University of New Brunswick
Faculty of Computer Science
CS2333: Computability and Formal Languages
*Homework Assignment 5, **Due Time, Date** 5:00 PM, March 4, 2022*

Student Name: ____________________________ Matriculation Number: ________________

Instructor: Rongxing Lu
The marking scheme is shown in the left margin and [100] constitutes full marks.

[**10**] 1. Verify whether the Grammar $S \to 0B|1A$, $A \to 0|0S|1AA|\varepsilon$, $B \to 1|1S|0BB$ generates the strings i) 001101011, and ii) 1100101001.

[**20**] 2. Give context-free grammars that generate the following languages. In all parts the alphabet Σ is $\{0, 1\}$.

[5] (a) $\{w|w$ contains at least three 1s$\}$.

[5] (b) $\{w|w$ starts and ends with the same symbol$\}$.

[5] (c) $\{w|$ the length of w is odd$\}$.

[5] (d) $\{w|$ the length of w is odd and its middle symbol is a $0\}$.

[**15**] 3. Find a reduced grammar equivalent to the grammar G, having the production rule $P : S \to AC|B$, $A \to a$, $C \to c|BC$, $E \to aA|e$, $F \to bB|e$.

[**15**] 4. Remove the unit productions from the grammar whose production rule is given by $P : S \to AB$, $A \to a$, $B \to C|b$, $C \to D$, $D \to E$, $E \to e$.

[**15**] 5. Remove the Null productions from the following grammar

$$S \to XYXZ, X \to aX|\varepsilon, Y \to bY|\varepsilon, Z \to c$$

[**25**] 6. Convert the following context-free grammar to the Chomsky normal form.

$$P : S \to ASA|1B, A \to B|S, B \to 0|\varepsilon$$

Solutions.

1. Verify whether the Grammar $S \to 0B|1A$, $A \to 0|0S|1AA|\varepsilon$, $B \to 1|1S|0BB$ generates the strings i) 001101011, and ii) 1100101001.

i)

S	$\to$	0B	$(S \to 0B)$
	$\to$	00BB	$(B \to 0BB)$
	$\to$	001B	$(B \to 1)$
	$\to$	0011S	$(B \to 1S)$
	$\to$	00110B	$(S \to 0B)$
	$\to$	001101S	$(B \to 1S)$
	$\to$	0011010B	$(S \to 0B)$
	$\to$	00110101S	$(B \to 1S)$
	$\to$	001101011A	$(S \to 1A)$
	$\to$	001101011ε	$(A \to \varepsilon)$
	$\to$	001101011	Yes!

ii)

S	$\to$	1A	$(S \to 1A)$
	$\to$	11AA	$(A \to 1AA)$
	$\to$	110A	$(A \to 0)$
	$\to$	1100S	$(A \to 0S)$
	$\to$	11001A	$(S \to 1A)$
	$\to$	110010S	$(A \to 0S)$
	$\to$	1100101A	$(S \to 1A)$
	$\to$	11001010S	$(A \to 0S)$
	$\to$	110010100B	$(S \to 0B)$
	$\to$	1100101001	$(B \to 1)$ Yes!

2. Give context-free grammars that generate the following languages. In all parts the alphabet Σ is $\{0,1\}$.

   (a) $\{w|w$ contains at least three 1s$\}$.

$$S \to R1R1R1R, \qquad R \to 0R|1R|\varepsilon$$

   (b) $\{w|w$ starts and ends with the same symbol$\}$.

$$S \to 0R0|1R1|\varepsilon, \qquad R \to 0R|1R|\varepsilon$$

   (c) $\{w|$ the length of w is odd$\}$.

$$S \to 0|1|00S|01S|10S|11S$$

   (d) $\{w|$ the length of w is odd and its middle symbol is a $0\}$.

$$S \to 0|0S0|0S1|1S0|1S1$$

3. Find a reduced grammar equivalent to the grammar G, having the production rule $P : S \to AC|B$, $A \to a$, $C \to c|BC$, $E \to aA|e$, $F \to bB|e$.

   Phase 1:

$$T = \{a, c, e\}$$

$$W_1 = \{A, C, E, F\}, W_2 = \{A, C, E, F, S\}, W_3 = \{A, C, E, F, S\}$$

   Therefore,

$$G' = \{\{A, C, E, F, S\}, \{a, c, e\}, P, (S)\}$$

$$P : S \to AC, A \to a, C \to c, E \to aA|e, F \to bB|e$$

   Phase 2:

$$Y_1 = \{S\}, Y_2 = \{S, A, C\}, Y_3 = \{S, A, C, a, c\}, Y_4 = \{S, A, C, a, c\}$$

   Therefore,

$$G'' = \{\{S, A, C\}, \{a, c\}, P, (S)\}$$

$$P : S \to AC, A \to a, C \to c$$

4. Remove the unit productions from the grammar whose production rule is given by $P : S \to AB$, $A \to a$, $B \to C|b$, $C \to D$, $D \to E$, $E \to e$.

   Because $E \to e$ and $D \to E$, we add $D \to e$ in the production rule to replace $D \to E$.

$$P : S \to AB, A \to a, B \to C|b, C \to D, D \to e, E \to e$$

   Because $D \to e$ and $C \to D$, we add $C \to e$ in the production rule to replace $C \to D$.

$$P : S \to AB, A \to a, B \to C|b, C \to e, D \to e, E \to e$$

   Because $C \to e$ and $B \to C$, we add $B \to e$ in the production rule to replace $B \to C$.

$$P : S \to AB, A \to a, B \to c|b, C \to e, D \to e, E \to e$$

   Remove unreachable symbols

$$Y_1 = \{S\}, Y_2 = \{S, A, B\}, Y_3 = \{S, A, B, a, b\}, Y_4 = \{S, A, B, a, b\}$$

$$P : S \to AB, A \to a, B \to c|b$$

5. Remove the Null productions from the following grammar

$$S \to XYXZ, X \to aX|\varepsilon, Y \to bY|\varepsilon, Z \to c$$

Step 1: to eliminate $X \to \varepsilon$

$$S \to XYXZ \Rightarrow S \to XYZ, S \to YXZ, S \to YZ$$
$$X \to aX \Rightarrow X \to a$$

New production rule becomes

$$S \to XYXZ|XYZ|YXZ|YZ, X \to aX|a, Y \to bY|\varepsilon, Z \to c$$

Step 2: to eliminate $Y \to \varepsilon$

$$S \to XYXZ|XYZ|YXZ|YZ \Rightarrow S \to XXZ|XZ|Z$$

$$Y \to bY \Rightarrow Y \to b$$

New production rule becomes

$$S \to XYXZ|XYZ|YXZ|YZ|XXZ|XZ|Z, X \to aX|a, Y \to bY|b, Z \to c$$

6. Convert the following context-free grammar to the Chomsky normal form.

$$P : S \to ASA|1B, A \to B|S, B \to 0|\varepsilon$$

Step 1: Since S appears in RHS, we need to add a new state S' and $S' \to S$ is added to the production

$$P : S' \to S, S \to ASA|1B, A \to B|S, B \to 0|\varepsilon$$

Step 2: Remove the Null Production

to eliminate $B \to \varepsilon$

$$S \to 1B \Rightarrow S \to 1$$
$$A \to B \Rightarrow A \to \varepsilon$$

The production rule becomes

$$P : S' \to S, S \to ASA|1B|1, A \to B|S|\varepsilon, B \to 0$$

to eliminate $A \to \varepsilon$

$$S \to ASA \Rightarrow S \to AS|SA|S$$

The production rule becomes

$$P : S' \to S, S \to ASA|1B|1|AS|SA|S, A \to B|S, B \to 0$$

Step 3: Remove the Unit Production $S \to S$, $S' \to S$, $A \to B$, $A \to S$

After removing $S \to S$

$$P : S' \to S, S \to ASA|1B|1|AS|SA, A \to B|S, B \to 0$$

After removing $S' \to S$

$$P : S' \to ASA|1B|1|AS|SA, S \to ASA|1B|1|AS|SA, A \to B|S, B \to 0$$

After removing $A \to B$

$$P : S' \to ASA|1B|1|AS|SA, S \to ASA|1B|1|AS|SA, A \to 0|S, B \to 0$$

After removing $A \to S$

$$P : S' \to ASA|1B|1|AS|SA, S \to ASA|1B|1|AS|SA, A \to 0|ASA|1B|1|AS|SA, B \to 0$$

Step 4: Now find out the productions that have more than two variables in RHS

$$S' \to ASA, \qquad S \to ASA, \qquad A \to ASA$$

After removing these, we have

$$\begin{aligned} P : \quad & S' \to AX|1B|1|AS|SA \\ & S \to AX|1B|1|AS|SA \\ & A \to 0|AX|1B|1|AS|SA \\ & B \to 0 \\ & X \to SA \end{aligned}$$

Step 5: Now change the productions

$$S' \to 1B, \qquad S \to 1B, \qquad A \to 1B$$

Finally, we get

$$\begin{aligned} P : \quad & S' \to AX|YB|1|AS|SA \\ & S \to AX|YB|1|AS|SA \\ & A \to 0|AX|YB|1|AS|SA \\ & B \to 0 \\ & X \to SA \\ & Y \to 1 \end{aligned}$$