2)a) $\left[\begin{array}{ccc|c} 8 & 8 & 7 & 23 \\ 8 & 7 & 6 & 21 \\ 7 & 6 & 5 & 18 \end{array}\right]$ $R_2 = R_2 - R_1$

$R_3 = R_3 - \frac{7}{8}R_1 \rightarrow 0.875$

$0.875 \times 8 = 7$

$0.875 \times 7 = 6.125 \approx 6.13$

$0.875 \times 23 = 20.125 \approx 20.1$

$6 - 7 = -1$

$5 - 6.13 = -1.13$

$18 - 20.1 = -2.1$

$\left[\begin{array}{ccc|c} 8 & 8 & 7 & 23 \\ 0 & -1 & -1 & -2 \\ 0 & -1 & -1.13 & -2.1 \end{array}\right]$ $R_3 = R_3 - R_2$

$\left[\begin{array}{ccc|c} 8 & 8 & 7 & 23 \\ 0 & -1 & -1 & -2 \\ 0 & 0 & -0.13 & -0.1 \end{array}\right]$

$-0.13x_3 = -0.1$ $\qquad x_3 = \frac{\cancel{-}0.1}{\cancel{-}0.13} = 0.769$

$-x_2 - 1(0.769) = -2$ $\qquad x_2 = 1.23$

$8x_1 + 8(1.23) + 7(0.769) = 23$

$x_1 = \frac{23 - 7(0.769) - 8(1.23)}{8}$

$x_1 = 0.972$

2) b) $r = b - Ax_a$

$$\begin{bmatrix} 23 \\ 21 \\ 18 \end{bmatrix} - \begin{bmatrix} 8 & 8 & 7 \\ 8 & 7 & 6 \\ 7 & 6 & 5 \end{bmatrix} \begin{bmatrix} 0.972 \\ 1.23 \\ 0.769 \end{bmatrix}$$

$$\begin{bmatrix} 23 \\ 21 \\ 18 \end{bmatrix} - \begin{bmatrix} 22.999 \\ 21 \\ 18.029 \end{bmatrix} = \begin{bmatrix} 0.001 \\ 0 \\ -0.29 \end{bmatrix}$$

$\|r\| = 0.291$

$\|b\| = 23 + 21 + 18 = 62$

$\|r\| / \|b\| = \frac{0.291}{62} = 0.004694 \approx 4.7 \times 10^{-3}$

residual is almost 0, therefore x_a is very close to the exact answer x.

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2) c)

$$\left[\begin{array}{ccc|c} 8 & 8 & 7 & 23 \\ 8 & 7 & 6 & 21 \\ 7 & 6 & 5 & 18 \end{array}\right]$$

$R_2 = R_2 - R_1$
$R_3 = R_3 - 0.875 R_1$

$$\left[\begin{array}{ccc|c} 8 & 8 & 7 & 23 \\ 0 & -1 & -1 & -2 \\ 0 & -1 & -1.125 & -2.125 \end{array}\right]$$

$R_3 = R_3 - R_2$

$$\left[\begin{array}{ccc|c} 8 & 8 & 7 & 23 \\ 0 & -1 & -1 & -2 \\ 0 & 0 & -0.125 & -0.125 \end{array}\right]$$

$-0.125 x_3 = -0.125$

$x_3 = 1$

$-x_2 - 1) = -2$

$x_2 = 1$

$8x_1 + 8 + 7 = 23$

$8x_1 = 8 \quad x_1 = 1$

$||x|| = 1+1+1 = 3$
$||x_a|| = 0.972 + 1.23 + 0.769 = 2.971$

$$\frac{||x|| - ||x_a||}{||x||} = \frac{3 - 2.971}{3} = 0.0096666$$
$$\simeq 9.7 \times 10^{-3}$$

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2)d) $A = \begin{bmatrix} 8 & 8 & 7 \\ 8 & 7 & 6 \\ 7 & 6 & 5 \end{bmatrix}$ to get A^{-1}:

$$\left[\begin{array}{ccc|ccc} 8 & 8 & 7 & 1 & 0 & 0 \\ 8 & 7 & 6 & 0 & 1 & 0 \\ 7 & 6 & 5 & 0 & 0 & 1 \end{array}\right] \quad R_2 = R_2 - R_1$$

$$\left[\begin{array}{ccc|ccc} 8 & 8 & 7 & 1 & 0 & 0 \\ 0 & -1 & -1 & -1 & 1 & 0 \\ 7 & 6 & 5 & 0 & 0 & 1 \end{array}\right] \quad R_2 = -1 \times R_2$$

$$\left[\begin{array}{ccc|ccc} 8 & 8 & 7 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & -1 & 0 \\ 7 & 6 & 5 & 0 & 0 & 1 \end{array}\right] \quad R_1 = R_1 - R_3$$

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 2 & 1 & 0 & -1 \\ 0 & 1 & 1 & 1 & -1 & 0 \\ 7 & 6 & 5 & 0 & 0 & 1 \end{array}\right] \quad R_1 = R_1 - 2R_2$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -1 & 2 & -1 \\ 0 & 1 & 1 & 1 & -1 & 0 \\ 7 & 1 & 0 & 0 & 0 & 1 \end{array}\right] \quad R_3 = R_3 - 5R_2$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -1 & 2 & -1 \\ 0 & 1 & 1 & 1 & -1 & 0 \\ 7 & 1 & 0 & -5 & 5 & 1 \end{array}\right] \quad R_3 = R_3 - 7R_1$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -1 & 2 & -1 \\ 0 & 1 & 1 & 1 & -1 & 0 \\ 0 & 1 & 0 & 2 & -9 & 8 \end{array}\right] \quad R_2 = R_2 - R_3$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -1 & 2 & -1 \\ 0 & 0 & 1 & -1 & 8 & -8 \\ 0 & 1 & 0 & 2 & -9 & 8 \end{array}\right] \quad R_2 \leftrightarrow R_3$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -1 & 2 & -1 \\ 0 & 1 & 0 & 2 & -9 & 8 \\ 0 & 0 & 1 & -1 & 8 & -8 \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} -1 & 2 & -1 \\ 2 & -9 & 8 \\ -1 & 8 & -8 \end{bmatrix}$$

$$\text{cond } A = \|A\| \times \|A^{-1}\| = 23 \times 19$$
$$= 437$$
$$= 437.0 = 4.37 \times 10^2$$

We will have 2 digits that could be lost - maximum - during calc process.

The relative error is $9.7 \times 10^{-3} \approx 0.01 = \frac{1}{100}$

4.37×10^2 is the upper bound as 0.01 error is less than the loss of 2 digits