Ans to the question no. 5

(1) $T(n) = T(n/2) + n - 1 \,;\; T(1) = 0$

Here,

$$T(n) = T(n/2) + n - 1$$
$$= [T(n/2^2) + \frac{n}{2}] + n - 1$$
$$= T(n/2^2) + \frac{n}{2} + n - 1$$
$$= [T(n/2^3) + \frac{n}{2^2}] + \frac{n}{2} + n - 1$$
$$= T(\frac{n}{2^3}) + \frac{n}{2^2} + \frac{n}{2} + n - 1$$
$$= \vdots$$
$$= T(n/2^k) + \frac{n}{2^{k-1}} + \frac{n}{2^{k-2}} + \dots + \frac{n}{2} + n - 1$$

Here, $n/2^k = 1$

$\Rightarrow n = 2^k$

$\Rightarrow k = \log n$

Now,

$$T(n) = T(1) + n\left[\frac{1}{2^{k-1}} + \frac{1}{2^{k-2}} + \dots + \frac{1}{2} + 1\right] - 1$$

$$= 0 + n[1+1] - 1$$

$$= 0 + 2n - 1$$

$$= 2n-1 \approx O(n)$$

(Ans)

(2) $T(n) = T(n-1) + n - 1 \;;\; T(1) = 0$

Here,

$$T(n) = T(n-1) + n-1$$

$$= [T(n-2) + n-1] + n-1$$

$$= T(n-2) + (n-1) + n-1$$

$$= [T(n-3) + n-2] + (n-1) + n-1$$

② $T(n) = T(n-1) + n - 1;\ T(1) = 0$

Now,

$$T(n) = T(n-1) + (n-1)$$

$$= [T(n-2) + n - 2] + n - 1$$

$$= T(n-2) + 2n - 3$$

$$= T(n-4) + 4n - 10$$

$$\vdots$$

$$T(n) = T(n-k) + kn - \cancel{10}\ c$$

[c = 10]
*constant

Now, $\cancel{k =}\ n - k = 1$

$\Rightarrow k = n - 1$

$$T(n) = T(n-(n-1)) + (n-1)n - c$$

$$= T(1) + n^2 - n - c$$

$$= 0 + n^2 - n - c \approx n^2 - n \approx n^2$$

[C is Neglegable]

So, ~~Time~~ wrost case complexity $O(n^2)$

③ $T(n) = T(n/3) + 2T(n/3) + n$

$= 3T(n/3) + n$

$= 3\left(3T\left(\frac{n}{3^2}\right) + \frac{n}{3}\right) + n$

$= 9T\left(\frac{n}{3^2}\right) + 2n$

$= 9\left(3T\left(\frac{n}{3^3}\right) + \frac{n}{3^2}\right) + 2n$

$= 27T\left(\frac{n}{3^3}\right) + 3n$

$= \vdots$

$= 3^k T\left(\frac{n}{3^k}\right) + kn$

Here, $\frac{n}{3^k} = 1$

$\Rightarrow k = \log_3 n$

$\cancel{T(n) = 3^k T(1)}$

Again, $3^k T(1) = 3^{\log_3 n} T(1) = n.1 = n$

Therefore,

$$T(n) = 3^k T\left(\frac{n}{3^k}\right) + kn$$

$$= 3^{\log_3 n} T(1) + \log_3 n \cdot n$$

$$= n + \log_3 n \cdot n$$

$$\approx O(n \log n)$$

(4) $T(n) = 2T(n/2) + n^2$

Here, Using the master theorm we get,

the recurrence form is;

$$T(n) = aT\left(\frac{n}{b}\right) + \theta(n^k \log^p n)$$

where $a \geq 1$, $b > 1$, $k \geq 0$ and p is real number.

~~Here, Now, Wil~~

we can see that, $a=2, b=2, K=2, P=0$

so, ~~mas~~ Here $a < b^k$ and $p = 0$.

Therefore by the master theorm we get $T(n) = O(n^2)$

[Proved]