

Report (6) AER 123; Gas Dynamics B.Sc. 2nd. Year, 2014/2015

# **Report 6: Oblique Shock Wave**

Submitted to:

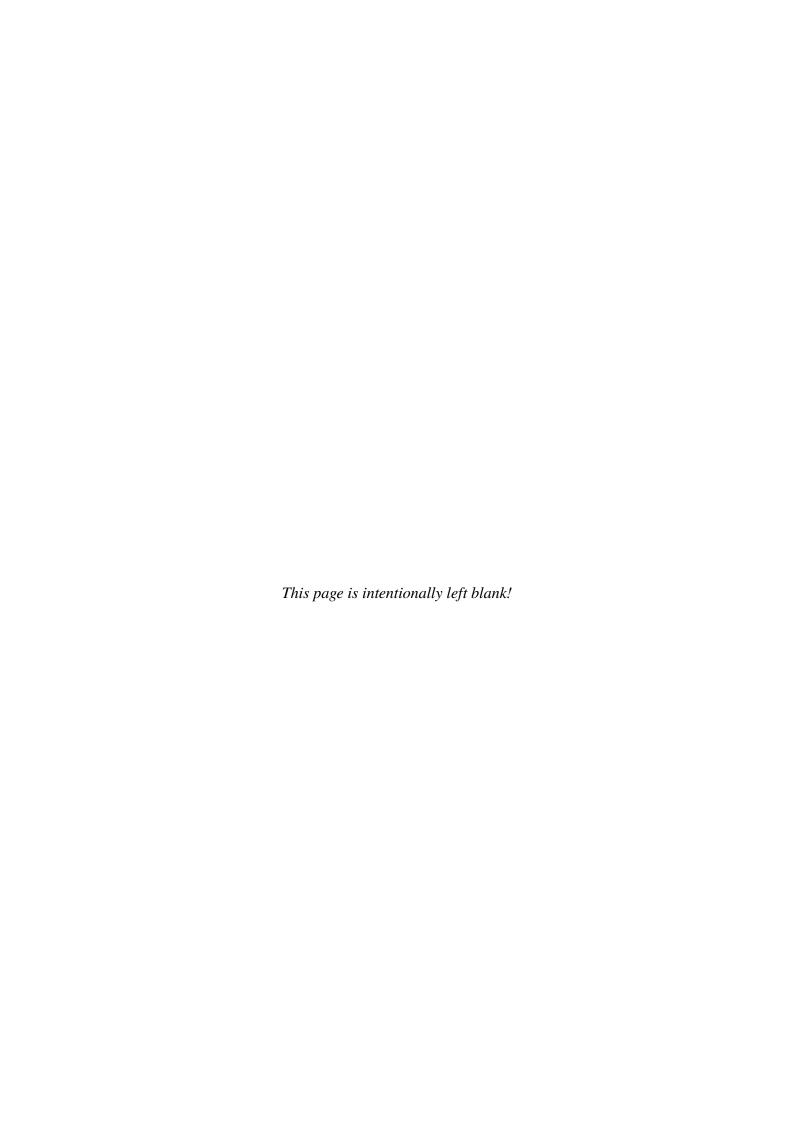
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### 1 Problem Statement

Produce charts that describe the change of supersonic flow properties when it turned away from itself.

### 2 Mathematical Model

The most general governing equation is N.S. equation. This is any dummy text just to show the capabilities of nomenclatures of LyX w.r.t. LATEX.

$$\mu = \sin^{-1}\left(\frac{1}{M}\right) \tag{1}$$

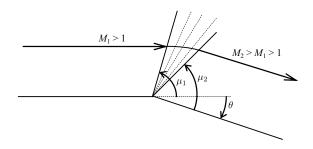
$$\left(\frac{a_0}{a}\right)^2 = 1 + \frac{\gamma - 1}{2}M^2 \tag{2}$$

### 3 Assumptions

- 1. Steady Flow
- 2. Quasi-dimensional flow; (area is variable with x only).
- 3. Body forces can be neglected; (weight of fluid).
- 4. Viscous stresses are absent.

- 5. Changes in potential energy are neglected.
- 6. Perfect gas.
- 7. Thermally perfect gas.
- 8. Adiabatic flow with no external work.

### 4 Analysis



We can derive the formula that governs super flow expansion as:

$$\nu(M_1) = \nu(M_2) + \theta \tag{3}$$

$$\nu(M) = \sqrt{\frac{\gamma + 1}{\gamma - 1}} \tan^{-1} \left( \sqrt{\frac{\gamma - 1}{\gamma + 1} (M^2 - 1)} \right) \tan^{-1} \left( \sqrt{M^2 - 1} \right)$$
 (4)

### 4.1 Working Procedure

If you know  $M_1$  &  $\theta$  know you can use equation (3) to solve for  $M_2$  using Newton Raphson iteration scheme as described below:

$$M_2 = M_2 - \frac{f(M_2)}{f'(M_2)} \tag{5}$$

$$f(M_2) = \sqrt{\frac{\gamma + 1}{\gamma - 1}} \tan^{-1} \left( \sqrt{\frac{\gamma - 1}{\gamma + 1} (M^2 - 1)} \right) - \tan^{-1} \left( \sqrt{M^2 - 1} \right) - \theta - \nu(M_1)$$
 (6)

$$f'(M_2) = \frac{M_2}{\sqrt{M_2^2 - 1} \left(1 + \frac{\gamma - 1}{\gamma + 1} \left(M_2^2 - 1\right)\right)} - \frac{1}{M_2 \sqrt{M_2^2 - 1}}$$
(7)

After you get  $M_2$  you can get the pressure and temperature using the isentropic relations:

$$\frac{T_2}{T_1} = \frac{1 + \frac{\gamma - 1}{2} M_1^2}{1 + \frac{\gamma - 1}{2} M_2^2} \tag{8}$$

$$\frac{P_2}{P_1} = \left(\frac{T_2}{T_1}\right)^{\frac{\gamma}{\gamma - 1}} \tag{9}$$

$$\frac{\rho_2}{\rho_1} = \left(\frac{T_2}{T_1}\right)^{\frac{1}{\gamma - 1}} \tag{10}$$

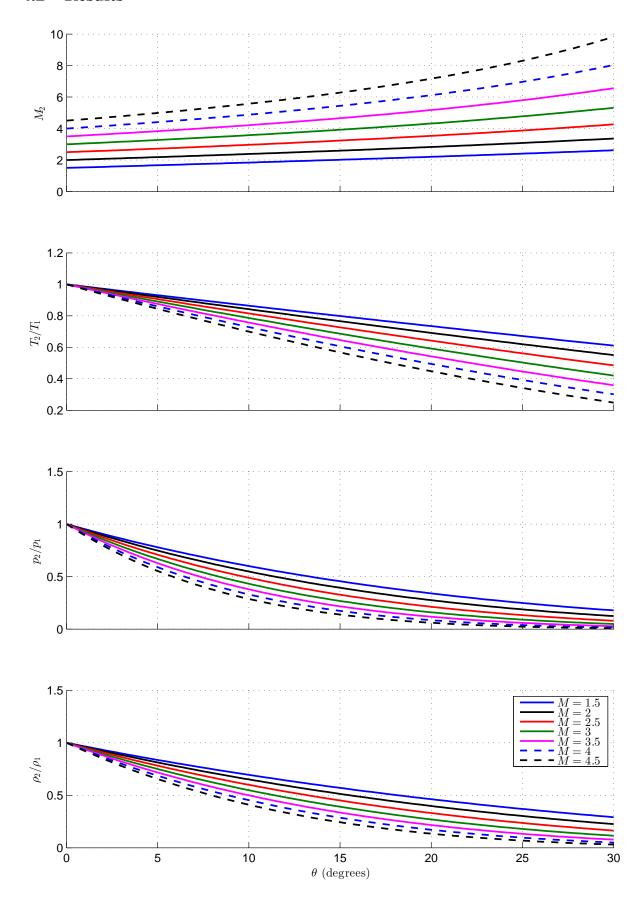
$$P_{01} = P_{02} = P_0 \tag{11}$$

function  $M_2o.m$  [code 2] illustrates this procedure. Report6\_main.m [code 1] was used to plot the figures shown in section 4.2.

هذا هو بعض الكلمات العربية في سطر انجليزي. This is a line with some Arabic words و هذا سطر عربي به بعض الكلمات الانجليزية.Thus is some English words in an Arabic. و هذا سطر عربي به بعض الكلمات الانجليزية

و بناءً عليه فقد تم اثبات المطلوب. و بناءً عليه فقد تم اثبات المطلوب.

## 4.2 Results



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The following table is just for demonstration. It doesn't provide any useful information.

$Angle(\theta)$	Temperature	Pressure	Density
0	123	111	444
20	500	222	444
40	640	222	444
90	200	222	444

### 5 Conclusion

We see that we can still obtain solutions for  $M_2$  for  $\theta>90^\circ$ . But, however I think the solutions for  $\theta>90^\circ$  aren't practical.

Pressure, temperature & density increases as the kinetic energy increase as in [1].

# **Appendices**

#### A Matlab Codes

### Code 1: Report6\_main.m

```
1 clc
 2 close all
   clearvars
 3
4
 5
  set(groot, 'DefaultAxesColorOrder', [0,0,1;0,0,0;1,0,0;0,0.5,0;1,0,1])
   set(groot, 'DefaultAxesLineStyleOrder', '-|--|-.')
6
7
   set(groot, 'DefaultLineLineWidth', 1.5);
8
9
   gamma=1.4;
10
   theta_d_vec=linspace(0,30);
11
12 \parallel M_1_{vec=1.5:.5:4.5}
13
  yAxesTitles_tex={'M_{2}','T_{2}/T_{1}','p_{2}/p_{1}','\rho_{2}/\rho_{1}'
14
   for ii=1:length(yAxesTitles_tex)
15
16
       subplot(length(yAxesTitles_tex),1,ii);
17
       hold on;
18
       grid;
19
       if ii~=length(yAxesTitles_tex), set(gca,'XTickLabel',[]);end
20
21
       ylabel(['$',yAxesTitles_tex{ii},'$'],'interpreter','latex');
22
   end
23
   xlabel('$\theta$<sub>□</sub>(degrees)', 'interpreter', 'latex'); %This applies to the
       bottom subplot
24
25
   M_1_legend_tex=cell(length(M_1_vec),1);
   |ii=1;
26
27
   for M_1=M_1_vec
28
       M_2=M_2o(M_1,theta_d_vec,gamma);
29
       subplot(4,1,1);plot(theta_d_vec,M_2);
30
31
       T_2T_1=(1+(gamma-1)/2*M_1*M_1)./(1+(gamma-1)/2*M_2.^2);
32
       subplot(4,1,2);plot(theta_d_vec,T_2_T_1);
33
34
       p_2_p_1=T_2_T_1.^(gamma/(gamma-1));
35
       subplot(4,1,3);plot(theta_d_vec,p_2_p_1);
36
37
       rho_2_rho_1=T_2_T_1.^(1/(gamma-1));
```

```
38
       subplot(4,1,4);plot(theta_d_vec,rho_2_rho_1);
39
40
       M_1_legend_tex{ii}=['$M=',num2str(M_1),'$'];
41
42
       ii=ii+1;
43
   end
44
   legend(M_1_legend_tex, 'interpreter', 'latex');
45
46
47
   set(groot, 'DefaultAxesColorOrder', 'remove')
   set(groot, 'DefaultAxesLineStyleOrder', 'remove')
48
49
   set(groot, 'DefaultLineLineWidth', 'remove');
50
   export_figure(gcf,'||',{'figures'},[],{'pdf','emf'})
51
```

#### Code 2: function M\_2o.m

```
1 | function M_2_vec=M_2o(M_1,theta_d_vec, ...
                       gamma) %optional arguments
2
  if nargin<3
3
       gamma=1.4;
4
5
   end
6
7
   theta_vec=deg2rad(theta_d_vec);
   n1=sqrt((gamma+1)/(gamma-1))*atan(sqrt((gamma-1)/(gamma+1)*(M_1^2-1)))-
8
      atan(sqrt(M_1^2-1));
9
   %Newton Raphson iteration
10
   M_2_vec=1.1*ones(size(theta_d_vec));
11
   for ii=1:length(theta_d_vec)
12
13
       f=sqrt((gamma+1)/(gamma-1))*atan(sqrt((gamma-1)/(gamma+1)*(M_2_vec(ii)
          ^2-1)))-atan(sqrt(M_2_vec(ii)^2-1))-theta_vec(ii)-n1;
       fdash=1/(M_2vec(ii)^2-1)^(1/2)*M_2vec(ii)/(1+(gamma-1)/(gamma+1)*(
14
          M_2_{vec(ii)^2-1)}-1/(M_2_{vec(ii)^2-1)^(1/2)/M_2_{vec(ii)};
15
       M_2_n=M_2_vec(ii)-f/fdash;
16
       while abs(M_2_vec(ii)-M_2_n)>=100*eps "This is dangerous. Infinte loop
17
           can occur!!
          M_2_vec(ii)=M_2_n;
18
          f=sqrt((gamma+1)/(gamma-1))*atan(sqrt((gamma-1)/(gamma+1)*(M_2_vec
19
              (ii)^2-1)))-atan(sqrt(M_2_vec(ii)^2-1))-theta_vec(ii)-n1;
          fdash=1/(M_2vec(ii)^2-1)^(1/2)*M_2vec(ii)/(1+(gamma-1)/(gamma+1)
20
              *(M_2\_vec(ii)^2-1))-1/(M_2\_vec(ii)^2-1)^(1/2)/M_2\_vec(ii);
21
          M_2_n=M_2_vec(ii)-f./fdash;
22
       end
       M_2_{vec(ii)}=M_2_n;
23
```

24 | end

## References

- [1] J. D. Anderson, Modern Compressible Flow, McGraw-Hill, New York, 1990.
- [2] Report (1).
- [3] Report (3).