ECE 484
Digital Control Systems
Ball and Beam Lab Project
Student Handout

CONTENTS

LAB SAFETY ORIENTATION

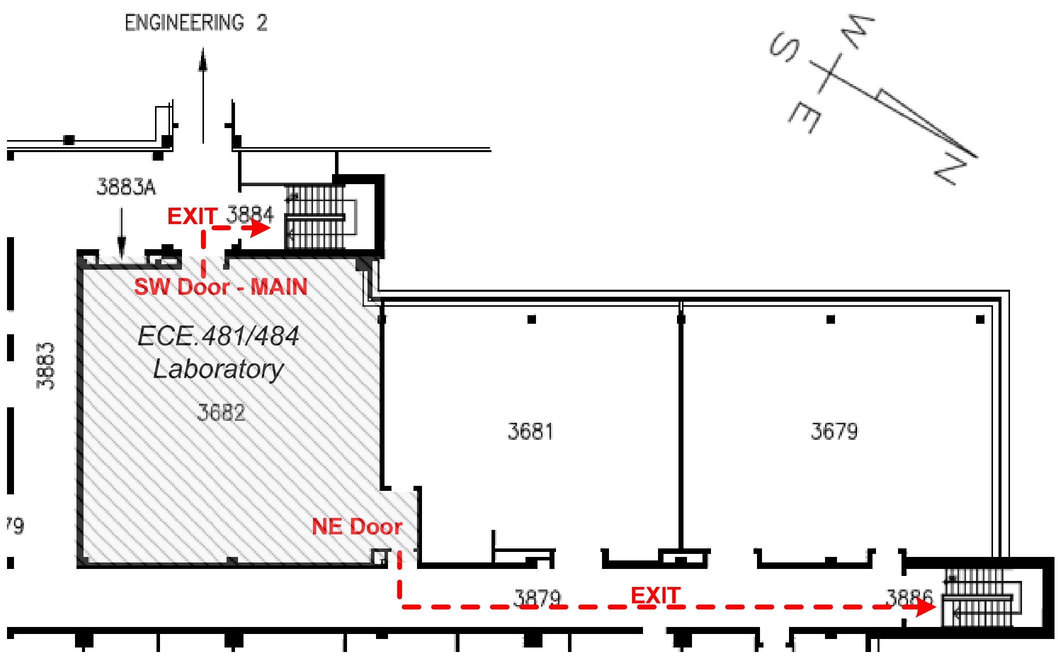
Under the Occupational Health and Safety Act and uWaterloo's Health Safety & Environment Management System (HSEMS), supervisors, i.e., Lab Instructor and Teaching Assistants, are responsible for providing a safe work environment in the lab, and Students are responsible for working in compliance with the regulations outlined in the Lab Safety Orientation and also for reporting any contravention of OHSA or lab hazard of which he/she knows.

Planning for Emergencies

Everyone in the lab should know what to do in emergency situations. Step-by-step emergency procedures, **First Aid, Fire/Evacuation, Lockdown, are posted on the inside of the laboratory door**. Please take the time to familiarize yourself with each procedure.

FIRE

The CPH-3682 lab, depicted below, can be accessed through the **main** door which has a combo lock. Students registered in this course are provided with the access code.



Two fire extinguishers exist in this lab, one by each door. In case of emergency, either door can be used to exit the lab. Follow the Fire/Evacuation procedure.

PHYSICAL INJURIES - A First Aid kit exists in the lab, and it is located by the main door.

LOCKDOWN - Follow the Lockdown procedure.

WatSAFE – a free safety and emergency notification app that enhances existing emergency notification systems in place across campus. Its alert and push notification features allow you to stay informed of major campus emergency events.

CPH-3682 is an open-lab with 24/7 access, which means that students will work unsupervised in the lab most of the time. Names and phone numbers of the relevant contacts for a lab emergency are posted on the main door.

Specific Safety Aspects for the ECE.484 Laboratory Project

a) Understand the hardware setup of the 484 lab project, which is presented in the lab manual, and also introduced during the GWiz Tutorial.
b) The GWiz Overview and Getting Started with GWiz files are available on LEARN.
c) Before starting your lab work, assess your surroundings. Report to the Lab Instructor any unsafe conditions you may notice.
d) Do not run cables on the floor or string them across walkways from bench to bench.
e) Work deliberately and carefully on your project. Always ask when in doubt.
f) **No food or drinks are allowed in the lab**. Spills on live equipment can

jeopardize your safety by causing the wet surroundings to become live/energized.

g) The experiment is powered through a power bar located within arm's reach from the control computer. In case of emergency, cut off power to the experiment from the power bar.
h) Keep fingers, hair, ties, etc. away from the gears of the lab apparatus.

General Safety Precautions

Electricity can kill or severely injure people and cause damage to property. The following provides some basic measures to help you control the risks from your use of electricity.

a) Never work alone.
b) Pay 100% attention to the work you are doing.
c) Connect the power source **last**.
d) Turn the power off before you start working on a circuit.
e) Check circuit power supply voltages for proper value and for type (ac/dc, frequency) before energizing the circuit.
f) Wear personal protective equipment adequate for the work you do.
g) If you are working with exposed circuits:
    - Remove conductive bracelets, wristwatches, chains, rings, etc.
    - Do not use metallic pencils, metal rulers

NOTES ON CHEATING, PLAGIARISM, AND OTHER FUN THINGS

Occasionally there has been confusion about what is allowed and what is not allowed in labs. To make things clear, the following rules apply to ECE 484. These rules have been created for one simple reason: so that each of you *learns* something about control engineering, and you will best learn by doing the work yourself. It is farcical having a lab where the only person who actually did the work is some guy from 1998, while everyone else copies his report. So, please note that:

1. You are responsible for knowing what constitutes an "academic offense" according to Policy 71 of the university. The complete policy is available at https://uwaterloo.ca/secretariat-general-counsel/policies-procedures-guidelines/policy-71. In particular, according to the policy:
    a. Cheating is an academic offence. Cheating includes copying from another student's work or allowing another student to copy from one's own work, submitting another person's work as one's own, fabrication of data, and use of unauthorized aids.
    b. Plagiarism (the act of presenting the ideas, words or other intellectual property of another as one's own) is an academic offence. The use of other people's work must be completely and unambiguously acknowledged and referenced in all written material, including laboratory reports and computer programs.

2. You and your lab partner should work together on all aspects of the lab, but you are not allowed to do significant work with any other students in the class. For example, you may talk with students in other groups about minor aspects of the lab ("I forgot—how do you start GWiz?"; "Where did you find the published value of the ball's mass?"), but you are not allowed to write up lab reports jointly with other teams or to share computer code in any way.

3. Do not obtain or look at lab reports (either in hardcopy or softcopy) written by students in another group, whether they are taking ECE484/481 presently or have taken it in a previous term.

4. In your report, you must completely and unambiguously acknowledge and reference any person, website, report, book, or notes that you used to help you with your work. You should reference this lab manual and the course notes, for example. If you violated rule 2 or rule 3, in your lab report you must fully acknowledge the help you received.

5. You must not let any student, except for your lab partner, access any part of your lab report (either in hardcopy or softcopy).

6. You must include, and sign, one of the following statements in your report:
    a. "The authors of this report declare that, in doing the lab work and writing up the lab report for ECE484, we followed rules 2, 3, 4, and 5 described at the beginning of the lab manual."
    b. "The authors of this report declare that, in doing the lab work and writing up the lab report for ECE484, we violated at least one of rules 2, 3, 4, or 5. These violations are completely and specifically described as follows: …"

Lab Overview

This project deals with a classical control experiment where the objective is to position a ball on a beam by appropriately inclining the beam. Figure 1 shows the ball and beam apparatus that we will be using in this lab. As seen in the figure, a DC motor is used to control a lever arm, which in turn is used to raise or lower the right side of the beam to cause the ball to move. The beam itself consists of two parallel rods, which together with the ball form a potentiometer used to measure the ball's position. One of the rods has a resistive film glued onto it, while the other rod and the ball form the wiper of the potentiometer. The left side of the beam is pivoted on a support. The input to the ball and beam apparatus is the motor voltage; both the ball position and the motor gear angle are measurable outputs.

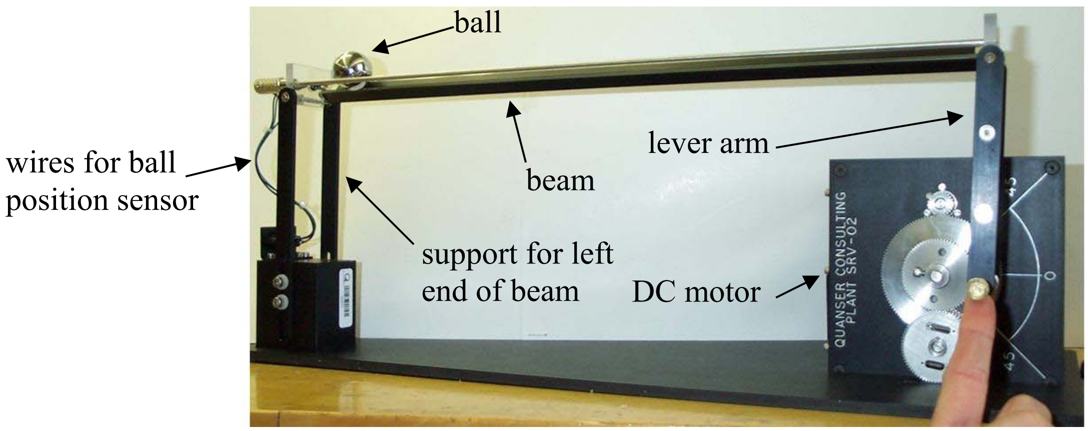


FIGURE 1: The plant: a "ball and beam" apparatus.

Figure 2 shows the main connections between the ball and beam apparatus, control computer (cRIO) and the lab Nexus computer. The cRIO has an FPGA, a real-time processor, an analog-to-digital converter and a dc servo motor drive and amplifier.

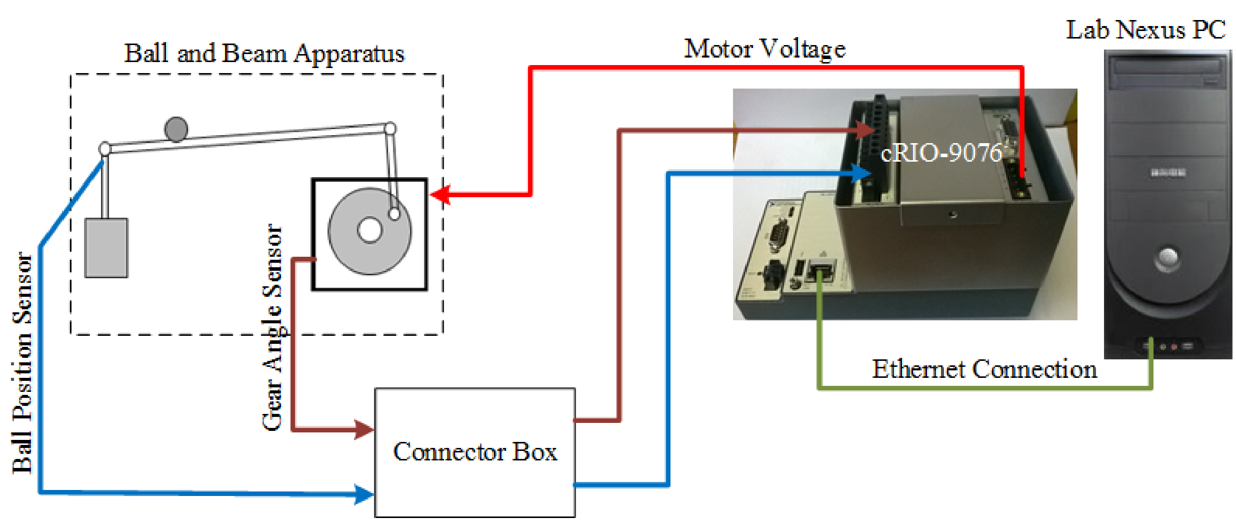


FIGURE 2: Key connections between the ball and beam apparatus, cRIO and lab Nexus computer

An application written in LabVIEW, GWiz, is used to access the measured signals and to set the desired motor voltage.
The main user interaction with GWiz is through a virtual instrument (vi) named cRIO-9076_RT.vi, shown in Figure 3. This vi is customizable; you can add or modify controls and indicators to suit your needs. Files giving users a tutorial on LabVIEW, an overview of GWiz, and how to get started, are provided on LEARN.

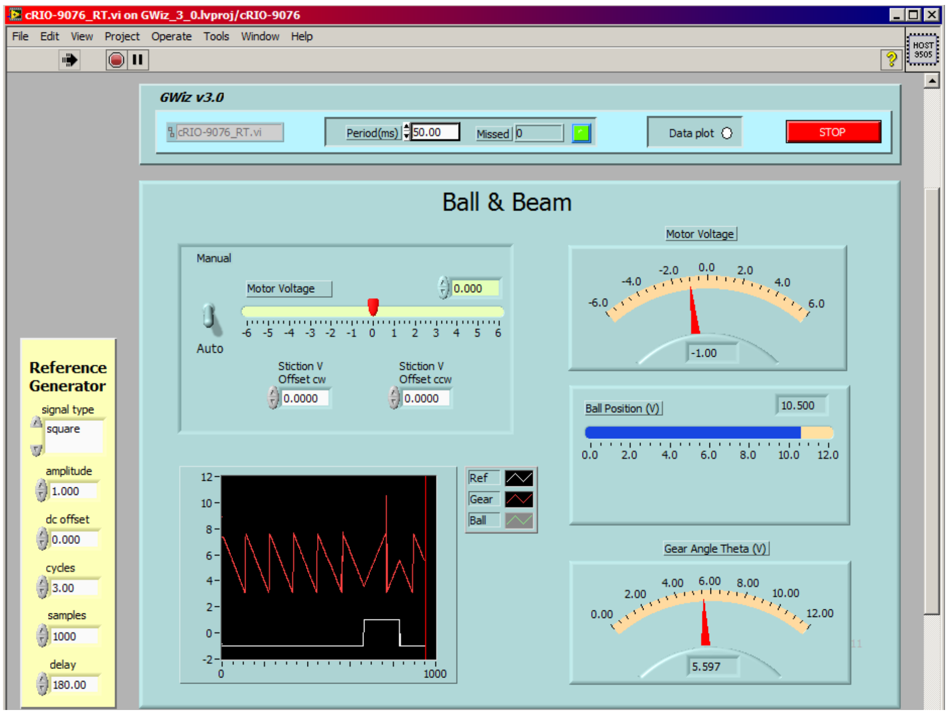


FIGURE 3: Front panel of cRIO-9076_RT.vi

Before attempting to control the ball and beam system, it is important to consider the control structure; that is, decide if a single control loop or an inner/outer-loop control structure is best, and consider what type of compensation is reasonable. As we will show later, the essential linearized ball and beam dynamics are summarized as follows:

$$\frac{\theta(s)}{V(s)} = \frac{K_1}{s(\tau s + 1)} \qquad \ldots(1)$$

$$\frac{\phi(s)}{\theta(s)} = K_2 \qquad \ldots(2)$$

$$\frac{Y(s)}{\phi(s)} = \frac{K_3}{s^2} \qquad \ldots(3)$$

where signals $V(t)$, $\theta(t)$, $\phi(t)$, and $y(t)$ (and some key geometric variables) are shown in Figure 4. The parameters K_1, K_2, K_3 and τ will be determined later. The datasheet of the ball and beam apparatus is available in Appendix 1.

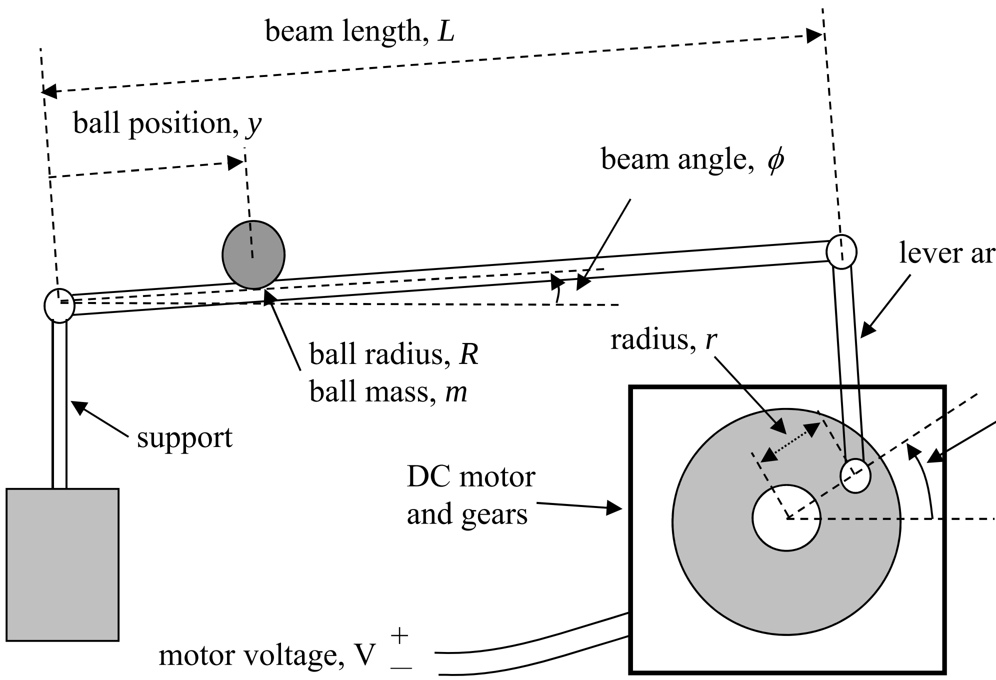


FIGURE 4: Geometry of the ball and beam system.

The simplest control topology is a single feedback control loop shown in Figure 5, where the plant is treated as the cascade connection of (1), (2), and (3). Although it is possible to design the compensator $C(s)$ such that the closed-loop system is stable, the triple integrator in the plant contributes 270° phase lag to the loop gain making it difficult to obtain "good" closed-loop performance. Another practical difficulty with this setup is that it is not clear how one can ensure that the motor does not rotate so much as to damage the lever arm mechanism. To avoid damage, you must ensure that $-\frac{\pi}{4}\,\text{rad} < \theta(t) < \frac{\pi}{4}\,\text{rad}$.

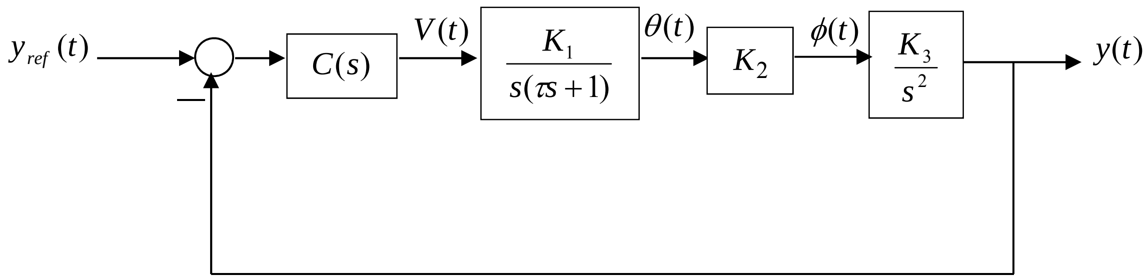


FIGURE 5: A single feedback control loop for the linearized plant. Note that a continuous-time controller is used in this diagram even though a sampled-data controller would be used in the end.

An alternative control arrangement is shown in Figure 6. In this setup, two feedback loops are used. The purpose of the inner loop is to control the motor gear angle position; controller $C_1(s)$ should be designed so that $\theta(t)$ tracks the reference signal $\theta_{ref}(t)$. The outer loop uses the inner feedback loop to control the ball position. From the point of view of controller $C_2(s)$, the "plant" is composed of everything enclosed by the dotted box in Figure 6. Clearly the inner loop must be designed before the outer loop. We will also see that the inner/outer loop control setup in Figure 6 avoids the two problems associated with the single loop control configuration in Figure 5.

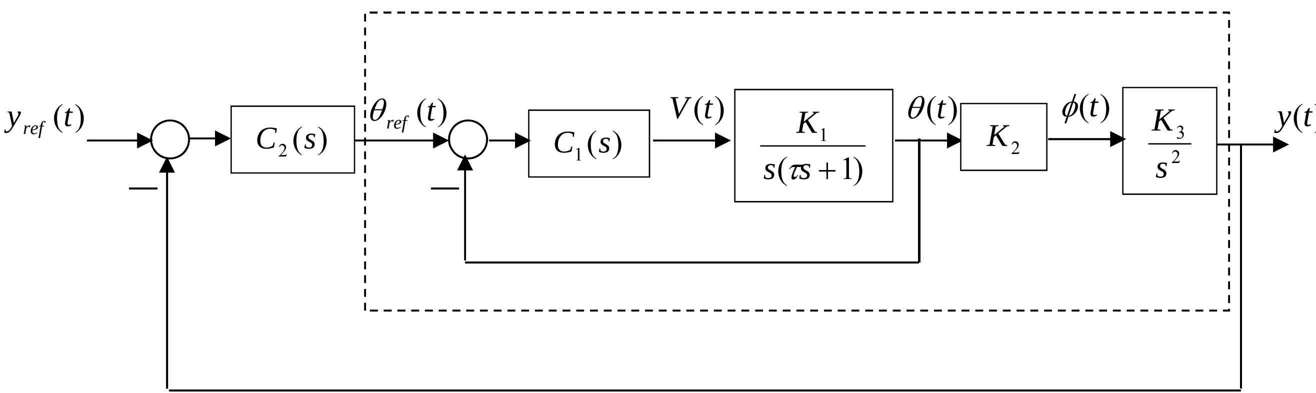


FIGURE 6: Inner/outer loop control configuration for the linearized plant. As in Figure 5, continuous-time controllers are used in this diagram.

GENERAL REPORT REQUIREMENTS

Write up your reports applying the following requirements:

- Submit your report to the appropriate drop-box on LEARN.
- Include and sign one of the statements from “rule 6” on page 4 of this lab manual.
- At the beginning of the report, specify the station number you have used. This should be the same as the station assigned to your group.
- Include introduction and conclusions sections.
- Your report should be self-contained, and complete. Be sure to comment on every lettered section from the lab manual.
- Be sure to include enough data and information so that your results are reproducible.
- The report should use a font no smaller than 12pt Times New Roman, and single line spacing.
- Your report must be typed, although you can write in equations if your handwriting is very neat. Handwriting the entire analysis is not acceptable.
- Always point out unexpected results and provide possible explanations. In general, your actual system data will probably not match the simulated data exactly. Discuss any differences and indicate where they came from, whether or not they are significant enough to pose a problem, and, if appropriate, what you could do to mitigate them. This should be *qualitative* analysis, not quantitative. *Do not* include pages of percent error calculations.
- Discuss the validity of any assumptions you made.
- Clearly define all variables in your report.
- Always indicate units when giving numerical data.
- Indicate performance measurements on your plots using cursors and/or labels.
- Formatting and presentation are important. Make your report readable and easy to follow. For example, do not break tables across pages, if possible, and do not put all figures at the end of your report since this is very tedious for the reader.
- Quality matters more than quantity. Do not put “filler” in your report.

LAB 1: MODELING AND VALIDATION OF THE MOTOR SYSTEM PLANT

For Lab 1, we focus on the inner loop in Figure 6, where the motor gear angle is controlled. The loop is duplicated in Figure 7(a); Figure 7(b) shows the later addition of a saturator to limit the motor operating angle range needed when connecting the beam; Figure 7(c) shows the discretized controller acting on the continuous plant.

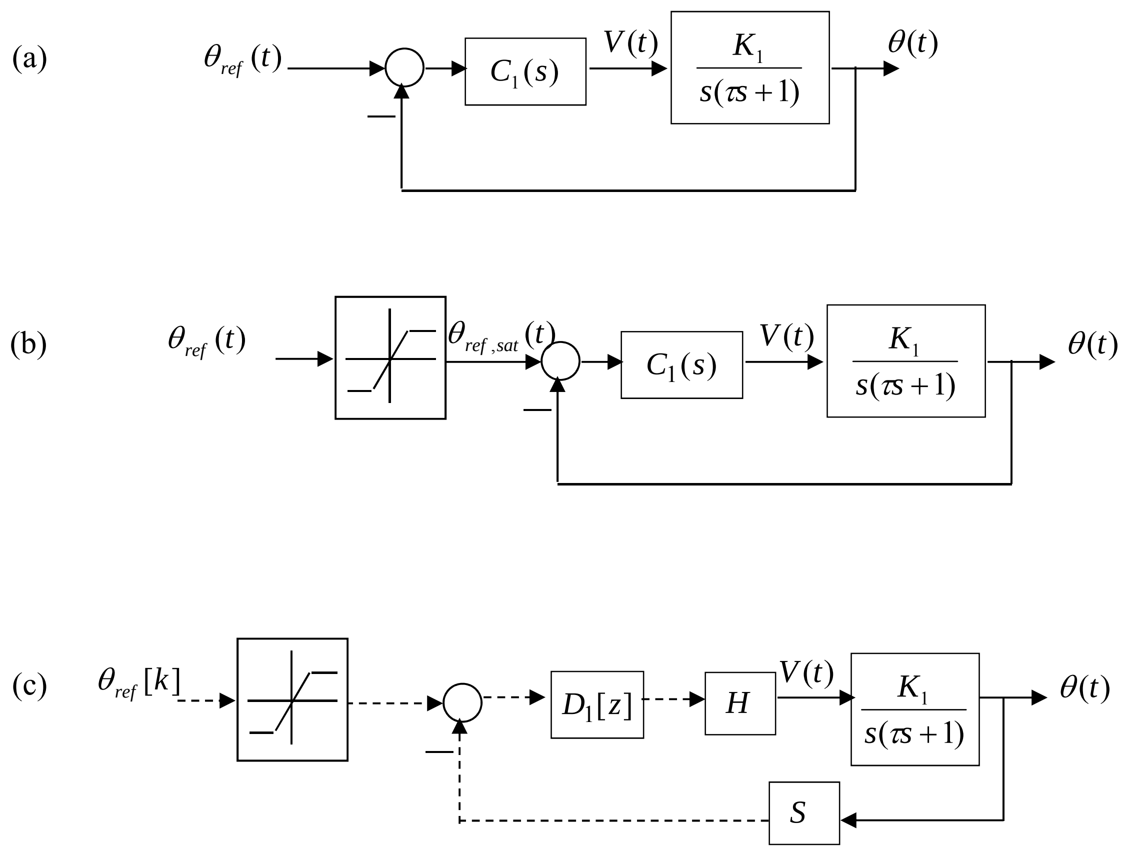


FIGURE 7: (a) continuous-time inner loop, (b) continuous-time inner loop with a saturator (c) sampled-data inner loop with a saturator (the dashed arrows represent discrete-time signals, the *S* block represents an ideal sampler, and the *H* block represents an ideal zero-order hold)

(a) Familiarize yourself with the GWiz application. From GWiz_3_0.lvproj open cRIO-9076_RT.vi. It is strongly suggested that BOTH partners compile the files on their account, and keep a copy of ongoing project work, as a measure of redundancy, and availability of files for submissions and demo.
Warning: Make sure that the beam is not attached to the gear at this point. You need a working controller for the inner loop to avoid damaging the (surprisingly expensive) ball and beam apparatus!

(b) Check out your experimental station. Figure 8 shows a detail of the motor system plant and how to correctly measure the gear angle θ.

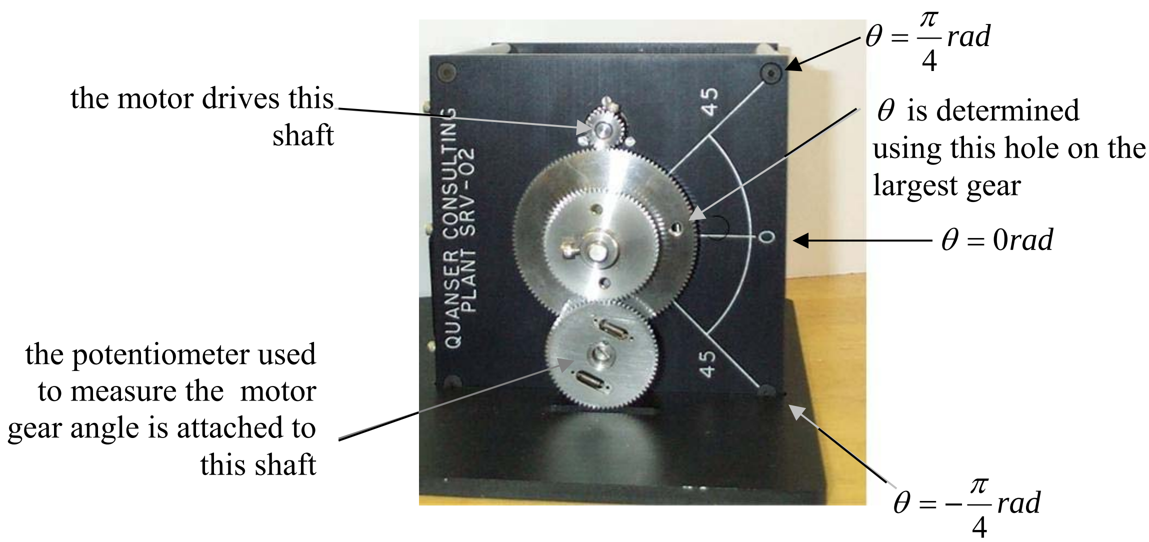


FIGURE 8: The motor system plant for Lab 1

(c) The motor gear angle is measured using a potentiometer. It is necessary to convert the potentiometer voltage to the corresponding gear angle in **radians**. This relationship is nonlinear due to an offset, as shown in Figure 9. You should invert this nonlinearity in software (referred to as "sensor scaling" in cRIO-9076_RT.vi), so that the overall system is linear. To implement the gear angle scaling, first experimentally determine the gain and offset for your apparatus. Then, write the necessary code in the formula node of the cRIO-9076_RT.vi file to compute the gear angle in radians.

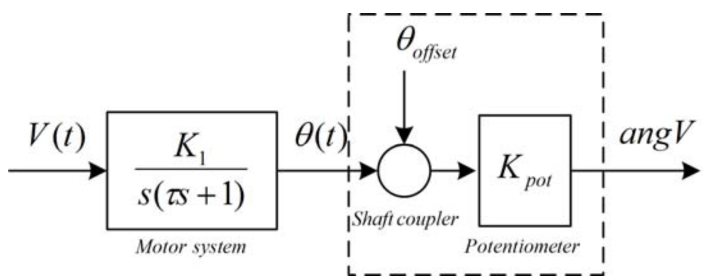


FIGURE 9: Nonlinearity introduced by servo-angle potentiometer

(d) Any motor has some amount of static friction, often called "stiction". From a control point of view, stiction is undesirable since it can make fine motor motions very difficult. One strategy to deal with stiction is to (approximately) cancel it out:

- Experimentally determine the motor voltage required to overcome stiction. The following tips may help:

- Be sure that the “Manual/Auto” toggle switch on the cRIO-9076_RT.vi front panel is set to “Manual” so that you can manually adjust the motor voltage.
- The motor stiction offset should be selected so that, ideally, the dead zone is completely eliminated.
- Stiction may be nonsymmetrical, so you need to determine the stiction offset for each direction: clockwise and counter-clockwise.

- A simple stiction compensator is provided in the block diagram of cRIO-9076_RT.vi.

In Lab 1 report, explain briefly how you determined suitable stiction offset values.

From now on, it will be assumed that the stiction compensation is implemented in your experimental setup.

(e) If you are not familiar with DC motor models, consult an undergraduate control textbook to verify that (1) is a standard model.
For the DC motor model shown in equation (1), the following *nominal* parameter values are provided:

$$K_1 = 1.53\ \mathrm{rad/(V \cdot s)}$$
$$\tau = 0.025\mathrm{s}$$

Using an experimental method, determine the specific values of K_1 and τ for your lab apparatus. Note the following:

- The plant is open-loop unstable, and therefore it is difficult to model the plant directly. Instead, stabilize the plant (how?), and use system identification techniques (e.g., use step response overshoot and time to first-peak to deduce the unknown system parameters, or perhaps a frequency response) to determine a model of the *closed-loop* system. Then use these results to compute K_1 and τ. Pay special attention to signs, as some values may be negative.
- With a stable inner-loop, it is a good opportunity to verify that the stiction value you selected in (d) is adequate: the stiction offset should be high enough so that the step response first peak is well rounded with no flat portions, but low enough that the step response settles to its steady-state value with no gear oscillations. Another check is that there should be little or no steady-state error.
- Be sure to set the sampling rate appropriately. If you do not explicitly account for the finite sampling rate in your modeling calculations, then set the sampling rate to be sufficiently fast (1,000Hz) when obtaining data. In your report, indicate what sampling rate you used and why.
- Use several sets of data and different system setups, e.g., different controller gains, to increase the probability that your parameter estimates are accurate.
- Be sure that you do not saturate the motor voltage by applying more than ±*6V*. Include in your report a plot of the motor voltage along with the plot of signals used for system modeling, as evidence that you have avoided saturation.

(f) To help prevent damage to the experimental apparatus when the beam is reconnected to the motor in Lab 2, it is critical that the motor gear angle satisfies $-\frac{\pi}{4}\text{rad} < \theta(t) < \frac{\pi}{4}\text{rad}$ at all times. To help ensure this, prefilter $\theta_{ref}(t)$ using a saturator, as shown in Figure 7(b). In Lab 2/(a) we will design a controller to achieve an overshoot no higher than 5%; when this controller is operating, the condition $-0.7\text{rad} < \theta_{ref,sat}(t) < 0.7\text{rad}$, will ensure $-\frac{\pi}{4}\text{rad} < \theta(t) < \frac{\pi}{4}\text{rad}$. Use Simulink to simulate the system in Figure 7(b) to verify that the saturator works as expected. Include in your report the Simulink diagram used and a simulated plot showing the effect of the saturator.

Warning: Do not attach the ball and beam to the motor at this point.

Include the saturator in your formula node, and perform an experiment to show that the saturator is functioning as desired. Include the plot in your Lab 1 report.

(g) Draw a block diagram for the inner-loop showing the experimental signals, the variable names used in the formula node, and their units. Be sure to include nonlinear effects, such as offsets and saturation, as well as sensor scaling. Correct signs should also be shown. Indicate which blocks, specifically, are part of the apparatus. Analog and digital signals should be indicated.

In addition to the General Report Requirements, follow these directions:

- Attempt, the best you can, to fit your Lab 1 report within 15 pages from the front cover to the end, including appendices.
- Include in your report the code from your formula node.

LAB 2: DESIGN OF THE INNER-LOOP CONTROLLER AND MODELING OF THE BALL AND BEAM

INNER LOOP CONTROLLER DESIGN

(a) We will be using the emulation design approach for Lab 2. Design a controller $C_1(s)$ in Figure 7(a) to satisfy (in simulation) the following specifications:

- The (linearized) feedback loop is stable
- When the reference signal steps from $\theta_{ref} = -0.7\text{rad}$ to $\theta_{ref} = 0.7\text{rad}$:
    - o The step response steady-state tracking error is zero
    - o The step response 2% settling time is no more than 0.50 seconds
    - o The step response overshoot is no more than 5%
    - o The motor voltage does not saturate.
- A pole placement design (ECE.484 course notes) or classical lead-compensator design (knowledge from your pre-requisite control course) or a different method, of your choice, may be used.

Verify the effectiveness of your compensator in simulation, but do not try it out on the real experiment yet. If your controller does not work as expected, one possible reason could be an incorrect sign.

(b) We will use the emulation approach to design the discrete-time controller $D_1[z]$ in Figure 7(c):

- Choose a sampling period suitable for the emulation approach and provide justification.
- Use any reasonable discretization method to find the discretized controller, $D_1[z]$. Use Simulink to simulate the sampled data system shown in Figure 7(c). Confirm that the closed-loop performance is similar to that of the system in Figure 7(b). Note that if you make $D_1[z]$ a discrete transfer function, then Simulink will automatically insert the sample and hold operators shown explicitly in Figure 7(c). Do not insert your own sample and hold, and be sure to correctly set the sampling period in the discrete transfer function.

(c) Implement your controller $D_1[z]$ on the real experiment. A guide on discrete controller implementation can be found in Appendix 2.

It is important to put as many decimals as you can on your controller coefficients; in Matlab, `format long` will give you 15 digits. Numerical errors, associated with rounded up coefficients, may cause your controlled system to go unstable. To extract the coefficients of your controller, `SYS`, you may use `[NUM,DEN]=tfdata(SYS,'v')`.

Be sure to include the saturator, as shown in Figure 7(c), to ensure that $-0.7\text{rad} < \theta_{ref,sat}[k] < 0.7\text{rad}$.

Warning!	**Do not attach the ball and beam to the motor until your controller for the inner loop is working satisfactorily, i.e., $-0.7\text{rad} < \theta(t) < 0.7\text{rad}$. You could damage the (surprisingly expensive) ball and beam apparatus if your motor controller is not working properly!**

Caution: be sure to avoid feeding back outputs from nonlinear blocks (like you might do by using a shift register). Instead, keep nonlinear values in separate variables from those being fed back in your controller.

Perform an experiment and provide a plot to show that the inner-loop controller is functioning as desired. Compare the performance of the experimental setup to your simulation results from part (c). If the experimental performance differs from the simulation performance, explain why.

BALL AND BEAM MODELING

(d) You can connect the beam to the gear now! Obviously, attaching the beam to the motor plant will influence the inner-loop model. Technically, the inner-loop should be remodeled, with the beam attached, and a new controller should be designed. However, due to the high gearing of the motor, the model does not change significantly. So for simplicity, we will assume that the inner loop model has not changed. By changing the stiction offset, you should find that your original inner-loop controller will still meet at least the steady-state error specification. Verify that this is so in both directions.

(e) Convert the ball position potentiometer voltage to the corresponding position in meters using the y-origin convention shown in Figure 4; implement it in the formula node. Show in your Lab 2 report the scaling for the ball position, and explain how you determined it.

(f) Draw a block diagram for the outer-loop showing the experimental signals, the variable names used in the formula node, and their units. Be sure to include nonlinear effects, such as offsets and saturation, as well as sensor scaling. Correct signs should also be shown. Indicate which blocks, specifically, are part of the apparatus. Distinguish between analog and digital signals.

(g) A model for the lever arm and the beam, with the structure shown in equations (2) and (3), is provided below:

$$\frac{\phi(s)}{\theta(s)} = K_2, \quad \textit{where } K_2 = 0.062 \qquad \ldots(4)$$

$$\frac{Y(s)}{\phi(s)} = \frac{K_3}{s^2}, \quad \textit{where } K_3 = 4.78\frac{\text{m}}{\text{s}^2} \qquad \ldots(5)$$

(h) Validate the ball and beam model by performing an experiment on the plant (i.e., the system in the dotted box in Figure 6). Note that the plant is unstable, so validation can be tricky, but do your best. If you feel that the constant K_3 needs to be adjusted based on your experimental results, do so. Provide, in your report, evidence of the validation you have performed.

Write up your results for Lab 2 following the General Report Requirements; also provide:

- The values from Lab 1 for K_1, τ, stiction values, scaling of the gear angle, and provide a short justification for any changes made.
- Attempt, the best you can, to fit your Lab 2 report within 15 pages from the front cover to the end, including appendices.
- Include in your report a screen capture of your final cRIO-9076_RT.vi block diagram (may use File > Print Window or Snipping Tool).
- Include in your report the code of your formula node.

Submit your final cRIO-9076_RT.vi including the motor position controller to the Lab 2 dropbox in LEARN, so that it can be tested by the TAs. Ensure that values needed for testing (including those in the Reference Generator) are set as default values.

LAB 3: DESIGN OF THE OUTER LOOP CONTROLLER

In Lab 3, we focus on controlling the ball position. The basic setup is the inner/outer loop control configuration shown in Figure 6. Note that, from the point of view of the outer loop controller $C_2(s)$ in Figure 6, the "plant" is the system within the dotted box.

Due to nonlinearities inherent to the apparatus, we may not get the expected performance with a simple controller. In the following steps, we will explore different controllers, step sizes, and configurations; progressive changes will be applied to improve performance. If you prefer a different design strategy that fits the final design objectives, feel free to implement it instead, while documenting in your report its progression and results.

(a) Simulate, on your plant model, a 0.10m peak-to-peak square wave (from 0.15m to 0.25m) using the following lead controller:

$$C_{2LD}(s) = 7\frac{s+0.35}{s+2.5}$$

Note that the input to the controller C_{2LD} is in meters and the output is in radians; this controller is based on the assumption that $y=0$ is at the left end of the beam (away from the motor). The sign of this controller's gain may need to be opposite, depending on the sign conventions used in the inner loop.

(b) **With the gear angle saturator in place**, discretize and implement this controller in the lab, and compare it to the simulation for a repetitive square wave alternating between 0.15m to 0.25m.

Caution: as in Lab 2, be sure to avoid feeding back the output from a nonlinear block into your controller.

As in Lab 2, it is important to put as many decimals as you can on your controller coefficients; in Matlab, `format long` will give you 15 digits. Numerical errors, associated with rounded up coefficients, may cause your controlled system to go unstable. To extract the coefficients of your controller, `SYS`, you may use `[NUM,DEN]=tfdata(SYS,'v')`.

At a minimum we would have expected:

- a symmetrical response, and
- zero steady-state error, since the plant model has a double-integrator.

If either of these characteristics were not observed in the lab, explain why. Attributing them to random nonlinearities is insufficient. If you think that nonlinearities are at fault, specify which ones and explain why your choice makes sense. *Hint:* try drawing a block diagram - think about why an integrator provides tracking and how (in a nonlinear system) the location of the integrator might affect your results.

(c) In an attempt to overcome poor tracking, design a controller C_2 that has an integrator. When choosing the order of your controller, look at the inner loop specifications as compared to the outer loop. Is there any reasonable approximation you could make? If so, provide an explanation for your choice in the lab report.

You can use either the emulation or the direct design approach to design the outer loop discrete-time controller. In either case, design C_2 so that the final sampled-data control system satisfies the following specifications **in simulation**:

- For a square wave input from 0.15m to 0.18m:
  - The step response steady-state error is zero.
  - The step response 2% settling time is no more than 7 seconds
  - The step response overshoot is less than 45%
  - $\theta_{ref}(t)$ does not saturate.

Use Simulink to verify that your controller meets these specifications. Include in your Lab 3 report a copy of your Simulink diagram and simulation output, showing that the specifications are met. Simulation plots should include $\theta_{ref}(t)$, as evidence that you have avoided saturation. If the specifications are not met, explain what aspects of the specifications were challenging.

(d) Here we will investigate the performance of C_2 for a larger square wave from 0.10m to 0.25m. First, simulate the response of the system, and be sure to include the θ_{ref} saturator. Then, modify your formula node in cRIO-9076_RT.vi to include the outer-loop controller. After carefully checking your implementation, cautiously run the experiment with C_2. It is unlikely that all the specifications of (c) will be met, but verify this experimentally before moving to the next step. Include simulated and experimental plots in your report, that also show $\theta_{ref}(t)$. If the specifications are not met, explain what aspects of the specifications were challenging. Do *not* redesign C_2 at this point.

(e) You may have noticed that for small beam angles, the ball may not move; it “sticks”. The ball stiction is somewhat similar to the motor stiction encountered in the inner-loop. The strategy used there was to approximately cancel it out by adding a small voltage offset to the motor signal. A suggested configuration to overcome the ball stiction is shown in Figure 10.

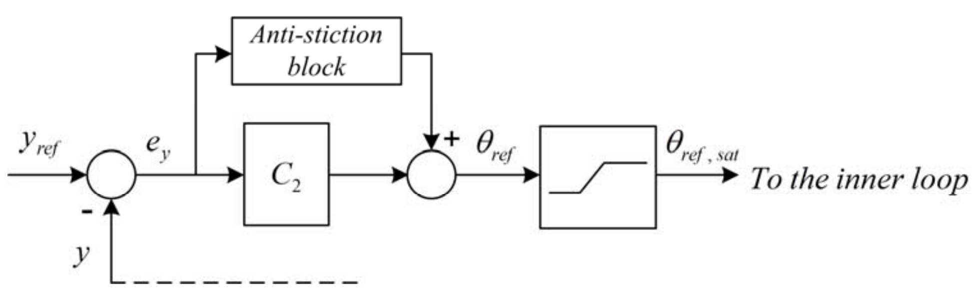


FIGURE 10: A possible ball anti-stiction implementation.

In conjunction with the above strategy, the control specifications for C_2 are slightly changed as follows:

- The (linearized) feedback system is stable.
- For a square wave input starting from 0.10m to 0.25m:
    - The step response steady-state error is less than 4%
    - The step response 2% settling time is no more than 10 seconds
    - The step response overshoot is less than 45%
    - $\theta_{ref}(t)$ saturation is permitted, because the step size is so large, but the saturator must be present.

If necessary, redesign C_2 to meet the above specifications.

Write up your final lab report using the General Report Requirements; also provide:

- From Lab 1 and Lab 2, restate the values used for K_1, τ, stiction, scaling of the gear angle, scaling of the ball position, inner loop controller, K_2, K_3 and explain if you made any modifications to them.
- Details on how you designed the outer-loop controller.
  If you used emulation, be sure to include:
    - The design of C_2 in Figure 6.
    - Your choice of discretization method.
    - A modified version of Figure 6, clearly showing how your discretized controller is connected. Clearly show all relevant sample-and-hold operators, and include the inner-loop saturator (like in Figure 7(c)).

  If you used direct design, be sure to include:
    - The mapping of the specifications to the z-plane.
    - The design of the discrete-time controller.
    - A modified version of Figure 6, clearly showing how your discrete-time controller is connected. Clearly show all relevant sample-and-hold operators, and include the inner-loop saturator (like in Figure 7(c)).
- Include in your report a screen capture of your final cRIO-9076_RT.vi block diagram (may use File > Print Window or Snipping Tool).
- Include in your report the code of your formula node.
- Attempt, the best you can, to fit your Lab 3 report into 25 pages.

Submit your final cRIO-9076_RT.vi including the inner and outer-loop controllers to the Lab3 dropbox in LEARN, so that it can be tested by the TAs. Ensure that values needed for testing (including those in the Reference Generator) are set as default values.

DEMO: DEMONSTRATION OF THE CONTROLLED BALL & BEAM SYSTEM

A short formal demonstration of Lab 3/(e) and question period covering the entire project will be scheduled for each group after the submission of Lab 3 and before the end of lectures. Partners are awarded demo participation marks individually. If a partner misses their reserved demo time without a valid reason, a mark of zero will be awarded to him/her for the demo.

The step from 0.10m to 0.25m is to be demonstrated; specifications will be tested on the best-performing period of the square input. The code used for the demonstration must be the one included in group's Lab 3 submission.

BIBLIOGRAPHY

[1] Quanser Consulting Inc., SRV02-Series Rotary Servo Plant User Manual, Revision 1, July 2004

[2] Quanser Consulting Inc., SRV02-Series Ball & Beam User Manual, Revision 1, July 2004

APPENDIX 1
BALL AND BEAM DATASHEET

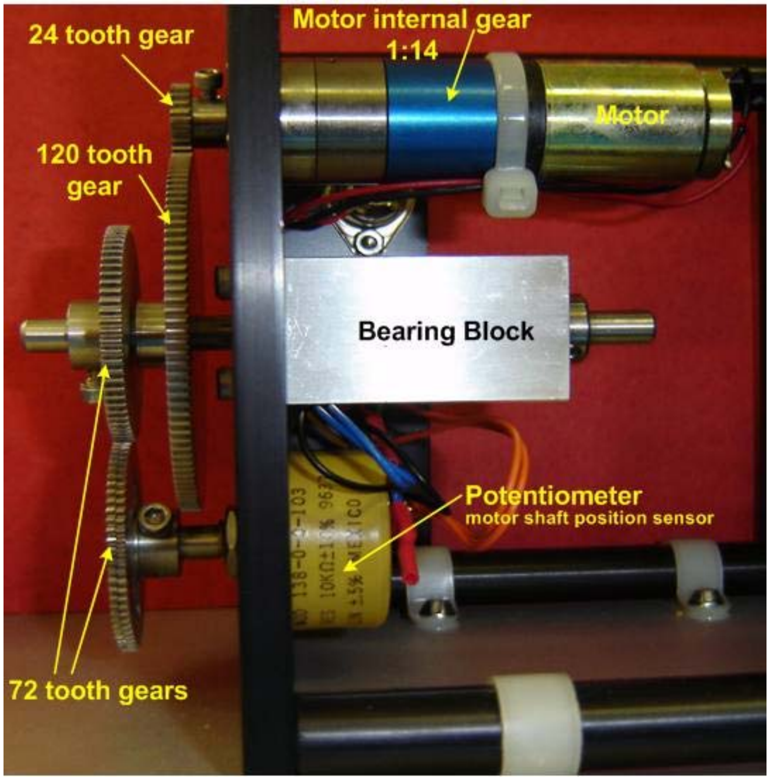


FIGURE A1: Motor, gear and gear sensor detail

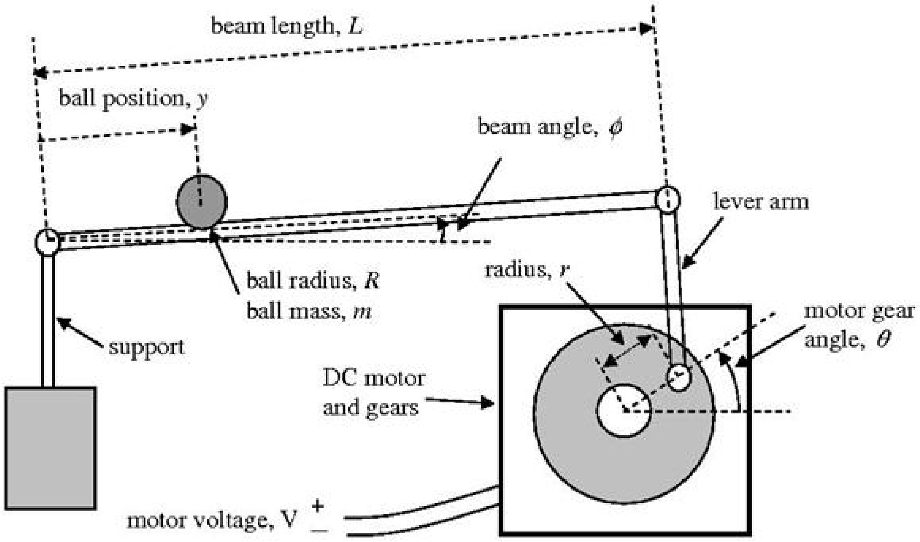


FIGURE A2: Geometry of the ball and beam system

Specification	Symbol	Value	Units
Motor and Gears Subsystem [1]			
Motor torque constant	K_T	7.67×10^{-3}	N m/A
Motor back e.m.f. constant	K_b	7.67×10^{-3}	V/(rad/s)
Armature resistance	R_a	2.6	Ω
Armature inductance	L_a	0.18	mH
Motor inertia	J_a	3.87×10^{-7}	kg m^2
Viscous damping coefficient	B	4.0×10^{-3}	N m/(rad/s)
Maximum motor voltage	V_{max}	6	V
Motor efficiency	η_m	69	%
Gear efficiency	η_g	85	%
Internal gear ratio	K_{gi}	14 : 1	-
External gear ratio	K_{ge}	120:24 = 5 : 1	-
Gear ratio	$K_g = K_{gi} \times K_{ge}$	70 : 1	-
Equivalent gear inertia	J_{eq}	2.0×10^{-3}	kg m^2
Motor shaft angle sensor resistance	R_{1s}	10	kΩ
Motor shaft angle sensor bias resistors	R_{1b}	7.15	kΩ
Gear angle sensor bias voltage	V_{1b}	10.65	V
Motor shaft angle sensor measurement range	V_{theta}	3 to 7.5	V
Ball and Beam Subsystem [2]			
Beam length (interior)	L	41.7	cm
Lever arm length	-	12	cm
Lever end radius of rotation	r	2.54	cm
Support arm length	-	16	cm
Beam inertia (includes motor and gears)	J_{eq}	0.0029	kg m^2
Ball radius	R	1.27	cm
Ball and beam module mass	M_{tot}	0.65	kg
Ball mass	m	0.064	kg
Ball position sensor resistance	R_{2s}	2	kΩ
Ball position sensor bias resistors (one on each side)	R_{2b}	1.54	kΩ
Ball position sensor bias voltage	V_{2b}	10.65	V
Ball position sensor measurement range	V_y	3 to 7.5	V

APPENDIX 2
DISCRETE CONTROLLER IMPLEMENTATION

Consider a *generic* transfer function, T(z), which was obtained by discretizing the continuous transfer function T(s) at a sample period T_s.

$$T(z) = \frac{U(z)}{R(z)} = \frac{az + b}{cz + d}$$

The z-transform operator, z, is a prediction operator; the notation for samples at various time points is shown in Table 1.

Table 1.

z-operator and Discrete Value	**Description**	**Lecture Notes Notation**	**Formula Node Notation**
$U(z) \leftrightarrow u[kT_s]$	Current sample	$u[k]$	u
$zU(z) \leftrightarrow u[kT_s + T_s]$	Next sample	$u[k+1]$	Future value, hence not available
$\frac{1}{z}U(z) \leftrightarrow u[kT_s - T_s]$	Previous sample	$u[k-1]$	u_1

Applying the above to *T(z)*, we get:

$$U(z)(cz + d) = R(z)(az + b)$$

$$czU(z) + dU(z) = azR(z) + bR(z)$$

$$cu[k+1] + du[k] = ar[k+1] + br[k]$$

Since future values are not available to us, we can shift back one sample to obtain:

$$cu[k] + du[k-1] = ar[k] + br[k-1]$$

The output of this transfer function follows:

$$\boxed{\boldsymbol{u[k] = -\frac{d}{c}u[k-1] + \frac{a}{c}r[k] + \frac{b}{c}r[k-1]}}$$