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[ctb] count NilT = 0
[ctr] count (Node n x y) = (count x) + (count y) + 1
[htb] height NilT = 0
[htr] height (Node n x y) = (max (height x) (height y)) + 1
[mc1] |a >= b = a
[mc2] | otherwise = b
[I.H.t1.] count t1 $\leq 2^{(\text{height t1})} - 1$
[I.H.t2.] count t2 $\leq 2^{(\text{height t2})} - 1$

B.C.
count NilT $\leq 2^{(\text{height NilT})} - 1$

$0 \leq 2^{(\text{height NilT})} - 1$ by [ctb]
$0 \leq 2^{0} - 1$ by [htb]
$0 \leq 1 - 1$ by [exponent rule]
$0 \leq 0$ by [arithmetic]
holds.

W.T.S.
count (Node T t1 t2) $\leq 2^{(\text{height (Node T t1 t2)})} - 1$
(count t1) + (count t2) + 1 = $2^{(\text{height (Node T t1 t2)})} - 1$ by [ctr]
$2^{(\text{height t1})} - 1 +$(count t2) + 1 = $2^{(\text{height (Node T t1 t2)})} - 1$ by [I.H.t1.]
$2^{(\text{height t1})} - 1 + 2^{(\text{height t2})} - 1 + 1 = 2^{(\text{height (Node T t1 t2)})} - 1$ by [I.H.t2.]
$2^{(\text{height t1})} + 2^{(\text{height t2})} - 1 = 2^{(\text{height (Node T t1 t2)})} - 1$ by [arithmetic]

CaseI: t1≥t2
$2^{(\text{height t1})} + 2^{(\text{height t2})} - 1 = 2^{(\text{height t1})+1} - 1$ by[mc1]
$2^{(\text{height t1})} + 2^{(\text{height t1})} - 1 = 2^{(\text{height t1})+1} - 1$ by[logic] see the note below
$2*2^{(\text{height t1})} - 1 = 2^{(\text{height t1})+1} - 1$ by[factoring the 2s]
$2^{1}*2^{(\text{height t1})} - 1 = 2^{(\text{height t1})+1} - 1$ by[exponential rule]
$2^{(\text{height t1})+1} - 1 = 2^{(\text{height t1})+1} - 1$ by[exponential rule]

Note for CaseI:
if we now for a fact that t1≥t2 then from this it follows that
$2^{(\text{height t1})} + 2^{(\text{height t1})} - 1 \geq 2^{(\text{height t1})} + 2^{(\text{height t2})} - 1$.
Therefore in the L.H.S. we can substitute this $2^{(\text{height t1})} + 2^{(\text{height t2})} - 1$ with that $2^{(\text{height t1})} + 2^{(\text{height t1})} - 1$.

CaseII: t1<t2
$2^{(\text{height t1})} + 2^{(\text{height t2})} - 1 = 2^{(\text{height t2})+1} - 1$ by[mc2]
$2^{(\text{height t2})} + 2^{(\text{height t2})} - 1 = 2^{(\text{height t2})+1} - 1$ by[logic] see the note below
$2*2^{(\text{height t2})} - 1 = 2^{(\text{height t2})+1} - 1$ by[factoring the 2s]
$2^{1}*2^{(\text{height t2})} - 1 = 2^{(\text{height t2})+1} - 1$ by[exponential rule]
$2^{(\text{height t2})+1} - 1 = 2^{(\text{height t2})+1} - 1$ by[exponential rule]

Q.E.D.

Note for CaseII:
if we now for a fact that $t1<t2$ then from this it follows that
$2^{(\text{height } t2)} + 2^{(\text{height } t2)} - 1 \geq 2^{(\text{height } t1)} + 2^{(\text{height } t2)} - 1$.
Therefore in the L.H.S. we can substitute this $2^{(\text{height } t1)} + 2^{(\text{height } t2)} - 1$
with that $2^{(\text{height } t2)} + 2^{(\text{height } t2)} - 1$.