1 Conditional Dimension

Knowing a set of quadratic forms, what can be said about the underlying Gaussians? What is the dimension of their semi-algebraic set? When is this set a manifold?

2 Kuonen's Saddlepoint Approximation Zero Argument

Kuonen gave a saddlepoint approximation for the CCDF of a linear combination of chi-square variables. Recall that any central quadratic form can be expressed as a linear combination of chi-square variables, as well as any quadratic expression with a non-singular covariance matrix. Suppose that a quadratic form/expression is equal in distribution to a linear combination of chi-squares as follows:

$$Y = \sum_{i=1}^{N} \lambda_i \chi^2_{\nu_i}(h_i^2)$$

Then the cumulant generating function $K(\zeta)$ is defined for $\zeta < \frac{1}{2} min_i \lambda_i^{-1}$ and is given by:

$$K(\zeta) = -\frac{1}{2}\sum_{i=1}^{N} \nu_i \log(1 - 2\zeta\lambda_i) + \sum_{i=1}^{N} \frac{h_i^2 \lambda_i}{1 - 2\zeta\lambda_i}$$

Then the CCDF is approximated by:

$$P(Y > q) \approx 1 - \Phi \left\{ w + \frac{1}{w} \log\left(\frac{v}{w}\right) \right\}$$

where:

$$\begin{aligned} w &= \text{sign}(\hat{\zeta})[2\{\hat{\zeta}q - K(\hat{\zeta})\}]^{1/2} \\ v &= \hat{\zeta}\{K''(\hat{\zeta})\}^{1/2} \end{aligned}$$

Φ is the CDFof the standard normal distribution, and $\hat{\zeta}$ is the unique solution of the equation $K'(\zeta) = q$. Then $\hat{\zeta}$ is a function of q. A direct computation yields

$$K'(\zeta) = \sum_{i=1}^{N} \frac{\lambda_i \nu_i}{1 - 2\lambda_i\zeta} + 2\sum_{i=1}^{n} \frac{h_i^2 \lambda_i^2}{(1 - 2\lambda_i\zeta)^2}$$

and

$$K''(\zeta) = 2\sum_{i=1}^{N} \frac{\lambda_i^2 \nu_i}{(1 - 2\lambda_i\zeta)^2} + 8\sum_{i=1}^{N} \frac{h_i^2 \lambda_i^3}{(1 - 2\lambda_i\zeta)^3}$$

If $\hat{\zeta}$ is also a root of $\zeta q - K(\zeta)$, the method collapses.

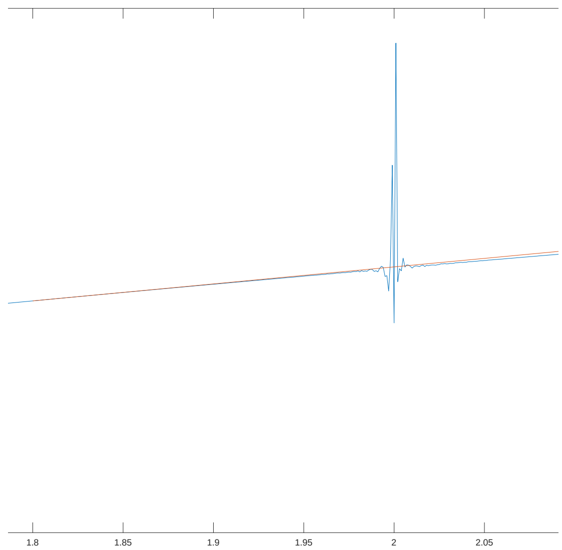

Figure 1: Argument by Kuonen and Taylor

This occurred for the first central form provided by Kuonen at $q = 2$.

Using the Taylor expansion at $q_0 = 1.8$, we can estimate the argument at $q = 2$ and have a meaningful result, as shown in the following figure 1:

For the non-central case, it covers an interval. It may render the methopd useless.

For the non-central form defined by $\lambda = [0.6, 0.3, 0.1]; \nu = [1, 2, 1]; h = [0.1, 0.2, 0.2];$, we obtain the following CDF

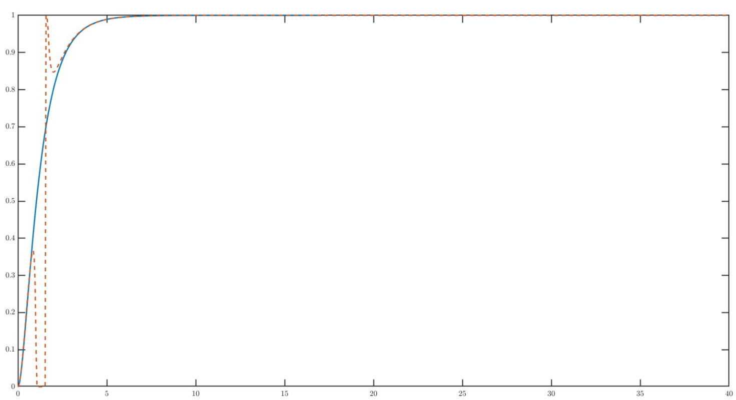

Figure 2: MC vs Kuonen CDF