Student Name: _____ Matriculation Number: _____

Instructor: Rongxing Lu
The marking scheme is shown in the left margin and [100] constitutes full marks.

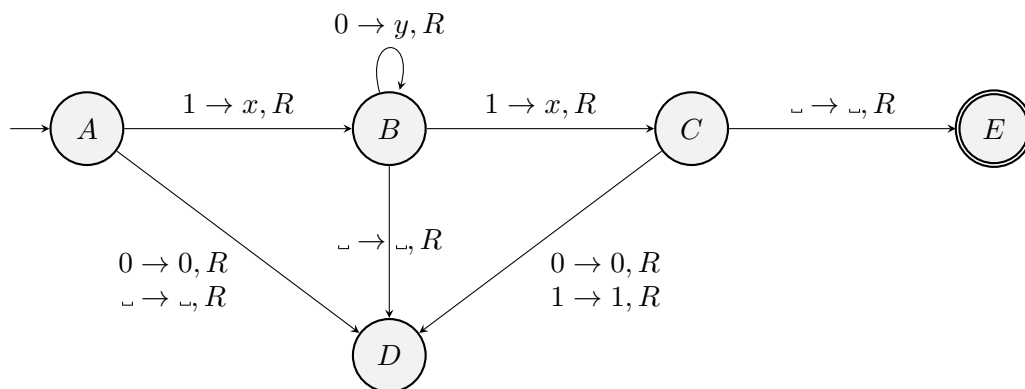[**30**] 1. Design a Turing machine which can recognize the language $L = 10^*1$ over $\Sigma = \{0, 1\}$.

[**30**] 2. Design a Turing machine which can recognize the language $L = \{1^n 0^n | n \in \mathbf{Z}^{nonneg}\}$ over $\Sigma = \{0, 1\}$.

[**40**] 3. Design a Turing machine to compute the function $f(x) = 2x + 1$, where x is a positive integer represented in unary notation, e.g., 6 is presented as 111111. (Hint: please refer to the tutorial question on computing the function $f(x) = 2x$).

Solutions.

1. Design a Turing machine which can recognize the language $L = 10^*1$ over $\Sigma = \{0, 1\}$.

   D is a reject state.

$$0 \to y, R$$

$$1 \to x, R \qquad 1 \to x, R \qquad \_ \to \_, R$$

A, B, C, E states; D reject state

$$0 \to 0, R$$
$$\_ \to \_, R$$

$$\_ \to \_, R$$

$$0 \to 0, R$$
$$1 \to 1, R$$

D

2. Design a Turing machine which can recognize the language $L = \{1^n 0^n | n \in \mathbf{Z}^{nonneg}\}$ over $\Sigma = \{0, 1\}$.

$$1 \to 1, R \qquad\qquad 1 \to 1, L$$
$$y \to y, R \qquad\qquad y \to y, L$$

$$1 \to x, R \qquad\qquad 0 \to y, L$$

A, B, C

$$x \to x, R$$

$$y \to y, R$$

$$\_ \to \_, L$$

$$D \qquad y \to y, R$$

$$\_ \to \_, L$$

E

2

3. Design a Turing machine to compute the function $f(x) = 2x + 1$, where x is a positive integer represented in unary notation, e.g., 6 is presented as 111111. (Hint: please refer to the tutorial question on computing the function $f(x) = 2x$).
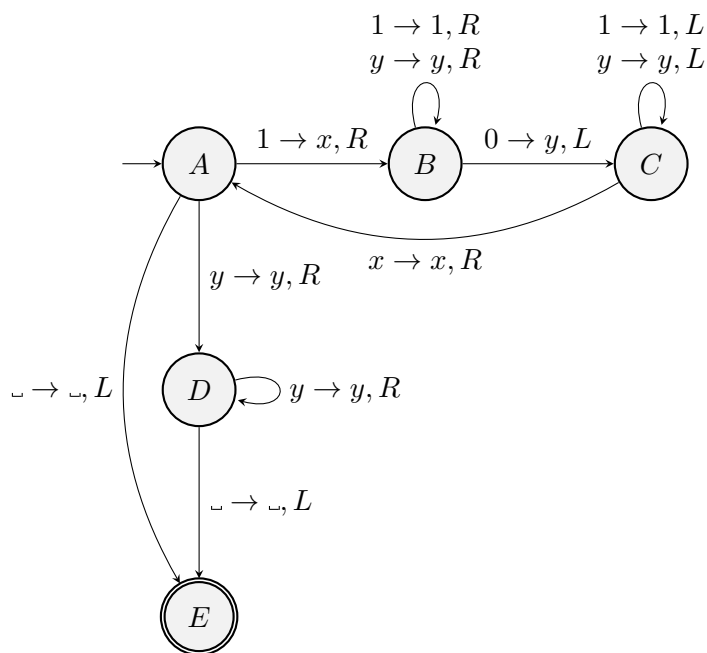
$$1 \to y, R \qquad\qquad 1 \to 1, L \qquad\qquad 1 \to 1, R$$

$$q_0 \xrightarrow{\ \sqcup \to \sqcup, L\ } q_1 \xrightarrow{\ y \to 1, R\ } q_2$$

$$\sqcup \to 1, L$$

$$\sqcup \to 1, L$$

$$q_3 \xrightarrow{\ \sqcup \to \sqcup, R\ } q_4$$