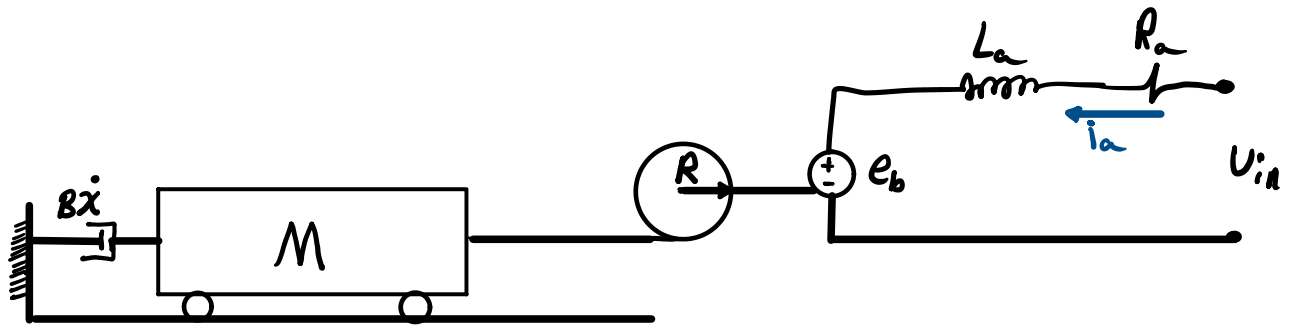


Mathematical model of the system



Given:

$$R = 1.1 \text{ cm} = 0.011 \text{ m}$$

$$L_a = 0.01 \text{ H}$$

$$V_{in} = 12 \text{ V}$$

$$R_a = \frac{V_{in}}{I_{stall}} = \frac{12}{1.5} = 8 \Omega$$

$$M = 107 \text{ gm} = 0.107 \text{ kg}$$

$$J_r = 0.0002 \text{ kg} \cdot \text{m}^2$$

$$K_t = 0.494 \text{ N} \cdot \text{m} / \text{A}$$

$$b = 5 \times 10^{-6} \frac{\text{N} \cdot \text{m}}{\text{r/s}}$$

$$\text{Friction} = 7.92 \text{ N}$$

For electrical system:

$$(R_a + \cancel{L_a s}) I(s) + e_b(s) = V_{in}(s)$$

neglect

$$e_b(s) = K_b s \theta_m(s)$$

$$T_m = K_t I(s)$$

$$\frac{R_a}{K_t} \times T_m(s) + K_b s \theta_m(s) = V_{in}(s)$$

$$16.19 T_m(s) + 0.494 s \theta_m(s) = V_{in}(s)$$

→ ①

For mechanical system:

$$J\ddot{\theta}_2 = T_{m4} - b\dot{\theta}_2$$

$$0.0002 \ddot{\theta}_2 = T_m - 5 \times 10^{-6} \dot{\theta}_2$$

$$T_m = 2 \times 10^{-4} \ddot{\theta}_2 + 5 \times 10^{-6} \dot{\theta}_2 \rightarrow \textcircled{2}$$

Sub 2 in 1

$$V_{in}(s) = 3.24 \times 10^{-3} \ddot{\theta}_2 + 8.1 \times 10^{-5} \dot{\theta}_2 + 0.494 \times \dot{\theta}_m$$

\leadsto Assume $\frac{N_1}{N_2} = 1$, $\theta_1 = \theta_2$

$$V_{in}(s) = 3.24 \times 10^{-3} \ddot{\theta}_m + 0.494 \dot{\theta}_m$$

$$V_{in}(s) = [3.24 \times 10^{-3} s^2 + 0.494 s] \theta_m(s)$$

$$\frac{\theta_m(s)}{V_{in}(s)} = \frac{1}{3.24 \times 10^{-3} s^2 + 0.494 s}$$

$$\frac{W(s)}{V_{in}(s)} = \frac{1}{3.24 \times 10^{-3} s + 0.494}$$