

2) a)

x

y

First

2nd

1

ln1

$$F[1\ 2] = \frac{\ln(2) - \ln(1)}{2 - 1} = 0.69314$$

2

ln2

$$F[2\ 3] = \frac{\ln(3) - \ln(2)}{3 - 2} = 0.40546$$

3

ln3

$$F[3\ 4] = \frac{\ln(4) - \ln(3)}{4 - 3} = 0.28768$$

4

ln4

F[1 2 3]

$$\frac{0.40546 - 0.69314}{3 - 1} = -0.14384$$

F[2 3 4]

$$\frac{0.28768 - 0.40546}{4 - 2} = -0.05889$$

$$= -0.05889$$

3rd

$$F[1\ 2\ 3\ 4] = \frac{-0.05889 - (-0.14384)}{4 - 1} = 0.02831$$

6)

$$2)b) p_2(x) = f[x_1] + f[x_1, x_2](x-x_1) + f[x_1, x_2, x_3] \frac{(x-x_1)(x-x_2)}{(x_3-x_1)(x_3-x_2)} \\ = \ln(1) + 0.69314(x-1) - 0.14384(x-1)(x-2)$$

2c) Polynomial from 1a)

$$(\ln(1)) \left( \frac{(x-2)(x-3)}{2} \right) + (\ln(2)) (-(x-1)(x-3))$$

$$+ (\ln(3)) \left( \frac{(x-2)(x-1)}{2} \right)$$

$$= 0 + 0.69314 (-(x-1)(x-3)) + 1.09861$$

$$\left( \frac{(x-2)(x-1)}{2} \right)$$

$$= -0.69314x^2 + 2.77256x - 2.07942 + 0.549305(x-2)(x-1)$$

$$= -0.69314x^2 + 2.77256x - 2.07942 + 0.549305x^2 - 1.647915x + 1.09861$$

$$= -0.143835x^2 + 1.124645x - 0.98081$$

$$2b) \text{ polynomial: } 0 + 0.69314x - 0.69314 \\ - 0.14384x^2 + 0.43152x - 0.28768 \\ = -0.14384x^2 + 1.12466x - 0.98082$$

As we can see the polynomials of 2b and 1a are approximately equal [would've been equal if I didn't round]   
  $\rightarrow$  simplified



(6)

$$2)d) \quad p_3(x) = p_2(x) + F[1 \ 2 \ 3 \ 4] (x-x_1)(x-x_2)(x-x_3)$$

$$= \ln(1) + 0.69314(x-1) - 0.14384(x-1)(x-2) + 0.02831(x-1)(x-2)(x-3)$$

$$p_2(1.5) = \ln(1) + 0.69314(1.5-1) - 0.14384(1.5-1)(1.5-2) = 0.38253$$

$$p_3(1.5) = 0.38253 + 0.02831(1.5-1)(1.5-2)(1.5-3) = 0.39314625$$

$$\text{error} = |p_3(1.5) - p_2(1.5)| = |0.39314625 - 0.38253| = 0.01061625$$

---