7) $f(x) = e^{-x^2}$

$f'(x) = (e^{-x^2})(-2x)$

$f''(x) = (e^{-x^2})(-2x)(-2x) + (-2)(e^{-x^2}) = e^{-x^2}(4x^2-2)$

$f'''(x) = (e^{-x^2})(-2x)(4x^2-2) + (e^{-x^2})(8x) = e^{-x^2}((-8x^3+4x)$

$+ (8x)) = e^{-x^2}(-8x^3+12x)$

$f(0) = 1 \quad f'(0) = 0 \quad f''(0) = -2 \quad f'''(0) = 0$

$P(0.1)_2 = 1 + 0(0.1-0) + \frac{-2}{2!}(0.1-0)^2 = 1 - \frac{\cancel{2}(0.1)^2}{\cancel{2}}$

$= 0.99$

absolute error $= |0.99 - 0.99004 98| = 0.0000498$

$f'''(c) < f'''(x) \qquad \frac{f'''(c)(x-x_0)}{3!}$

$$\boxed{\begin{array}{l} x_0 < c < x \\ 0 < c < 0.1 \end{array}}$$

$\frac{e^{-0.1^2}(-8(0.1)^3 + 12(0.1))}{3!} = \underline{0.01848}$

$\downarrow$

upper bound

$0.01848 > 0.0000498$

There is a difference of 0.0184302 between the upper bound & the absolute error