Combinatorics and pigeonhole principle:

Question 01: How many 6 letter passwords (assume all are small letters and letter repetition is allowed if not mentioned specifically)-

a) End with y ?   Ans: 11881376
b) Start with r and end with b? (Letter repetition is not allowed)   Ans: 14950
c) Contain only vowel?   Ans: 15625
d) Do not contain z,e,r,o? Ans: 113379904
e) Contain "est"?   Ans: 421824

Question 02: How many positive integer < 1000 are

a) Divisible by 5?   Ans: 199
b) Divisible by 5 but not 7?   Ans: 171
c) Divisible by both 5 and 7?   Ans: 28
d) Divisible by either 5 or 7?   Ans: 313
e) Divisible by only 5 or only 7?   Ans: 285
f) Have distinct digits?   Ans: 739

Question 03: A box has 10 blue toys and 10 red toys inside it. A kid selects toys at random without looking at them.

a) How many toys must she choose to have at least three toys of the same color?   Ans: 5
b) How many toys must she choose to have at least three toys of red? Ans: 13
c) In how many ways can she arrange all the toys?   Ans: 184756

Question 04: Assume, Dhaka's population is 22M.

a) Prove that, there exists at least 4 people who has same three initials (ex: AIA) who were born in the same day of a year (year can be different)?

Question 05: Suppose there are 30 students in your section. Each student's student ID's last digit is in between 0-8.

a) Show that your class must has at least 25 female or 25 male students.Correction: 15 female or 15 male
b) Show that your class must has 17 male or 13 female students.
c) If 5 different digits is selected from 0 to 8, there must be a pair made out of these integers with a sum equal to 9.
d) Is the conclusion in part (c) is true if 4 digits are selected instead of 5?

Question 06: In how many ways can 21 Women and 10 men to stand in a line so that, no man stand next to each other?   Ans: 2.35 X 10^12

Question 07: Sixteen people want to participate in a game.

a) In how many ways, a team of 10 players can be formed?   Ans: 8008
b) In how many ways, a team of 10 players can be formed, if they're given jersey based on their position?   Ans: 2.9 X 10^10

c) Assume, six of sixteen participants are women. In how many ways team can be formed if the team has at least one woman player?   Ans: 8007
d) In how many ways, can we choose the 1st, 2nd and 3rd prize owners from 10 players?   Ans: 4320

Question 08: 6 women and 8 men are waiting to get on micro bus and a bus arrives which has 8 seats . How many ways are there to select who can get on to the bus if,

a) At least 3 passenger has to be woman? Ans: 2534
b) Exactly 1 passenger has to be women and at least 4 passenger has to be men? Ans: 2638 , Correction: at least 1 women & at least 4 men

Question 09: A string of 14 binary digits exists

a) How many ways that string can be formed with exactly three 0's and eleven 1's if every 0 is followed by two 1's?   Ans: 56
b) How many ways that string can be formed with at least five 0's and five 1's?   Ans: 13442

Question 10: A multinational organization will be formed with 18 countries. How many ways are there to select 8 countries from Asia, 6 countries from America and 4 countries from Europe? (There are 48, 35 and 44 countries in Asia, America and Europe respectively)   Ans: 8.31 X 10^19

Question 11: Given two strings:

(i)"EVERGREEN"

(ii) "00001101110"

a) How many 7 or more character can be formed from (i)?   Ans: 17955
b) In how many different ways the letter can be arranged from (i) if all the consonants are glued together? Ans: 300
c) In how many different ways the letter can be arranged from (ii) excluding (ii)?   Ans: 461

Question 12: A password can be of 6,7 or 8 length. The password can contain any small letter or decimal digit. However, the password must contain one decimal digit. In how many ways the password can be formed? Ans: 2.68 X 10^12

Question 13: A shop has T-shirt of 4 different color (pink, purple, white, black) and 5 different sizes (S,M,L). However, S size comes in all colors, M size comes in pink and white and L size comes in pink, purple and white. How many T-shirt the shop owner should keep in stock so that there is at least one of each available size and color is found?   Ans: 9   Correction: 3 different sizes

Question 14: In how many ways can 10 friends sit around Bonfire in circular order? Ans: 362880

Question 15: In how many ways can 7 pearls be arranged to form a necklace? Ans: 20160

Question 16: 7 cards are drawn from standard deck of 52 cards. In how many ways we can choose if

a) Exactly 3 of the cards are king? Ans: 778320
b) All 5 cards are hearts? Ans: 953667
c) Exactly 4 cards are face cards? Ans: 4890600