University of New Brunswick
Faculty of Computer Science
CS2333: Computability and Formal Languages
Homework Assignment 3, **Due Time, Date** *5:00 PM, February 11, 2022*

Student Name: ____________________________ Matriculation Number: ________________

Instructor: Rongxing Lu
The marking scheme is shown in the left margin and [100] constitutes full marks.

[**20**] 1. Give NFAs with the specified number of states recognizing each of the following languages.

[4] (a) The language $\{w|w \in \{0,1\}^*$ end with 00 with three states$\}$.

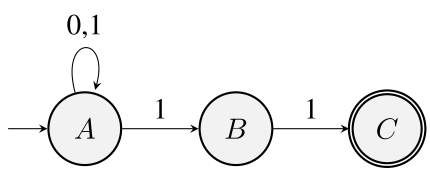
[4] (b) The language $\{0\}$ with two states.

[4] (c) The language $0^*1^*0^*0$ with three states.

[4] (d) The language $\{\varepsilon\}$ with one state.

[4] (e) The language 0^* with one state.

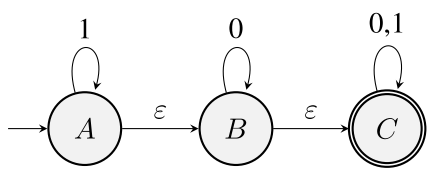
[**10**] 2. Given below is the NFA for a language



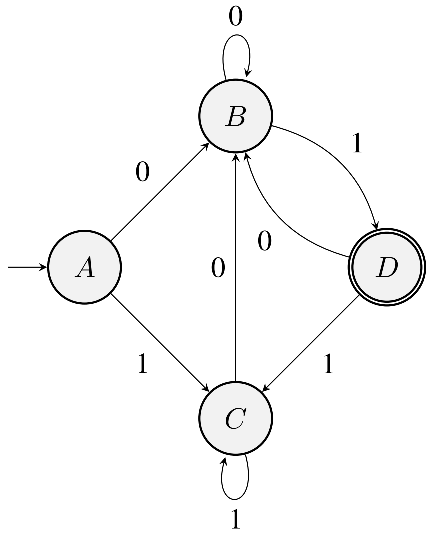
$$L = \{ \text{ set of all strings over } \{0,1\} \text{ that end with '11'}\}$$

Construct its equivalent DFA.

[**10**] 3. Convert the following ε-NFA to its equivalent NFA.



[**10**] 4. Minimize the following DFA with reduced states.

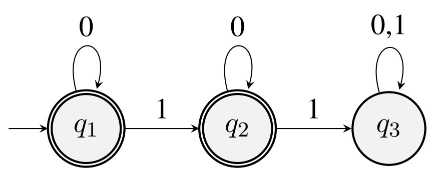


[**10**] 5. Design Regular Expression for the following languages over $\{a, b\}$.

[5] (a) Language accepting strings of length at least 1.

[5] (b) Language accepting strings of length at most 3.

[**10**] 6. Find the Regular Expression for the following DFA.



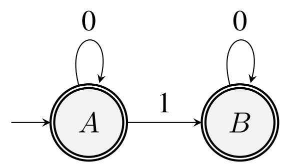
[**10**] 7. Covert each of the following Regular Expression to its equivalent Finite Automata.

[5] (a) $0^* + 0^*10^*$

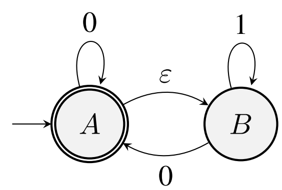
[5] (b) $10 + (1 + 00)1^*0$

[**20**] 8. Convert each of the following NFAs to an equivalent DFA.

[10] (a)



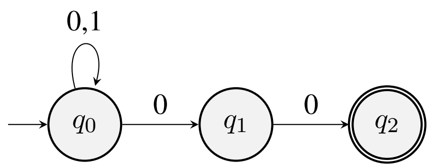
[10] (b)



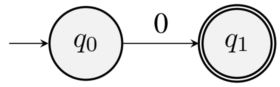
Solutions.

1. Give NFAs with the specified number of states recognizing each of the following languages.

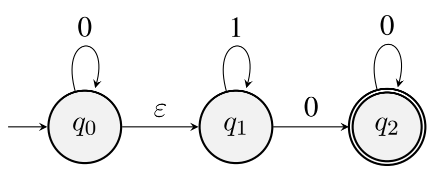
   (a) The language $\{w|w \in \{0,1\}^*$ end with 00 with three states$\}$.



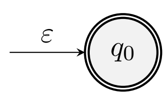
   (b) The language $\{0\}$ with two states.



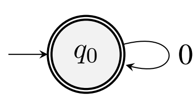
   (c) The language $0^*1^*0^*0$ with three states.



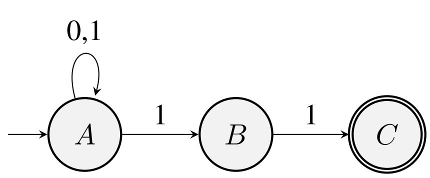
   (d) The language $\{\varepsilon\}$ with one state.



   (e) The language 0^* with one state.



2. Given below is the NFA for a language



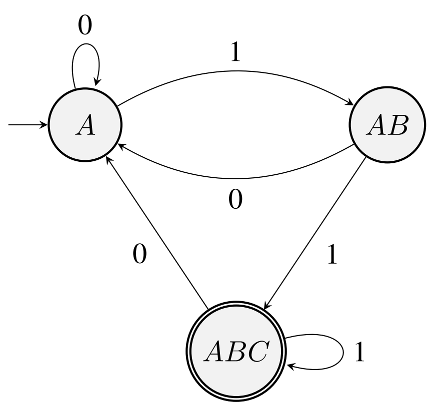
$$L = \{\text{ set of all strings over } \{0,1\} \text{ that end with '11'}\}$$

Construct its equivalent DFA.

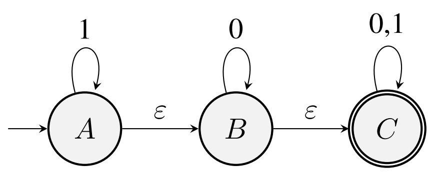
	0	1
→A	A	A,B
B	∅	C
C	∅	∅

Set A as the initial state,

	0	1
→A	A	AB
AB	A	ABC
ABC	A	ABC



3. Convert the following ε-NFA to its equivalent NFA.



	ε^*	0	ε^*
A	A	∅	
	B	B	B,C
	C	C	C

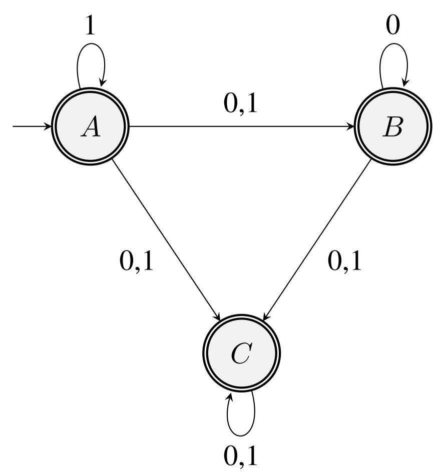
	ε^*	1	ε^*
A	A	A	A,B,C
	B	∅	
	C	C	C

	ε^*	0	ε^*
B	B	B	B,C
	C	C	C

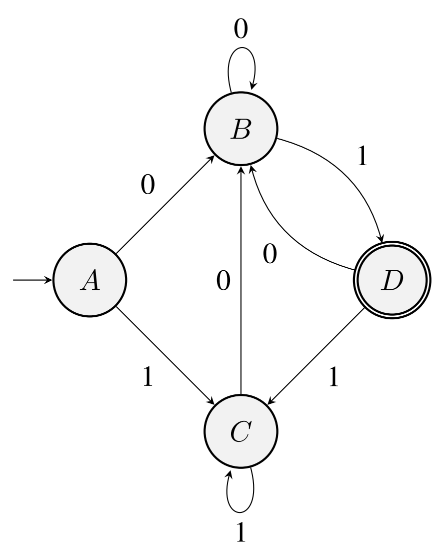
	ε^*	1	ε^*
B	B	$\emptyset$	
	C	C	C

A, B, C are all final states, since the ε can make them reach C.

	0	1
$\rightarrow\underline{\text{A}}$	B,C	A,B,C
$\underline{\text{B}}$	B,C	C
$\underline{\text{C}}$	C	C



4. Minimize the following DFA with reduced states.



	0	1
→A	B	C
B	B	D
C	B	C
<u>D</u>	B	C

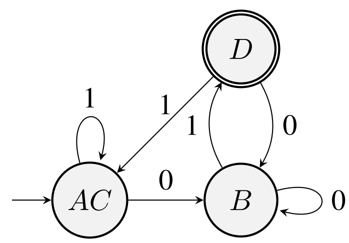
0-Equivalence $\{ABC\}$, $\{D\}$

1-Equivalence $\{AC\}$, $\{B\}$, $\{D\}$

2-Equivalence $\{AC\}$, $\{B\}$, $\{D\}$

As there is no difference between 1-Equivalence and 2-Equivalence, we have

	0	1
→AC	B	AC
B	B	D
<u>D</u>	B	AC



5. Design Regular Expression for the following languages over $\{a, b\}$.

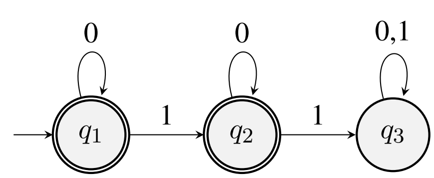
   (a) Language accepting strings of length at least 1.

$$L_1 = \{a, b, aa, ab, ba, bb, aaa, \cdots\}$$
$$R_1 = (a + b)(a + b)^*$$

   (b) Language accepting strings of length at most 3.

$$L_2 = \{\varepsilon, a, b, aa, ab, ba, bb, aaa, aab, aba, abb, baa, bab, bba, bbb \cdots\}$$
$$R_2 = (\varepsilon + a + b)(\varepsilon + a + b)(\varepsilon + a + b)$$

6. Find the Regular Expression for the following DFA.



$$q_1 = \varepsilon + q_1 \cdot 0 \tag{1}$$

$$q_2 = q_1 \cdot 1 + q_2 \cdot 0 \tag{2}$$

$$q_3 = q_2 \cdot 1 + q_3 \cdot 0 + q_3 \cdot 1 \tag{3}$$

Based on the Arden Theorem, we know if $R = Q + RP$, we have $R = QP^*$.

Final state q_1

From (1), we have

$$\underbrace{q_1}_{R} = \underbrace{\varepsilon}_{Q} + \underbrace{q_1}_{R} \cdot \underbrace{0}_{P}$$

and then $q_1 = \varepsilon 0^*$, that is $q_1 = 0^*$.

Final state q_2

From (2), we have

$$q_2 = q_1 \cdot 1 + q_2 \cdot 0 \qquad q_2 = 0^* \cdot 1 + q_2 \cdot 0 \qquad \underbrace{q_2}_{R} = \underbrace{0^* \cdot 1}_{Q} + \underbrace{q_2}_{R} \cdot \underbrace{0}_{P}$$

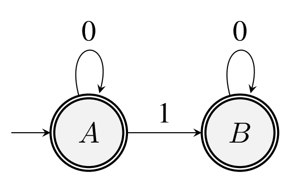
and then $q_2 = 0^*10^*$.

Then, R is the union of both final states

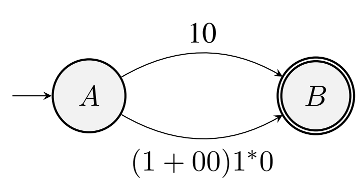
$$R = 0^* + 0^*10^* \qquad \text{or} \qquad R = 0^* + 0^*10^* = 0^*(\varepsilon + 10^*)$$

7. Covert each of the following Regular Expression to its equivalent Finite Automata.

   (a) $0^* + 0^*10^*$



   (b) $10 + (1 + 00)1^*0$



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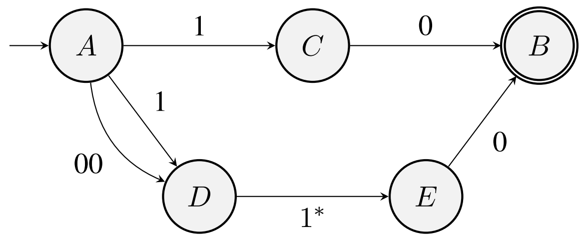

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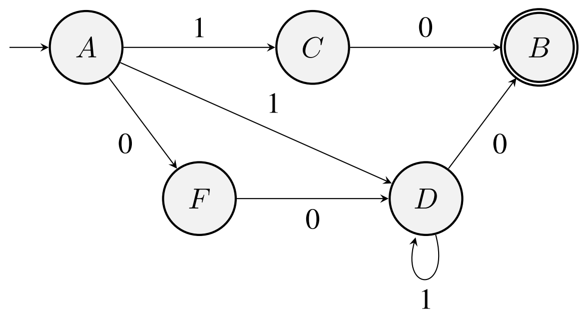
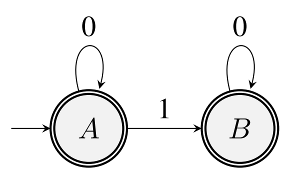
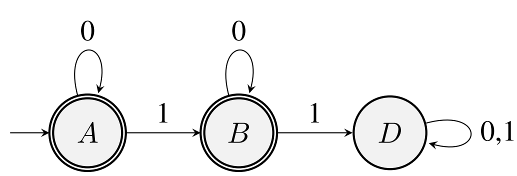


8. Convert each of the following NFAs to an equivalent DFA.

(a)



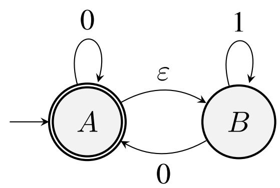
	0	1
→A	A	B
B	B	∅



	0	1
→A	A	B
B	B	D
D	D	D

D is a dead state.

(b)



Step 1: convert ε-NFA to NFA.

	ε^*	0	ε^*
A	A	A	A,B
	B	A	A,B

	ε^*	1	ε^*
A	A	$\emptyset$	
	B	B	B

	ε^*	0	ε^*
B	B	A	A,B

	ε^*	1	ε^*
B	B	B	B

Step 2:

	0	1
$\rightarrow\underline{\text{A}}$	A,B	B
B	A,B	B

Step 3: set AB as the initial state (Note that, if we set A as the initial state, we can use the minimization of DFA to get the same result.)

	0	1
$\rightarrow\underline{\text{AB}}$	AB	B
B	AB	B