

Problem Statement : Solving an ODE using Euler's Method

Consider the following first-order differential equation:

$$\frac{dy}{dt} = -2y + 4, \quad y(0) = 1$$

1. **Implement Euler's Method** to solve the differential equation numerically for t in the interval $[0, 5]$ with a step size of $h=0.1$

Plot the numerical solution $y(t)$ obtained from Euler's method.

2. If the analytical solution to this differential equation is:

$$y(t) = 2 - (2 - y_0)e^{-2t}$$

where $y_0=1$, compute the analytical solution for $y(t)$ at the same time points used in the numerical solution and compute the error of the numerical solution at each step.

3. Investigate the impact of varying the step size h (try values $h=0.05$, and $h = 0.2$) on the accuracy of Euler's method.