

Diferansiyel Denklemler

Hafta 10

$Q(x)$ | Karakteristik Denklemin Kökü DEĞİLDİR | $y_{\text{ö}}$

$$e^{mx} \quad | \quad m \quad | Ae^{mx}$$

$$P_n(x) \quad | \quad SIFIR \quad | A_0 + A_1x + \dots + A_nx^n$$

$$\sin \beta x, \cos \beta x \quad | \quad \pm i\beta \quad | A \cos \beta x + B \sin \beta x$$

$$P_n(x)e^{mx} \quad | \quad m \quad | e^{mx}(A_0 + A_1x + \dots + A_nx^n)$$

$$e^{\alpha x} \sin \beta x, e^{\alpha x} \cos \beta x \quad | \quad \alpha \pm i\beta \quad | e^{\alpha x}(A \cos \beta x + B \sin \beta x)$$

$$P_n(x) \cos \beta x, P_n(x) \sin \beta x \quad | \quad \pm i\beta \quad | (A_0 + A_1x + \dots + A_nx^n) \cos \beta x + (B_0 + B_1x + \dots + B_nx^n) \sin \beta x$$

Örnek1 $y'' - y = 3 \sin 2x$ 'in genel çözümünü bulunuz.

1. ADIM: $y_g = y_n + y_{\text{ö}}$

2. ADIM: Karakteristik denklem; $r^2 - 1 = 0$

$$r_1 = 1, \quad r_2 = -1$$

$$y_n = c_1 e^x + c_2 e^{-x}$$

3. ADIM: $y_{\text{ö}} = ?$

$\pm 2i$ köklerde var mı? YOK.

$y_{\ddot{o}} = A \cos 2x + B \sin 2x$ şeklinde aranır.

A ve B yi bul!

$$y'_{\ddot{o}} = -2A \sin 2x + 2B \cos 2x$$

$$y''_{\ddot{o}} = -4A \cos 2x - 4B \sin 2x$$

$$y'' - y = 3 \sin 2x$$

$$[-4A - A] \cos 2x + [-4B - B] \sin 2x = 3 \sin 2x$$

$$A = 0, \quad B = -\frac{3}{5}$$

$$y_{\ddot{o}} = -\frac{3}{5} \sin 2x$$

$$4. ADIM: y_g = c_1 e^x + c_2 e^{-x} - \frac{3}{5} \sin 2x$$

Örnek2 $y'' - y = (2x - 1)e^x$ genel çözümünü bulunuz.

$$1. ADIM: y_g = y_n + y_{\ddot{o}}$$

$$2. ADIM: Karakteristik denklem; $r^2 - 1 = 0$$$

$$r_1 = 1, \quad r_2 = -1$$

$$y_h = c_1 e^x + c_2 e^{-x}$$

$$3. ADIM: y_{\ddot{}} = ?$$

$m = 1$ köklerde var mı? VAR.

$$y_{\ddot{}} = x(Ax + B)e^{mx}$$

A ve B yi bul!

$$y_{\ddot{}} = (Ax^2 + Bx)e^x$$

$$y'_{\ddot{}} = (2Ax + B)e^x + (Ax^2 + Bx)e^x$$

$$= [Ax^2 + (B + 2A)x + B]e^x$$

$$y''_{\ddot{}} = [2Ax + (B + 2A)]e^x + [Ax^2 + (B + 2A)x + B]e^x$$

$$= [Ax^2 + (B + 4A)x + (2A + 2B)]e^x$$

$$y''_{\ddot{}} - y_{\ddot{}} = (2x + 1)e^x$$

$$(A - A)x^2 + ((B + 4A) - B)x + (2A + 2B)] e^x = (2x + 1)e^x$$

$$4A = 2 \Rightarrow A = \frac{1}{2}, \quad 2A + 2B = -1 \Rightarrow B = -1$$

$$y_g = c_1 e^x + c_2 e^{-x} + x \left(\frac{1}{2}x - 1 \right) e^x$$

Örnek3 $y'' - y = e^{2x} \cos 3x$ genel çözümünü bulunuz.

1. ADIM: $y_g = y_n + y_{\ddot{o}}$

2. ADIM: Karakteristik denklem; $r^2 - 1 = 0$

$$r_1 = 1, \quad r_2 = -1$$

$$y_n = c_1 e^x + c_2 e^{-x}$$

3. ADIM: $y_{\ddot{o}} = ?$

$y_{\ddot{o}} = e^{2x}(A \cos 3x + B \sin 3x)$ şeklinde aranır.

(Yazarsanız 5 puanı kaptınız.)

A ve B'yi bul!

$$y = e^{2x}(A \cos 3x + B \sin 3x)$$

$$y' = 2e^{2x}(A \cos 3x + B \sin 3x) + e^{2x}(-3A \sin 3x + 3B \cos 3x)$$

$$= e^{2x}[(2A + 3B) \cos 3x + (2B - 3A) \sin 3x]$$

$$y''_{\ddot{o}} = e^{2x} \{ [(4A + 6B) + (6B - 9A)] \cos 3x \\ + [(4B - 6A) - (6A + 9B) \sin 3x] \}$$

$$y'' - y = e^{2x} \cos 3x$$

Aslında görmemiz gereken şekliyle;

$$y''_{\ddot{o}} - y_{\ddot{o}} = e^{2x} \cos 3x$$

$$\underbrace{[(12B - 5A) - A]}_1 \cos 3x + \underbrace{[(-5B - 12A) - B]}_0 \sin 3x = \cos 3x$$

$$-6A + 12B = 1$$

$$-12A - 6B = 0$$

$$A = -\frac{1}{30} , \quad B = \frac{1}{15}$$

$$y_g = c_1 e^x + c_2 e^{-x} + e^{2x} \left(-\frac{1}{30} \cos 3x + \frac{1}{15} \sin 3x \right)$$

$$\boxed{\text{Örnek4}} \quad y'' - y' = x \sin 4x$$

$$1. ADIM: y_g = y_n + y_{\ddot{o}}$$

$$2. ADIM: \text{Karakteristik denklem; } r^2 - r = 0$$

$$\text{kökler: } r_1 = 0 , \quad r_2 = 1$$

$$y_n = c_1 e^{0x} + c_2 e^{-x} \implies y_n = c_1 + c_2 e^{-x}$$

$$3. ADIM: y_{\ddot{o}} = ?$$

$$\pm 4i \text{ köklerde var mı? YOK}$$

$$y_{\ddot{0}} = (A_0 + A_1 x) \cos 4x + (B_0 + B_1 x) \sin 4x$$

Türev alınır

$$\begin{aligned} y'_{\ddot{0}} &= A_1 \cos 4x - 4(A_0 + A_1 x) \sin 4x + B_1 \sin 4x + 4(B_0 + B_1 x) \cos 4x \\ &= [4B_1 x + (A_0 + 4B_0)] \cos 4x + [-4A_1 x + (B_1 - 4A_0)] \sin 4x \end{aligned}$$

Tekrar türev alınır

$$y''_{\ddot{0}} = [-16A_1 x + (8B_1 - 16A_0)] \cos 4x + [-16B_1 x + (-8A_1 - 16B_0)] \sin 4x$$

$$y''_{\ddot{0}} - y'_{\ddot{0}} = x \sin 4x$$

$$\begin{aligned} &\left[\underbrace{(-16A_1 - 4B_1)}_0 x + \underbrace{(8B_1 - 16A_0) - (A_1 + 4B_0)}_0 \right] \cos 4x + \\ &\left[\underbrace{(-16B_1 + 4A_1)}_1 x + \underbrace{(-8A_1 - 16B_0) - (B_1 - 4A_0)}_0 \right] \sin 4x = x \cos 4x \end{aligned}$$

A ları ve B leri biz hesaplayacağız.

NOT $Q(x) = Q_1(x) + Q_2(x) + \cdots + Q_n(x)$ ise

Her $Q_i(x)$ için bir $y_{\ddot{0}i}$ aranır.

$y_{\ddot{0}} = y_{\ddot{0}1} + y_{\ddot{0}2} + \cdots + y_{\ddot{0}n}$ yazılır.

$$\boxed{\text{Örnek6}} \quad y'' + y = e^x + x^2 + 2 \sin 2x$$

$$y_g = y_n + y_{\ddot{o}1} + y_{\ddot{o}2} + y_{\ddot{o}3}$$

$$y_n = ?$$

$$r^2 + 1 = 0 \Rightarrow r = \pm i$$

$$y_n = c_1 \cos x + c_2 \sin x$$

$$y_{\ddot{o}} = y_{\ddot{o}1} + y_{\ddot{o}2} + y_{\ddot{o}3},$$

$$y_{\ddot{o}1} \rightarrow Q_1(x) = e^x$$

$$y_{\ddot{o}2} \rightarrow Q_2(x) = x^2$$

$$y_{\ddot{o}3} \rightarrow Q_3(x) = \sin 2x \Rightarrow \pm 2i \Rightarrow y_{\ddot{o}} = E \sin 2x + F \cos 2x$$

$$m = 1 \Rightarrow y_{\ddot{o}1} = Ae^x$$

$$y_{\ddot{o}2} = Bx^2 + Cx + D$$

$$y_{\ddot{}} = Ae^x + Bx^2 + Cx + D + E \sin 2x + F \cos 2x$$

$$y'_{\ddot{}} = Ae^x + 2Bx + C + 2E \cos 2x - 2F \sin 2x$$

$$y''_{\ddot{}} = Ae^x + 2B - 4E \sin 2x - 4F \cos 2x$$

$$y''_{\ddot{}} + y_{\ddot{}} = e^x + x^2 + 2 \sin 2x$$

$$y''_{\ddot{}} + y_{\ddot{}} = \underbrace{2A}_1 e^x + \left(\underbrace{B}_1 x^2 + \underbrace{C}_0 x \underbrace{(D + 2B)}_0 \right) \\ + \underbrace{(-3F)}_0 \cos 2x - \underbrace{3E}_1 \sin 2x = e^x + x^2 + \sin 2x$$

$$A = \frac{1}{2} , \quad B = 1, C = 0, D = -2, \quad E = -\frac{1}{3}, \quad F = 0$$

$$y_{\ddot{}} = \frac{1}{2} e^x + (x^2 - 2) - \frac{1}{3} \sin 2x$$

Problemler

- 1) $y'' + y = e^{-5x}$
- 2) $y''' - y'' = x^2 + 7$
- 3) $y'' - 4y' + 4y = e^{2x}$

$$4) y^{(7)} - y^{(5)} = \sin x$$

$$5) y'' - y = x \cos 2x$$

$$6) y'' + y = x \sin x$$