

1) a) Algorithm: `isPalindrome(stringToCheck)`
`length = stringToCheck.length()`
`if (length == 0)`
`return "yes"`
`return isPalindromeRecursive(stringToCheck,`
`0, length-1)`

Algorithm: ^{Recursive} `isPalindrome(stringToCheck, startingIndex,`
`endingIndex)`
`if (startingIndex == endingIndex)`
`return "yes"`
`if ((stringToCheck.charAt(startingIndex)) !=`
`(stringToCheck.charAt(endingIndex)))`
`return "No"`
`if (startingIndex < endingIndex+1)`
`return isPalindromeRecursive(stringToCheck,`
`startingIndex+1,`
`endingIndex-1)`
`return "yes"`

2)c) iteration : $\begin{array}{c|c|c|c|c} 1 & 2 & 3 & 4 & \dots & k^{\text{th}} \\ j & 2^1 & 2^2 & 2^3 & 2^4 & 2^k \end{array}$

$$2^k = n$$

$$\log_2 2^k = \log_2 n$$

$$k = \log_2 n$$

$x = 2$	1
$j = 2$	1
while ($j \leq n$)	$\log n$
$x = x + 1$	$\log n$
$j = j \times 2$	$\log n$

$$\Theta(\log n)$$

2)d)

$x = 0$	1	
$j = n$	1	
while ($j \geq 1$)	$\log n$	$\Theta(\log n)$
$x = x + 1$	$\log n$	
$j = 2j/3$	$\log n$	

iteration : $\begin{array}{c|c|c|c|c} 1 & 2 & 3 & 4 & \dots & k^{\text{th}} \\ j & n & (2/3)^1 n & (2/3)^2 n & (2/3)^3 n & (2/3)^{k-1} n \\ & & & & & = 1 \end{array}$

$$(2/3)^{k-1} n = 1$$

$$(2/3)^{k-1} = \frac{1}{n}$$

$$k-1 = \log_{2/3} \frac{1}{n}$$

$$k = \log_{2/3} \frac{1}{n} + 1$$

$$(n^{1/2})^{1/2} = n^{1/4}$$

$$(n^{1/4})^{1/2} = n^{1/8}$$

2f)

```

x = 0
j = n
while (j >= 2)
    x = x + 1
    j = j / 2
    
```

```

1
1
log log n
log log n
log log n
    
```

$$\Theta(\log \log n)$$

iteration: 1 2 3 4 ... k

$n^{1/2^0}$ $n^{1/2^1}$ $n^{1/2^2}$ $n^{1/2^3}$... $n^{1/2^{k-1}}$

$$n^{1/2^{k-1}} = 2$$

$$\log_2 n^{1/2^{k-1}} = \log_2 2$$

$$1/2^{k-1} \log_2 n = 1$$

$$\log_2 n = 2^{k-1}$$

$$\log_2 \log_2 n = k-1 \log_2 2 \quad \log_a a = 1$$

$$\log_2 \log_2 n + 1 = k$$

2a)

```

p = 1
for i = 1 to 5n^2 do
    p = p * i
return p
    
```

```

1
n^2
n^2
1
    
```

$$\Theta(n^2)$$

iteration: 1 2 3 4 ... k

 1 2 3 4 ... k

$$k = 5n^2$$

2e)

$x = 0$

$j = 2$

while ($j \leq n$)

$x = x + 1$

$j = j^3$

1

1

$\log \log n$

$\log \log n$

$\log \log n$

$\Theta(\log \log n)$

iteration:	1	2	3	4	...	k^{th}
j	2	2^3	2^{3^2}	2^{3^3}		$2^{3^{k-1}}$

3^{k-1}

$2 = n$

$$3^{k-1} \log_2 2 = \log_2 n$$

$$3^{k-1} = \log_2 n$$

$$k-1 = \log_3 \log_2 n$$

$$k = \log_3 \log_2 n + 1$$

2)g)

```

x = 0
j = n
while (j >= 1)
    for i = j to n
        x = x + 1
    j = j - 1
return x

```

1
1
n
n²
n²
n
1

$\Theta(n^2)$

iteration:	1	2	3	4	...	k
j	n	n-1	n-2	n-3		n-(k-1)

$$n - (k - 1) = 1$$

$$-(k - 1) = 1 - n$$

$$k - 1 = -1 + n$$

$$k = n$$

~~for i = 1 to n~~

j:	n	n-1	n-2	n-3	...	n-(k-1)
for loop:	0	1	2	3		k-1

$$k - 1 = n$$

$$\frac{n(n+1)}{2} = n^2$$

2b)

```

s = 0
for i = 1 to n
  for j = i to n
    s = s + 1
return s

```

$\Theta(n^2)$

iter:	1	2	3	4	...	k
i:	1	2	3	4		k
j:	1	2	3	4		k

$k = n$

$$\text{inner loop} = \frac{n(n+1)}{2} = n^2$$

4a & c) Algorithm: repeatNums (array)

```

n = array.length
for (i ← 0; i < n)
    m = array[i] % n
    array[m] += n
for (j ← 0; j < n)
    if (array[j] ≥ 2 * n)
        print(j)

```

$\Theta(n)$

iteration:	1	2	3	4	...	k
i	0	1	2	3		n-1

$$k = n - 1$$

iteration:	1	2	3	4	...	k
j	0	1	2	3		n-1

$$k = n - 1$$

3a)

```

s ← 0
for i ← 0 to n-1
  for j ← i to n-1
    p ← 1
    for k ← i to j
      p ← p × A[k]
    s ← s + p
return s

```

1
n
n
n
n
n
1

$O(n^3)$

iteration:	1	2	3	4	...	k
i	0	1	2	3		k-1
j	0	1	2	3		k
k	0	1	2	3		k

2nd loop: ~~0~~ | ~~1~~ | ~~2~~ | ... | ~~k~~

i	0	1
j	0	1
3rd loop:	0	1

i	0	1	2	...	k
1st loop:	n-1	n-2	n-3		0

i, j, k are incremented by a constant