ÖR1 $y = c_1x + c_2x^2 + c_3x^3$, $c_1, c_2, c_3 \in \mathbb{R}$ ol. üz. bu eğri ailesini çözüm kabul eden en düşük mertebeli dif. denk.?

$y = c_1x + c_2x^2 + c_3x^3$

$y' = c_1 + 2c_2x + 3c_3x^2$

$y'' = 2c_2 + 6c_3x$

$y''' = 6c_3 \Rightarrow c_3 = \dfrac{y'''}{6}$

$y'' = 2c_2 + xy''' \Rightarrow c_2 = \dfrac{y''}{2} - \dfrac{x}{2}y'''$

$c_1 = y' - (y'' - xy''')x - x^2\dfrac{y'''}{2} = y' - xy'' + \left(x^2 - \dfrac{x^2}{2}\right)y''' = y' - xy'' + \dfrac{x^2}{2}y'''$

$$y = x\left(y' - xy'' + \frac{x^2}{2}y'''\right) + x^2\left(\frac{y''}{2} - \frac{x}{2}y'''\right) + x^3\left(\frac{y'''}{6}\right)$$

$$y = xy' + \left(-x^2 + \frac{x^2}{2}\right)y'' + \left(\cancel{\frac{x^3}{2}} - \cancel{\frac{x^3}{2}} + \frac{x^3}{6}\right)y'''$$

$$\boxed{y = xy' - \frac{x^2}{2}y'' + \frac{x^3}{6}y'''}$$

$$y \overset{?}{=} x(\cancel{c_1} + 2c_2x + 3c_3x^2) - \frac{x^2}{2}(2c_2 + 6c_3x) + \frac{x^3}{6}(6c_3) \overset{?}{=} \cancel{c_1x} + c_2x^2 + c_3x^3$$

$$2c_2x^2 - c_2x^2 + 3c_3x^3 - 3c_3x^3 + c_3x^3 \overset{?}{=} c_2x^2 + c_3x^3 \quad ✓$$

ÖR2 $(x + \sqrt{x})y' = y + \sqrt{y}$ genel çözümü?

$$(x+\sqrt{x})\frac{dy}{dx} = y + \sqrt{y} \Rightarrow \frac{dy}{y+\sqrt{y}} = \frac{dx}{x+\sqrt{x}} \Rightarrow \int \frac{dy}{y+\sqrt{y}} = \int \frac{dx}{x+\sqrt{x}} + 2\ln c$$

$$J = \int \frac{dy}{y+\sqrt{y}}; \quad \left[\sqrt{y} = t \Leftrightarrow y = t^2,\ dy = 2t\,dt\right] \quad J = \int \frac{2t\,dt}{t^2+t} = 2\int \frac{dt}{t+1} = 2\ln|t+1| = 2\ln|\sqrt{y}+1| \text{ old.}$$

Aranan genel çözüm $\cancel{2}\ln|\sqrt{y}+1| = \cancel{2}\ln|\sqrt{x}+1| + \cancel{2}\ln c \Rightarrow \sqrt{y}+1 = (\sqrt{x}+1)c \Rightarrow$

$$y = (c(\sqrt{x}+1) - 1)^2 \ //$$

ÖR3 $x^2y' = y - xy \quad y(1) = 1 \Rightarrow y(x) = ?$

$$x^2\frac{dy}{dx} = y(1-x) \Rightarrow \frac{dy}{y} = \frac{1-x}{x^2}dx \Rightarrow \int\frac{dy}{y} = \int\left(\frac{1}{x^2} - \frac{1}{x}\right)dx + c \Rightarrow \ln y = -\frac{1}{x} - \ln x + c$$

$$y(1) = 1 \Rightarrow \cancel{\ln 1}^{0} = -\frac{1}{1} - \cancel{\ln 1}^{0} + c \Rightarrow c = 1 \Rightarrow \ln y = -\frac{1}{x} - \ln x + 1 \Rightarrow y = e^{1 - \frac{1}{x} - \ln x} = \frac{e\,e^{-1/x}}{x}$$

$$\boxed{y = \frac{e^{\frac{x-1}{x}}}{x}}$$

ÖR4 $y' = \left(\dfrac{2y+3}{4x+5}\right)^2 \Rightarrow$ genel çözüm?

$$\frac{dy}{dx} = \frac{(2y+3)^2}{(4x+5)^2} \Rightarrow \frac{dy}{(2y+3)^2} = \frac{dx}{(4x+5)^2} \Rightarrow \int\frac{dy}{(2y+3)^2} = \int\frac{dx}{(4x+5)^2}\ c$$

$$\left[\begin{matrix} 2y+3 = t, & 4x+5 = u \\ 2\,dy = dt, & 4dx = du \end{matrix}\right] \quad \frac{1}{2}\int\frac{dt}{t^2} = \frac{1}{4}\int\frac{du}{u^2}\ c \Rightarrow -\frac{1}{2t} = -\frac{1}{4u} - c$$

$$\frac{2}{t} = \frac{1}{u} + 4c \Rightarrow \frac{2}{2y+3} = \frac{1}{4x+5} + 4c \ //$$