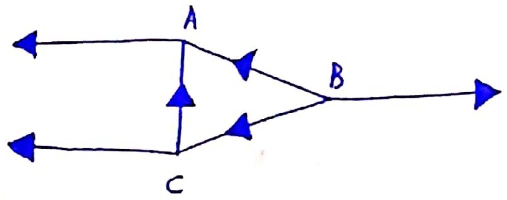
QUESTION # 11

Find general flow pattern of the network shown in figure. Assuming that flows are all non-negative. What is the longest possible values for x_3?



SOLUTION:

Node	Flow in		Flow out
A	$x_1 + x_3$	=	20
B	x_2	=	$x_3 + x_4$
C	80	=	$x_1 + x_2$
Total Flows:	80	=	$x_4 + 20$

Arranging Equations:

$$x_1 + x_3 = 20$$

$$x_2 - x_3 - x_4 = 0$$

$$x_1 + x_2 = 80$$

$$x_4 = 60$$

Matrix Form

$$\begin{bmatrix} 1 & 0 & 1 & 0 & 20 \\ 0 & 1 & -1 & -1 & 0 \\ 1 & 1 & 0 & 0 & 80 \\ 0 & 0 & 0 & 1 & 60 \end{bmatrix} \sim \xrightarrow{R_3 - R_1} \begin{bmatrix} 1 & 0 & 1 & 0 & 20 \\ 0 & 1 & -1 & -1 & 0 \\ 0 & 1 & -1 & 0 & 60 \\ 0 & 0 & 0 & 1 & 60 \end{bmatrix}$$

$$\xrightarrow{R_3 - R_2} \begin{bmatrix} 1 & 0 & 1 & 0 & 20 \\ 0 & 1 & -1 & -1 & 0 \\ 0 & 0 & 0 & 1 & 60 \\ 0 & 0 & 0 & 1 & 60 \end{bmatrix} \xrightarrow{\sim R_2 + R_3} \begin{bmatrix} 1 & 0 & 1 & 0 & 20 \\ 0 & 1 & -1 & 0 & 60 \\ 0 & 0 & 0 & 1 & 60 \\ 0 & 0 & 0 & 1 & 60 \end{bmatrix}$$

$$\xrightarrow{\sim R_4 - R_3} \begin{bmatrix} 1 & 0 & 1 & 0 & 20 \\ 0 & 1 & -1 & 0 & 60 \\ 0 & 0 & 0 & 1 & 60 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \longrightarrow \text{reduced echelon form}$$

$$\begin{cases} x_1 = 20 - x_3 \\ x_2 = 60 + x_3 \\ x_3 \text{ is free} \\ x_4 = 60 \end{cases}$$

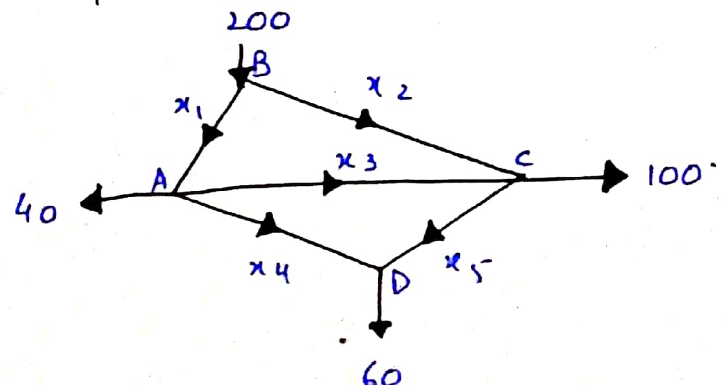
As x_1 cannot be negative so target value of x_3 is 20.

QUESTION # 12

(a) Find the general traffic pattern in subway network shown in figure.

(b) Describe the general traffic pattern when the road whose flow is x_4 is closed.

(c) When $x_4 = 0$, what is minimum value of x_1?



SOLUTION

Intersection		Flow in		Flow out
A	:	x_1	=	$x_3 + x_4 + 40$
B	:	200	=	$x_1 + x_2$
C	:	$x_2 + x_3$	=	$x_5 + 100$
D	:	$x_4 + x_5$	=	60
Total Flow :		200	=	$40 + 100 + 60 = 200$

Arranging Equations:

$$x_1 - x_3 - x_4 = 40$$

$$x_1 + x_2 = 200$$

$$x_2 + x_3 - x_5 = 100$$

$$x_4 + x_5 = 60$$

Matrix form:

$$\begin{bmatrix} 1 & 0 & -1 & -1 & 0 & 40 \\ 1 & 1 & 0 & 0 & 0 & 200 \\ 0 & 1 & 1 & 0 & -1 & 100 \\ 0 & 0 & 0 & 1 & 1 & 60 \end{bmatrix} \xrightarrow{\sim R_2 - R_1} \begin{bmatrix} 1 & 0 & -1 & -1 & 0 & 40 \\ 0 & 1 & 1 & 1 & 0 & 160 \\ 0 & 1 & 1 & 0 & -1 & 100 \\ 0 & 0 & 0 & 1 & 1 & 60 \end{bmatrix}$$

$$\xrightarrow{\sim R_3 - R_2} \begin{bmatrix} 1 & 0 & -1 & -1 & 0 & 40 \\ 0 & 1 & 1 & 1 & 0 & 160 \\ 0 & 0 & 0 & -1 & -1 & -60 \\ 0 & 0 & 0 & 1 & 1 & 60 \end{bmatrix} \xrightarrow{\sim R_3 \times -1} \begin{bmatrix} 1 & 0 & -1 & -1 & 0 & 40 \\ 0 & 1 & 1 & 1 & 0 & 160 \\ 0 & 0 & 0 & 1 & 1 & 60 \\ 0 & 0 & 0 & 1 & 1 & 60 \end{bmatrix}$$

$$\xrightarrow{\sim R_1 + R_3} \begin{bmatrix} 1 & 0 & -1 & 0 & 1 & 100 \\ 0 & 1 & 1 & 1 & 0 & 160 \\ 0 & 0 & 0 & 1 & 1 & 60 \\ 0 & 0 & 0 & 1 & 1 & 60 \end{bmatrix} \xrightarrow{\sim R_2 - R_3} \begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 40 \\ 0 & 1 & 1 & 0 & -1 & 100 \\ 0 & 0 & 0 & 1 & 1 & 60 \\ 0 & 0 & 0 & 1 & 1 & 60 \end{bmatrix}$$

$$\xrightarrow{\sim R_4 - R_3} \begin{bmatrix} 1 & 0 & -1 & 0 & 1 & 40 \\ 0 & 1 & 1 & 0 & -1 & 100 \\ 0 & 0 & 0 & 1 & 1 & 60 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$ Reduced Echelon Form

$$\begin{cases} x_1 = 100 + x_3 - x_5 \\ x_2 = 100 - x_3 + x_5 \\ x_3 \text{ is free} \\ x_4 = 60 - x_5 \\ x_5 \text{ is free} \end{cases}$$

(b)

If x_4 will be 0 then x_5 must be 60. The general pattern will be :

$$\begin{cases} x_1 = 40 + x_3 \\ x_2 = 160 - x_3 \\ x_3 \text{ is free} \\ x_4 = 0 \\ x_5 = 60 \end{cases}$$

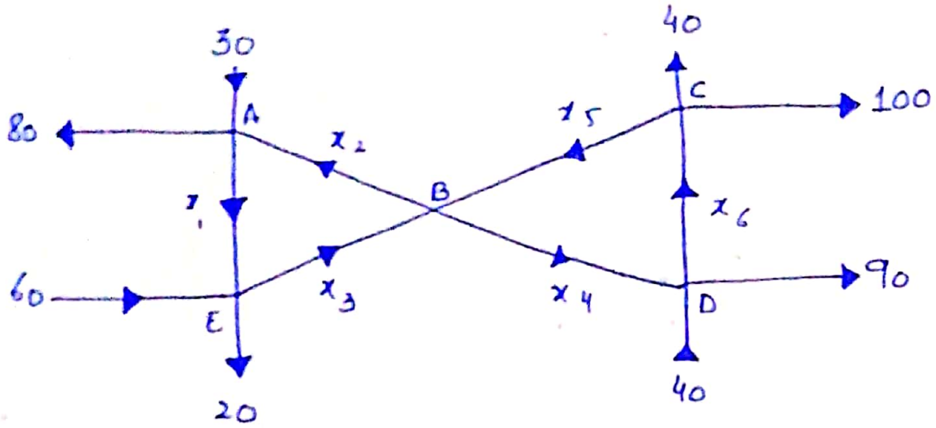
(c)

Mimum value of x_1 is 40 cars/minute, because x_3 cannot be negative.

QUESTION # 13

(a) Find the general flow pattern in network show in figure

(b) Assuming that flow must be in branches denoted by x_2, x_3, x_4, x_5



SOLUTION

(a)

Intersection	Flow in		Flow out
A	$x_2 + 30$	=	$x_1 + 80$
B	$x_3 + x_5$	=	$x_2 + x_4$
C	$x_6 + 100$	=	$x_5 + 40$
D	$x_4 + 40$	=	$x_6 + 90$
E	$x_1 + 60$	=	$x_3 + 20$
Total Flow :	230	=	230

Arranged Equation:

$$x_1 - x_2 = -50$$
$$x_2 - x_3 + x_4 - x_5 = 0$$
$$x_5 - x_6 = 60$$
$$x_4 - x_6 = 50$$
$$x_1 - x_3 = -40$$

Matrix form:

$$\begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 & -50 \\ 0 & 1 & -1 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & 60 \\ 0 & 0 & 0 & 1 & 0 & -1 & 50 \\ 1 & 0 & -1 & 0 & 0 & 0 & -40 \end{bmatrix} \xrightarrow{\sim R_5 - R_1} \begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 & -50 \\ 0 & 1 & -1 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & 60 \\ 0 & 0 & 0 & 1 & 0 & -1 & 50 \\ 0 & 1 & -1 & 0 & 0 & 0 & 10 \end{bmatrix}$$

$$\xrightarrow{\sim R_1 + R_2} \begin{bmatrix} 1 & 0 & -1 & 1 & -1 & 0 & -50 \\ 0 & 1 & -1 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & 60 \\ 0 & 0 & 0 & 1 & 0 & -1 & 50 \\ 0 & 1 & -1 & 0 & 0 & 0 & 10 \end{bmatrix} \xrightarrow{R_5 - R_2} \begin{bmatrix} 1 & 0 & -1 & 1 & -1 & 0 & -50 \\ 0 & 1 & -1 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & 60 \\ 0 & 0 & 0 & 1 & 0 & -1 & 50 \\ 0 & 0 & 0 & -1 & 1 & 0 & 10 \end{bmatrix}$$

$$\xrightarrow[R_3 \text{ \& } R_4]{\sim \text{Swap}} \begin{bmatrix} 1 & 0 & -1 & 1 & -1 & 0 & -50 \\ 0 & 1 & -1 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1 & 50 \\ 0 & 0 & 0 & 0 & 1 & -1 & 60 \\ 0 & 0 & 0 & -1 & 1 & 0 & 10 \end{bmatrix} \xrightarrow{\sim R_1 - R_3} \begin{bmatrix} 1 & 0 & -1 & 0 & -1 & 1 & -100 \\ 0 & 1 & -1 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1 & 50 \\ 0 & 0 & 0 & 0 & 1 & -1 & 60 \\ 0 & 0 & 0 & -1 & 1 & 0 & 10 \end{bmatrix}$$

$$\xrightarrow{\sim R_2 - R_3} \begin{bmatrix} 1 & 0 & -1 & 0 & -1 & 1 & -100 \\ 0 & 1 & -1 & 0 & -1 & 1 & -50 \\ 0 & 0 & 0 & 1 & 0 & -1 & 50 \\ 0 & 0 & 0 & 0 & 1 & -1 & 60 \\ 0 & 0 & 0 & -1 & 1 & 0 & 10 \end{bmatrix} \xrightarrow{\sim R_5 + R_3} \begin{bmatrix} 1 & 0 & -1 & 0 & -1 & 1 & -100 \\ 0 & 1 & -1 & 0 & -1 & 1 & -50 \\ 0 & 0 & 0 & 1 & 0 & -1 & 50 \\ 0 & 0 & 0 & 0 & 1 & -1 & 60 \\ 0 & 0 & 0 & 0 & 1 & -1 & 60 \end{bmatrix}$$

$$\xrightarrow{\sim R_1 + R_4} \begin{bmatrix} 1 & 0 & -1 & 0 & 0 & 0 & -40 \\ 0 & 1 & -1 & 0 & -1 & 1 & -50 \\ 0 & 0 & 0 & 1 & 0 & -1 & 50 \\ 0 & 0 & 0 & 0 & 1 & -1 & 60 \\ 0 & 0 & 0 & 0 & 1 & -1 & 60 \end{bmatrix} \xrightarrow{\sim R_2 + R_4} \begin{bmatrix} 1 & 0 & -1 & 0 & 0 & 0 & -40 \\ 0 & 1 & -1 & 0 & 0 & 0 & 10 \\ 0 & 0 & 0 & 1 & 0 & -1 & 50 \\ 0 & 0 & 0 & 0 & 1 & -1 & 60 \\ 0 & 0 & 0 & 0 & 1 & -1 & 60 \end{bmatrix}$$

$$\xrightarrow{\sim R_5 - R_4} \begin{bmatrix} 1 & 0 & -1 & 0 & 0 & 0 & -40 \\ 0 & 1 & -1 & 0 & 0 & 0 & 10 \\ 0 & 0 & 0 & 1 & 0 & -1 & 50 \\ 0 & 0 & 0 & 0 & 1 & -1 & 60 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

General Solution:

$$\begin{cases} x_1 = x_3 - 40 \\ x_2 = x_3 + 10 \\ x_3 \text{ is free} \\ x_4 = x_6 + 50 \\ x_5 = x_6 + 60 \\ x_6 \text{ is free} \end{cases}$$

(b) As x_1 cannot be negative so $x_3 \geq 40$ x_2 will be $x_2 \geq 50$ As x_6 cannot be negative so $x_4 \geq 50$ & $x_5 \geq 60$. Minimum flows are:

$x_2 = 50$

$x_3 = 40$

$x_4 = 50$

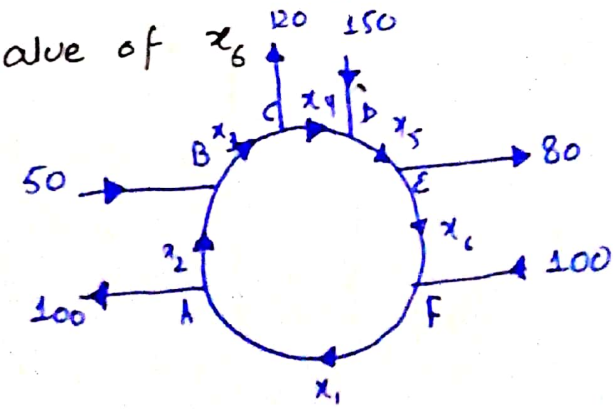
$x_5 = 60$

where $x_1 = x_6 = 0$

QUESTION # 14

Intersections in England are often constructed as one-way "roundabouts". Assume that traffic must travel in show directions. Find general solution and smallest possible value of x_6

SOLUTION



Intersection	Flow in		Flow out
A	x_1	=	$x_2 + 100$
B	$x_2 + 50$	=	x_3
C	x_3	=	$x_4 + 120$
D	$x_4 + 150$	=	x_5
E	x_5	=	$x_6 + 80$
F	$x_6 + 100$	=	x_1

Arranged Equations

$$x_1 - x_2 = 100$$
$$x_2 - x_3 = -50$$
$$x_3 - x_4 = 120$$
$$x_4 - x_5 = -150$$
$$x_5 - x_6 = 80$$
$$-x_1 + x_6 = -100$$

Matrix Form:

$$\begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 & 100 \\ 0 & 1 & -1 & 0 & 0 & 0 & -50 \\ 0 & 0 & 1 & -1 & 0 & 0 & 120 \\ 0 & 0 & 0 & 1 & -1 & 0 & -150 \\ 0 & 0 & 0 & 0 & 1 & -1 & 80 \\ -1 & 0 & 0 & 0 & 0 & 1 & -100 \end{bmatrix} \xrightarrow{\sim R_6 + R_1} \begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 & 100 \\ 0 & 1 & -1 & 0 & 0 & 0 & -50 \\ 0 & 0 & 1 & -1 & 0 & 0 & 120 \\ 0 & 0 & 0 & 1 & -1 & 0 & -150 \\ 0 & 0 & 0 & 0 & 1 & -1 & 80 \\ 0 & -1 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

$$\xrightarrow{\sim R_1 + R_2} \begin{bmatrix} 1 & 0 & -1 & 0 & 0 & 0 & 50 \\ 0 & 1 & -1 & 0 & 0 & 0 & -50 \\ 0 & 0 & 1 & -1 & 0 & 0 & 120 \\ 0 & 0 & 0 & 1 & -1 & 0 & -150 \\ 0 & 0 & 0 & 0 & 1 & -1 & 80 \\ 0 & -1 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \xrightarrow{\sim R_6 + R_2} \begin{bmatrix} 1 & 0 & -1 & 0 & 0 & 0 & 50 \\ 0 & 1 & -1 & 0 & 0 & 0 & -50 \\ 0 & 0 & 1 & -1 & 0 & 0 & 120 \\ 0 & 0 & 0 & 1 & -1 & 0 & -150 \\ 0 & 0 & 0 & 0 & 1 & -1 & 80 \\ 0 & 0 & -1 & 0 & 0 & 1 & -50 \end{bmatrix}$$

$$\xrightarrow[\sim R_2 + R_3]{\sim R_1 + R_3} \begin{bmatrix} 1 & 0 & 0 & -1 & 0 & 0 & 170 \\ 0 & 1 & 0 & -1 & 0 & 0 & 70 \\ 0 & 0 & 1 & -1 & 0 & 0 & 120 \\ 0 & 0 & 0 & 1 & -1 & 0 & -150 \\ 0 & 0 & 0 & 0 & 1 & -1 & 80 \\ 0 & 0 & -1 & 0 & 0 & 1 & -50 \end{bmatrix} \xrightarrow{\sim R_6 + R_3} \begin{bmatrix} 1 & 0 & 0 & -1 & 0 & 0 & 170 \\ 0 & 1 & 0 & -1 & 0 & 0 & 70 \\ 0 & 0 & 1 & -1 & 0 & 0 & 120 \\ 0 & 0 & 0 & 1 & -1 & 0 & -150 \\ 0 & 0 & 0 & 0 & 1 & -1 & 80 \\ 0 & 0 & 0 & -1 & 0 & 1 & 70 \end{bmatrix}$$

$$\xrightarrow[\sim R_2 + R_4 \;\; \sim R_3 + R_4]{\sim R_1 + R_4} \begin{bmatrix} 1 & 0 & 0 & 0 & -1 & 0 & 20 \\ 0 & 1 & 0 & 0 & -1 & 0 & -80 \\ 0 & 0 & 1 & 0 & -1 & 0 & -30 \\ 0 & 0 & 0 & 1 & -1 & 0 & -150 \\ 0 & 0 & 0 & 0 & 1 & -1 & 80 \\ 0 & 0 & 0 & 0 & 0 & 1 & -80 \end{bmatrix} \xrightarrow{\sim R_6 + R_4} \begin{bmatrix} 1 & 0 & 0 & 0 & -1 & 0 & 20 \\ 0 & 1 & 0 & 0 & -1 & 0 & -80 \\ 0 & 0 & 1 & 0 & -1 & 0 & -30 \\ 0 & 0 & 0 & 1 & -1 & 0 & -150 \\ 0 & 0 & 0 & 0 & 1 & -1 & 80 \\ 0 & 0 & 0 & 0 & -1 & 1 & -80 \end{bmatrix}$$

$$\xrightarrow[\sim R_2 + R_5]{\sim R_1 + R_5} \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & -1 & 100 \\ 0 & 1 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & -1 & 0 & -30 \\ 0 & 0 & 0 & 1 & -1 & 0 & -150 \\ 0 & 0 & 0 & 0 & 1 & -1 & 80 \\ 0 & 0 & 0 & 0 & -1 & 1 & -80 \end{bmatrix} \xrightarrow[\sim R_4 + R_5]{\sim R_3 + R_5} \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & -1 & 100 \\ 0 & 1 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 & 50 \\ 0 & 0 & 0 & 1 & 0 & -1 & -70 \\ 0 & 0 & 0 & 0 & 1 & -1 & 80 \\ 0 & 0 & 0 & 0 & -1 & 1 & -80 \end{bmatrix}$$

$$\xrightarrow{\sim R_6 + R_5} \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & -1 & 100 \\ 0 & 1 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 & 50 \\ 0 & 0 & 0 & 1 & 0 & -1 & -70 \\ 0 & 0 & 0 & 0 & 1 & -1 & 80 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

General Form:

$$\begin{cases} x_1 = 100 + x_6 \\ x_2 = x_6 \\ x_3 = 50 + x_6 \\ x_4 = -70 + x_6 \\ x_5 = 80 + x_6 \\ x_6 \text{ is free} \end{cases}$$

As x_4 cannot be negative so the minimum positive value of x_6 is 70.