Diferansiyel Denklemler

Hafta 10

$$Q(x) \quad \Big| \; Karakteristik\ Denklemin\ Kökü\ DEĞİLDİR \; \Big| \; y_{ö}$$

$Q(x)$	Karakteristik Denklemin Kökü DEĞİLDİR	$y_{ö}$
e^{mx}	m	Ae^{mx}
$P_n(x)$	$SIFIR$	$A_0 + A_1x + \cdots + A_nx^n$
$\sin\beta x, \cos\beta x$	$\pm i\beta$	$A\cos\beta x + B\sin\beta x$
$P_n(x)e^{mx}$	m	$e^{mx}(A_0 + A_1x + \cdots + A_nx^n)$
$e^{\alpha x}\sin\beta x, e^{\alpha x}\cos\beta x$	$\alpha \pm i\beta$	$e^{\alpha x}(A\cos\beta x + B\sin\beta x)$
$P_n(x)\cos\beta x, P_n(x)\sin\beta x$	$\pm i\beta$	$(A_0 + A_1x + \cdots + A_nx^n)\cos\beta x + (B_0 + B_1x + \cdots + B_nx^n)\sin\beta x$

$\boxed{Örnek1}$ $y'' - y = 3\sin 2x$ $'in\ genel$ çözümünü $bulunuz.$

$$1.ADIM:\ y_g = y_h + y_{ö}$$

$$2.ADIM:\ Karakteristik\ denklem;\ \ r^2 - 1 = 0$$

$$r_1 = 1, \quad r_2 = -1$$

$$y_h = c_1e^x + c_2e^{-x}$$

$$3.ADIM:\ y_{ö} = ?$$

$$\pm 2i \textit{ köklerde var mı? YOK.}$$

$$y_{ö} = A\cos 2x + B\sin 2x \quad \textit{şeklinde aranır.}$$

$$\textit{A ve B yi bul!}$$

$$y'_{ö} = -2A\sin 2x + 2B\cos 2x$$
$$y''_{ö} = -4A\cos\ 2x - 4B\sin 2x$$

$$y'' - y = 3\sin 2x$$

$$[-4A - A]\cos 2x + [-4B - B]\sin 2x = 3\sin 2x$$

$$A = 0 \quad , \quad B = -\frac{3}{5}$$

$$y_{ö} = -\frac{3}{5}\sin 2x$$

$$4.\ ADIM:\ y_g = c_1e^{x} + c_2e^{-x} - \frac{3}{5}\sin 2x$$

$\boxed{\text{Örnek2}}$ $y'' - y = (2x - 1)e^{x}$ *genel* çözümünü *bulunuz*.

$$1.\ ADIM:\ y_g = y_h + y_{ö}$$

$$2.\ ADIM:\ \textit{Karakteristik denklem};\ r^2 - 1 = 0$$

$$r_1 = 1\,, \quad r_2 = -1$$

$$y_h = c_1 e^x + c_2 e^{-x}$$

$$3.\,ADIM\colon\; y_ö = ?$$

$$m = 1 \; köklerde \; var \; mı? \, VAR.$$

$$y_ö = x(Ax + B)e^{mx}$$

$$A \; ve \; B \; yi \; bul!$$

$$y_ö = (Ax^2 + Bx)e^x$$

$$y'_ö = (2Ax + B)e^x + (Ax^2 + Bx)e^x$$

$$= [Ax^2 + (B + 2A)x + B]e^x$$

$$y''_ö = [2Ax + (B + 2A)]e^x + [Ax^2 + (B + 2A)x + B]e^x$$

$$= [Ax^2 + (B + 4A)x + (2A + 2B)]e^x$$

$$y''_ö - y_ö = (2x + 1)e^x$$

$$(A - A)x^2 + ((B + 4A) - B)x + (2A + 2B)]\, e^x = (2x - 1)e^x$$

$$4A = 2 \;\Rightarrow\; A = \frac{1}{2}\,, \qquad 2A + 2B = -1 \;\Rightarrow\; B = -1$$

$$\boxed{y_g = c_1 e^x + c_2 e^{-x} + x\left(\frac{1}{2}x - 1\right) e^x}$$

$\boxed{Örnek3}$ $y'' - y = e^{2x}\cos 3x$ $genel$ çözümünü $bulunuz$.

$$1. ADIM:\ y_g = y_h + y_ö$$

$$2. ADIM:\ Karakteristik\ denklem;\ \ r^2 - 1 = 0$$

$$r_1 = 1\,,\quad r_2 = -1$$

$$y_h = c_1e^x + c_2e^{-x}$$

$$3. ADIM:\ y_ö = ?$$

$$y_ö = e^{2x}(Acos3x + Bsin3x)\ şeklinde\ aranır.$$

$$(Yazarsanız\ 5\ puanı\ kaptınız.)$$

$$A\ ve\ B'yi\ bul!$$

$$y = e^{2x}(A\cos 3x + B\sin 3x)$$

$$y' = 2e^{2x}(A\cos 3x + B\sin 3x) + e^{2x}(-3A\sin 3x + 3B\cos 3x)$$

$$= e^{2x}[(2A + 3B)\cos 3x + (2B - 3A)\sin 3x]$$

$$y''_ö = e^{2x}\{[(4A + 6B) + (6B - 9A)]\cos 3x + [(4B - 6A) - (6A + 9B)\sin 3x]\}$$

$$y'' - y = e^{2x}\cos 3x$$

$$Aslında\ görmemiz\ gereken\ şekliyle;$$

$$y''_ö - y_ö = e^{2x}\cos 3x$$

$$\underbrace{[(12B-5A)-A]}_{1}\cos 3x+\underbrace{[(-5B-12A)-B]}_{0}\sin 3x=\cos 3x$$

$$-6A+12B=1$$
$$-12A-6B=0$$

$$A=-\frac{1}{30}\ ,\quad B=\frac{1}{15}$$

$$\boxed{y_g=c_1e^{x}+c_2e^{-x}+e^{2x}\left(-\frac{1}{30}\cos 3x+\frac{1}{15}\sin 3x\right)}$$

$\boxed{\text{Örnek4}}$ $y''-y'=\text{xsin}\,4x$

$1.\,ADIM:\ y_g=y_h+y_ö$

$2.\,ADIM:\ Karakteristik\ denklem;\ \ r^2-r=0$

$kökler:\ r_1=0\ ,\quad r_2=1$

$$y_h=c_1e^{0x}+c_2e^{-x}\Longrightarrow y_h=c_1+c_2e^{-x}$$

$3.\,ADIM:\ y_ö=?$

$\pm 4i\ köklerde\ var\ mı?\,YOK$

$$y_ö=(A_0+A_1x)\cos 4x+(B_0+B_1x)\sin 4x$$

Türev alınır

$$y'_{\text{ö}} = A_1 \cos 4x - 4(A_0 + A_1 x) \sin 4x + B_1 \sin 4x + 4(B_0 + B_1 x) \cos 4x$$

$$= [4B_1 x + (A_0 + 4B_0)] \cos 4x + [-4\, A_1 x + (B_1 - 4A_0)] \sin 4x$$

Tekrar türev alınır

$$y''_{\text{ö}} = [-16A_1 x + (8B_1 - 16A_0)] \cos 4x + [-16B_1 x + (-8A_1 - 16B_0)] \sin 4x$$

$$y''_{\text{ö}} - y'_{\text{ö}} = x \sin 4x$$

$$\left[\underbrace{(-16A_1 - 4B_1)}_{0} x + \underbrace{(8B_1 - 16A_0) - (A_1 + 4B_0)}_{0}\right] \cos 4x +$$
$$\left[\underbrace{(-16B_1 + 4A_1)}_{1} x + \underbrace{(-8A_1 - 16B_0) - (B_1 - 4A_0)}_{0}\right] \sin 4x = x \cos 4x$$

A ları ve B leri biz hesaplayacağız.

$\boxed{NOT}$ $Q(x) = Q_1(x) + Q_2(x) + \cdots + Q_n(x)$ *ise*

Her $Q_i(x)$ *için bir* $y_{\text{ö}i}$ *aranır.*

$\boxed{y_{\text{ö}} = y_{\text{ö}1} + y_{\text{ö}2} + \cdots + y_{\text{ö}n}}$ *yazılır.*

$\boxed{\text{Örnek6}}$ $y'' + y = e^x + x^2 + 2 \sin 2x$

$$y_g = y_h + y_{\text{ö}1} + y_{\text{ö}2} + y_{\text{ö}3}$$

$$y_h = ?$$

$$r^2 + 1 = 0 \;\Rightarrow\; r = \pm i$$

$$y_h = c_1 \cos x + c_2 \sin x$$

$$y_ö = y_{ö1} + y_{ö2} + y_{ö3}$$

$$Problem\ 1: y'' + y = e^x$$

$$Problem\ 2:\ y'' + y = x^2$$

$$Problem\ 3:\ y'' + y = 2\sin 2x$$

$$m = 1 \;\Rightarrow\; y_{ö1} = Ae^x$$

$$y_{ö2} = Bx^2 + Cx + D$$

$$y_{ö3} = E \sin 2x + F \cos 2x$$

$$y_ö = y_{ö1} + y_{ö2} + y_{ö3}$$

$$y_ö = Ae^x + Bx^2 + Cx + D + E \sin 2x + F \cos 2x$$

$$y'_ö = Ae^x + 2Bx + C + 2E \cos 2x - 2F \sin 2x$$

$$y''_ö = Ae^x + 2B - 4E \sin 2x - 4F \cos 2x$$

Şimdi denklemde yerine yazalım.

$$y''_ö + y_ö = e^x + x^2 + 2 \sin 2x$$

$$y''_ö + y_ö = \underbrace{2A}_{1} e^x + \left(\underbrace{B}_{1} x^2 + \underbrace{C}_{0} x \underbrace{(D + 2B)}_{0}\right)$$
$$+ \underbrace{(-3F)}_{0} \cos 2x \underbrace{-3E}_{1} \sin 2x = e^x + x^2 + \sin 2x$$

$$A = \frac{1}{2}\ , \qquad B = 1, C = 0, D = -2, \qquad E = -\frac{1}{3}, \qquad F = 0$$

$$y_{ö} = \frac{1}{2}e^{x} + (x^2 - 2) - \frac{1}{3}\sin 2x$$

Problemler

1) $y'' + y = e^{-5x}$
2) $y''' - y'' = x^2 + 7$
3) $y'' - 4y' + 4y = e^{2x}$
4) $y^{(7)} - y^{(5)} = \sin x$
5) $y'' - y = x\cos 2x$
6) $y'' + y = x\sin x$