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IT-21019

### Assignment:

1) Is 1729 a Carmichael?

We know,

$$1729 = 7 \times 13 \times 19$$

Here, Each  $p \mid 1729 \rightarrow (p-1)$

1728:

$$* 7-1=6 \text{ and } 6 \mid 1728$$

$$* 13-1=12 \text{ and } 12 \mid 1728$$

$$* 19-1=18 \text{ and } 18 \mid 1728$$

$\therefore$  Yes, 1729 is a Carmichael number.

Ans.

2) Primitive root of  $\mathbb{Z}_{23}$

The powers of 5 modulo 23 generate all non-zero elements of  $\mathbb{Z}_{23}$ .

$$5^1 \equiv 5 \pmod{23}$$

$$5^2 \equiv 2 \pmod{23}$$

$$5^3 \equiv 3 \pmod{23}$$

$$5^4 \equiv 4 \pmod{23}$$

⋮

⋮

⋮

⋮

$$5^{22} \equiv 1 \pmod{23}$$

$\therefore 5$  is the ~~the~~ primitive root of modulo 23.

3) Is  $\langle \mathbb{Z}_n, + \rangle$  a ring?

" is prime and  $\mathbb{Z}_n$  is field

And it satisfies,

→ Commutative under both addition, multiplication

→ Associative

→ Has additive and multiplicative identity.

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So, yes.  $\langle \mathbb{Z}_n, + \rangle$  a ring  
Ans.

4) Are  $\langle \mathbb{Z}_{32}, + \rangle, \langle \mathbb{Z}_3, \cdot \rangle$  abelian?  
 $\rightarrow \langle \mathbb{Z}_{32}, + \rangle \rightarrow$  yes, its abelian  
 $\rightarrow \langle \mathbb{Z}_3, \cdot \rangle \rightarrow$  No, all elements invertible

5)  $GF(2^3)$  Polynomial

Let, irreducible polynomial,  
 $f(x) = x^3 + x + 1$

field:  $GF(2^3) = \{0, 1, x, x+1, x^2, x^2+1, x^2+x, x^2+x+1\}$

So,  
 $(x+1)(x^2+x) \equiv 1 \pmod{(x^3+x+1)}$