CS 2333
A 8

1) A_{CFG} Decidable

Case1: $w \in L(G)$ ✓ Finite steps

Case 2: $w \notin L(G)$ ✗ how many steps we need to run?

if G is a CFG, the we can convert it to CNF H

if $|w| = n$, we check & decide whether $w \in L(H)$, at $2n-1$ steps

if $w \in L(H) = w \in L(G)$ and vice versa

→ Finite number of steps

Therefore A_{CFG} is decidable

2) A_{TM} Undecidable

assume there is TM D can decide [halt / not halt]

$D(T, w) = \begin{cases} \text{halt}, & \text{if } (T, w) \text{ halt} \\ \text{not halt}, & \text{if } (T, w) \text{ not halt} \end{cases}$

TM C(s)

s: a program

encode → string
<s>

call D(s, <s>)

if(D(s, <s>) == halt)
return not halt
if(D(s, <s>) == not halt)
return halt

consider s ← c

C(c) { not halt ⇒ D(c, <c>) == halt
⇓
(c, <c>) halt
halt ⇒ D(c, <c>) == not halt
⇓
(c, <c>) not halt

contradiction

assumption not true
∴ A_{TM} is undecidable