CONCISE NOTE OF LECTURE 2- FUNCTION

Relations vs. Functions

A relation is just a relationship between x- and y-coordinates that means two variables or two equations.

Example 1: $y^2=x$

Example 2: If we consider two set like, A= {1,5} and B={3,4}

Then the product of the two sets are A×B= {(1,3), (1,4) (5,3) (5,4)} and the order should be (x,y). But if we say that x>y then the product answer should be A×B= {(5,3) (5,4)}. So relation depends on the given condition.

In the set of everything that is a relation, there's a smaller subset which we call functions.

A function tries to define these relationships. It tries to give the relationship a mathematical form. An equation is a mathematical way of looking at the relationship between concepts or items. These concepts or items are represented by what are called variables.

Example: $y=x+3$

A variable represents a concept or an item whose magnitude can be represented by a number, i.e. measured quantitatively. Variables are called variables because they vary, i.e. they can have a variety of values. Thus a variable can be considered as a quantity which assumes a variety of values in a particular problem. Many items in economics can take on different values. Mathematics usually uses letters from the end of the alphabet to represent variables.

Example1: Set variable A={2, 6, 8, 10} so the value of a set is 2 , 6, 8 and 10 which is changeable.

Example2: Equation Variable $Y=X+3$, Here Y and X is two variable and Y is changeable for the all distinct value of X.

Domain:

The "domain" of a function or relation is:

- the set of all values for which it can be evaluated
- the set of allowable "input" values
- the values along the horizontal axis for which a point can be plotted along the vertical axis

Example 1:

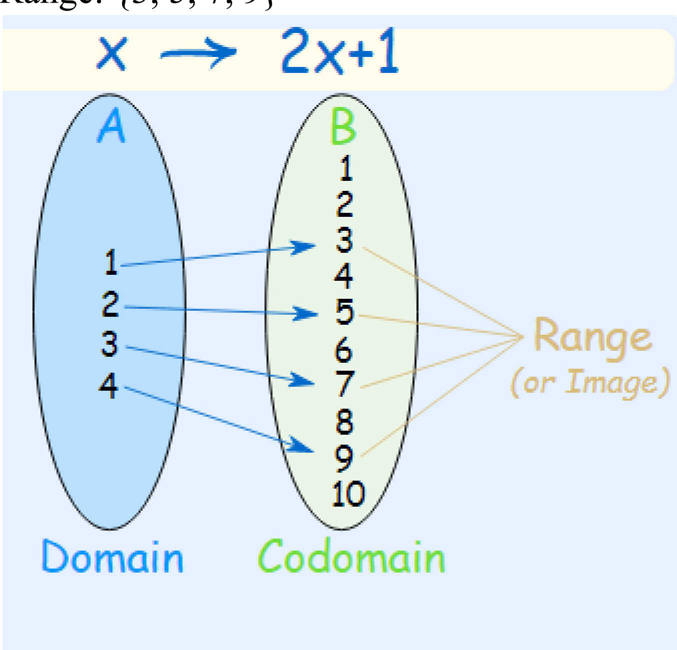
• The set "A" is the Domain,

• The set "B" is the Codomain,

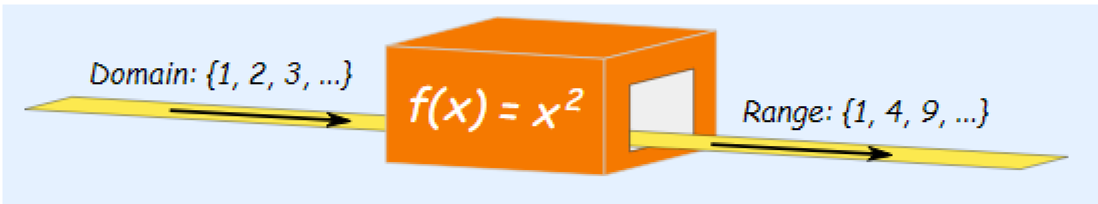
• And the set of elements that get pointed to in B (the actual values produced by the function) are the Range, also called the Image.

And we have:

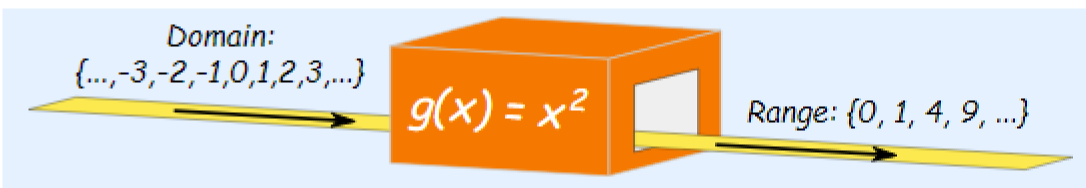
- Domain: {1, 2, 3, 4}
- Codomain: {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}
- Range: {3, 5, 7, 9}



Example 2: a simple function like $f(x) = x^2$ can have the domain (what goes in) of just the counting numbers {1,2,3,...}, and the range will then be the set {1,4,9,...}



And another function $g(x) = x^2$ can have the domain of integers {...,-3,-2,-1,0,1,2,3,...}, in which case the range is the set {0,1,4,9,...}



Range

The "range" of a function or relation is:

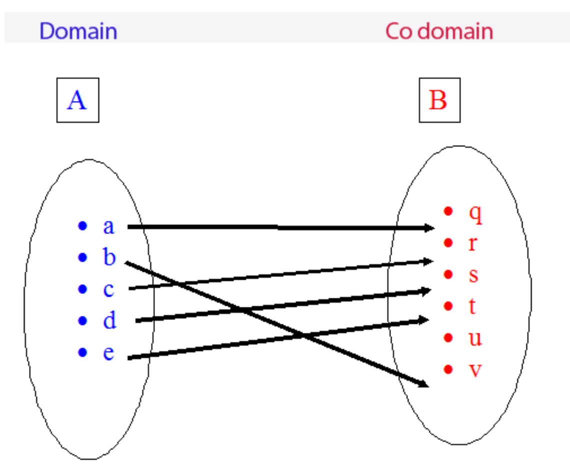
- the set of all values that it can produce
- its "output" set of values
- the set of values along the vertical axis for which a point can be plotted on its graph

Codomain

The “codomain” of a function or relation is a set of values that includes the Range as described above, but may also include additional values beyond those in the range.

Codomains can be useful when:

- You need to restrict the output of a function. For example, by specifying a codomain to be “the set of positive Real numbers”, you are instructing any who use the function to ignore any negative values it produces.
- The Range might be difficult to specify exactly, but a larger set of numbers that includes the entire Range can be specified. For example, a codomain could specify the set of all positive Real numbers, even though the function does not generate all possible positive Real numbers.
- More examples are given below:
  Example1:



$Domain\ A\{a,b,c,d,e\}$

$Codomain\ B\{q, r, s, t, u, v\}$

$Range\ \{q, r, s, t, v\}$

Example:2

$$f(x) = x^2 + 2,\ x \in \{-3,-2,-1,0,1,2,3\}$$

$a)$ State the domain

$b)$ State the range

a) The domain is $\{-3,-2,-1,0,1,2,3\}$

b) $f(x) = x^2 + 2$

$$f(-3) = (-3)^2 + 2 = 9 + 2 = 11$$

$$f(-2) = (-2)^2 + 2 = 4 + 2 = 6$$

$$f(-1) = (-1)^2 + 2 = 1 + 2 = 3$$

$$f(0) = (0)^2 + 2 = 0 + 2 = 2$$

$$f(1) = 1^2 + 2 = 1 + 2 = 3$$

$$f(2) = 2^2 + 2 = 4 + 2 = 6$$

$$f(3) = 3^2 + 2 = 9 + 2 = 11$$

The range is $\{2,3,6,11\}$

Example 3:

The function $f(x) = \cos x°$

has domain $\{\, x : \; 0 \le x \le 360, x \in \mathbb{R}\}$

$a)$ State the range

$b)$ If $f(a) = 1$, find a

a) The range is $\{f(x) : -1 \le f(x) \le 1, \; x \in \mathbb{R}\}$

Example 4:

The function $h(x) = x^2 + 1$

has domain $x \subset \mathbb{Z}$ and range $\{1,2,5,10\}$

List all possible elements of the domain.

$h(x) = x^2 + 1$ so set $h(x) = range = x^2 + 1$

$$\begin{aligned} h(x) &= x^2 + 1 \\ 1 &= x^2 + 1 \\ 0 &= x^2 \\ x &= \pm\sqrt{0} \\ x &= 0 \end{aligned}$$

$$\begin{aligned} h(x) &= x^2 + 1 \\ 2 &= x^2 + 1 \\ 1 &= x^2 \\ x &= \pm\sqrt{1} \\ x &= \pm 1 \end{aligned}$$

$$\begin{aligned} h(x) &= x^2 + 1 \\ 5 &= x^2 + 1 \\ 4 &= x^2 \\ x &= \pm\sqrt{4} \\ x &= \pm 2 \end{aligned}$$

$$\begin{aligned} h(x) &= x^2 + 1 \\ 10 &= x^2 + 1 \\ 9 &= x^2 \\ x &= \pm\sqrt{9} \\ x &= \pm 3 \end{aligned}$$

Domain $\{-3,-2,-1,0,1,2,3\}$

How to Find the Domain of a Rational Function: Examples with Solutions

Example 1

Find the domain of the function f defined by

$$f(x) = \frac{1}{x-2}$$

f(x) can take real values if the denominator of f(x) is NOT ZERO because division by zero is not allowed in mathematics

$x - 2 \neq 0$

Solve the above inequality for to obtain the domain: $x \neq 2$

Which in interval form may be written as follows

$(-\infty , 2) \cup (2 , +\infty)$

Example 2

Find the domain of the function f defined by

$$f(x) = \frac{x+3}{x^2+7}$$

Solution to Example 2

For f(x) to have real values, the denominator must be different from zero. Hence

$x^2 + 7 \neq 0$

Expression $x^2 + 7$ is always positive (square added to a positive number). Hence the domain of f is given by the interval

$(-\infty , +\infty)$

Example 3

Find the domain of the function f defined by:

$$f(x) = \frac{2x+9}{2x^2+x-15}$$

<u>Solution to Example 3</u>

For f(x) given above to be real, its denominator must be different from zero. Let us first find the roots of the denominator by solving the equation

$2x^2 + x - 15 = 0$

The roots are

-3 and $5/2$

The denominator $2x^2 + x - 15$ is not equal to zeros for all real values except -3 and $5/2$.

Hence the domain of the given function is given by

$(-\infty, -3) \cup (-3, 5/2) \cup (5/2, +\infty)$

Example 4

Find the domain of the function f given by:

$$f(x) = \frac{2}{2x - 6} - \frac{x}{4x + 7}$$

Solution to Example 4

For f(x) to be real, both denominators 2x - 6 and - 4x + 7 must not be equal to zero. Let us find the values of x that make the two denominators equal to zero

2x - 6 = 0 gives x = 3

4x + 7 gives x = - 7 / 4

f(x) is real for all real values except 3 and - 7/4. The domain of the above function is given by

$(-\infty , -7/4) \cup (-7/4 , 3) \cup (3 , +\infty)$

Examples on How to Find the Domain of Square Root Functions with Solutions

Example 1

Find the domain of function f defined by

$f(x) = \sqrt{(x - 1)}$

Solution to Example 1

- For f(x) to have real values, the radicand (expression under the radical) of the square root function must be positive or equal to 0. Hence
  $x - 1 \geq 0$
- The solution set to the above inequality is the domain of f(x) and is given by:
  $x \geq 1$
  or in interval form $[1 , +\infty)$

Example 2

Find the domain of function f defined by

$f(x) = \sqrt{[(x - 2)(x + 3)]}$

Solution to Example 2

- For f(x) to have real values, the radicand (x - 2)(x + 3) must be positive. Hence
  $(x - 2)(x + 3) \geq 0$
- Solve the above inequality to obtain the solution set, which is also the domain, in interval form as follows:
  $(-\infty , -3] \cup [2 , +\infty)$

Example 3

Find the domain of function f defined by:

$f(x) = \sqrt{[x^2 + 2 x - 1]}$

Solution to Example 3

- For $\sqrt{[x^2 + 2 x - 1]}$ to be real, the radicand must be positive or equal to 0. Hence the inequality
  $x^2 + 2x - 1 \geq 0$
- The solution set of the above inequality, which is also the domain, is given in interval form as follows:
  $(-\infty , -1-\sqrt{2}] \cup [-1+ \sqrt{2} , +\infty)$
- The domain of the given function is given by the interval $(-\infty , -1-\sqrt{2}] \cup [-1+\sqrt{2} , +\infty)$.

Example 4

Find the domain of function f defined by:

$f(x) = \sqrt{[(2x - 1)/(x + 3)]}$

Solution to Example 4

- The domain of this function is the set of all values of x such that $(2x - 1)/(x + 3) \geq 0$ which is an inequality to solve. The solution set of the above inequality which is also the domain is given by
$(-\infty , -3) \cup [1 / 2 , +\infty)$

Domain and Range of Logarithmic Functions

EXAMPLE 1: IDENTIFYING THE DOMAIN OF A LOGARITHMIC SHIFT

What is the domain of $f(x) = \log_2(x + 3)$?

SOLUTION

The logarithmic function is defined only when the input is positive, so this function is defined when $x + 3 > 0$. Solving this inequality,

$$\begin{cases} x + 3 > 0 & \text{The input must be positive.} \\ x > -3 & \text{Subtract 3.} \end{cases}$$

The domain of $f(x) = \log_2(x + 3)$ is $(-3, \infty)$.

EXAMPLE 2: IDENTIFYING THE DOMAIN OF A LOGARITHMIC SHIFT AND REFLECTION

What is the domain of $f(x) = \log(5 - 2x)$?

SOLUTION

The logarithmic function is defined only when the input is positive, so this function is defined when $5 - 2x > 0$. Solving this inequality,

$$\begin{cases} 5 - 2x > 0 & \text{The input must be positive.} \\ -2x > -5 & \text{Subtract 5.} \\ x < \frac{5}{2} & \text{Divide by } -2 \text{ and switch the inequality.} \end{cases}$$

The domain of $f(x) = \log(5 - 2x)$ is $\left(-\infty, \frac{5}{2}\right)$.

When we call relation as a function?

Example 1		
Problem	Is the relation given by the set of ordered pairs below a function? $\{(-3, -6),(-2, -1),(1, 0),(1, 5),(2, 0)\}$	
	<table><tr><td>x</td><td>y</td></tr><tr><td>−3</td><td>−6</td></tr><tr><td>−2</td><td>−1</td></tr><tr><td>1</td><td>0</td></tr><tr><td>1</td><td>5</td></tr><tr><td>2</td><td>0</td></tr></table>	Organizing the ordered pairs in a table can help. By definition, the inputs in a function have only one output. The input 1 has two outputs: 0 and 5.
Answer	The relation is not a function.	

Example 2		
Problem	Is the relation given by the set of ordered pairs below a function? $\{(-3, 4),(-2, 4),(-1, 4),(2, 4),(3, 4)\}$	
	<table><tr><td>x</td><td>y</td></tr><tr><td>−3</td><td>4</td></tr><tr><td>−2</td><td>4</td></tr><tr><td>−1</td><td>4</td></tr><tr><td>2</td><td>4</td></tr><tr><td>3</td><td>4</td></tr></table>	You could reorganize the information by creating a table.
	Each input has only one output.	Each input has only one output, and the

	fact that it is the same output (4) does not matter.
Answer This relation is a function.	

Identify Function from a graph:

The vertical line test can be used to determine whether a graph represents a function. If we can draw any vertical line that intersects a graph more than once, then the graph does *not* define a function because a function has only one output value for each input value.