - P_2(1.5)|$

$$= |0.40546 - 0.38253|$$
$$= 0.02293$$

$p(2.4)$ absolute error $= |f(2.4) - p_2(2.4)|$

$= 0.01438$

C) Theoretical error bound

$$\frac{(x-x_1)(x-x_2)(x-x_1)}{n!} \quad f^{(n)}(c)$$

$f(x) = \ln(x)$

$f'(x) = \frac{1}{x}$ $\qquad\qquad f'''(x) = \frac{2}{x^3}$

$f''(x) = \frac{-1}{x^2}$

let $c=1$: $\dfrac{(1.5-1)(1.5-2)(1.5-3)}{6} \times \dfrac{2}{1^3} = 0.125$

let $c=3$: $\dfrac{(1.5-1)(1.5-2)(1.5-3)}{6} \times \dfrac{2}{3^3} = 0.00462$

$1 < c < 3$

$0.00462 < p(1.5) \text{ error} < 0.125$