2) $\int_1^2 \frac{1}{x}\,dx$

2)a) $O(h^2)$ $h = 0.5, 0.25, 0.125$ trapezoid

1 —— 1.5 —— 2 $h = 0.5$ $m = 2$

$\int_1^2 \frac{1}{x}\,dx = 0.25\left(\frac{1}{1} + \frac{2}{1.5} + \frac{1}{2}\right) = 0.708333333$

1 — 1.25 — 1.5 — 1.75 — 2 $m = 4$ $h = 0.25$

$\int_1^2 \frac{1}{x}\,dx = 0.125\left(\frac{1}{1} + 2\left(\frac{1}{1.25} + \frac{1}{1.5} + \frac{1}{1.75}\right) + \frac{1}{2}\right)$

$= 0.697023809 5$

1 — 1.125 — 1.250 — 1.375 — 1.500 — 1.625
— 1.750 — 1.875 — 2 $m = 8$ $h = 0.125$

$\int_1^2 \frac{1}{x}\,dx = 0.0625\left(\frac{1}{1} + 2\left(\frac{1}{1.125} + \frac{1}{1.250} + \frac{1}{1.375}\right.\right.$

$\left.\left. + \frac{1}{1.5} + \frac{1}{1.625} + \frac{1}{1.75} + \frac{1}{1.875}\right) + \frac{1}{2}\right)$

$= 0.694121850 4$

integral (S) of $\frac{1}{x} = \ln(x)$

$\ln(2) - \ln(1) = 0.693147181$

$h \rightarrow$ $0.5 = 2^1(0.25) = 2^2(0.125)$

abs err of $h = 0.5$: $|0.7083333333 - 0.6931471811|$
$= 0.0151861523$

abs err of $h = 0.25$: $|0.6970238095 - 0.6931471811|$
$= 0.0038766295$

abs err of $h = 0.125$: $|0.6941218505 - 0.6931471811|$
$= 0.0009746695$

abs err $\rightarrow 0.0151861523 \approx 4(0.0038766295) \approx 16(0.0009746695)$

$\rightarrow 0.0151861523 \approx 0.0155065180 \approx 0.0155947120$

$O(h^2)$ [because of abs err & h]

2)b) Simpson $O(h^4)$ $h = 0.5, 0.25, 0.125$

$h = 0.5$ $\quad 1 \text{ —— } 1.5 \text{ — } 2 \quad$ $m = 1$

$$\int_1^2 \frac{1}{x}\,dx = (0.5/3)\left(1 + 4\left(\frac{1}{1.5}\right) + 0.5\right) = \frac{25}{36}$$
$$= 0.6944444444$$

$h = 0.25$ $\quad 1 \text{ — } 1.25 \text{ — } 1.5 \text{ — } 1.75 \text{ — } 2$
$m = 2$

$$\int_1^2 \frac{1}{x}\,dx = (0.25/3)\left(1 + 4\left(\frac{1}{1.25} + \frac{1}{1.75}\right) + 2\left(\frac{1}{1.5}\right) + \frac{1}{2}\right)$$
$$= \frac{1747}{2520} = 0.6932539683$$

$h = 0.125$

$1 - 1.125 - 1.25 - 1.375 - 1.5 - 1.625 - 1.75$
$- 1.875 - 2$

$m = 4$

$$\int_1^2 \frac{1}{x}\,dx$$

$$= (0.125/3)\left(1 + 4\left(\frac{1}{1.125} + \frac{1}{1.375} + \frac{1}{1.625} + \frac{1}{1.875}\right) + 2\left(\frac{1}{1.25} + \frac{1}{1.5} + \frac{1}{1.75}\right) + 0.5\right)$$

$$= 0.693154507$$

$h \to 0.5 = 2^1(0.25) + 2^2(0.125)$

abs err of $h = 0.5$: $|0.6944444444 - 0.693147181| = 0.0012972634$

abs err of $h = 0.25$: $|0.6932539683 - 0.693147181| = 0.0001067873$

abs err of $h = 0.125$: $|0.693154507 - 0.693147181| = 0.0000073497$

abs err $\to 0.0012972634 \simeq 16(0.0001067873)$
$\simeq 256(0.0000073497)$

$\to 0.0012972634 \simeq 0.0017085968 \simeq 0.0018815232$

$O(h^4)$