Practice Sheet #3

Continuity and Differentiability

(*a*)Test the continuity of the following functions:

1. $f(x)=\begin{cases}\cos x, & x\ge 0\\ -\cos x, & x<0\end{cases}$ at $x=0$.

2. $f(x)=\begin{cases}x\cos(1/x), & x\ne 0\\ 0 & ,x=0\end{cases}$ at $x=0$.

3. $f(x)=\begin{cases}e^{1/x} & ,x\ne 0\\ 1 & ,x=0\end{cases}$ at $x=0$.

4. $f(x)=\begin{cases}e^{\frac{-|x|}{2}} & ,-1<x<0\\ x^2 & ,0\le x<2\end{cases}$ at $x=0$.

5. $f(x)=\begin{cases}(x-a)\sin\left(\dfrac{1}{x-a}\right) & ,x\ne a\\ 0 & ,x=a\end{cases}$ at $x=a$.

6. $f(x)=\begin{cases}1 & ,x<0\\ 1+\sin x & ,0\le x<\pi/2\\ 2+(x-\pi/2)^2 & ,x\ge \pi/2\end{cases}$
at $x=0$ and $x=\pi/2$.

7. $f(x)=\begin{cases}x\sin(1/x) & ,x\ne 0\\ 0 & ,x=0\end{cases}$ at $x=0$.

8. $f(x)=\begin{cases}\dfrac{|x-3|}{x-3} & ,x\ne 3\\ 0 & ,x=3\end{cases}$ at $x=3$.

9. $f(x)=|x|+|x-1|$ at $x=0$ and $x=1$.

10. $f(x)=\begin{cases}(1+x)^{1/x} & ,x\ne 0\\ 1 & ,x=0\end{cases}$ at $x=0$.

(*b*)Test the differentiability of the following functions:

1. $f(x)=\begin{cases}\cos x & ,x\ge 0\\ -\cos x & ,x<0\end{cases}$ at $x=0$.

2. $f(x)=\begin{cases}x^2\sin(1/x) & ,x\ne 0\\ 0 & ,x=0\end{cases}$ at $x=0$.

3. $f(x)=|x|$ at $x=0$.

4. $f(x)=\begin{cases}x\cos(1/x) & ,x\ne 0\\ 0 & ,x=0\end{cases}$ at $x=0$.

(*c*) Let $f(x)=\begin{cases}x^2-16x & ,x<9\\ 12\sqrt{x} & ,x\ge 9\end{cases}$. Is *f*(*x*) continuous at $x=9$? Determine whether *f*(*x*) is differentiable at $x=9$.

(*d*) Let $f(x)=\begin{cases}x^2 & ,x\le 1\\ \sqrt{x} & ,x>1\end{cases}$. Is *f*(*x*) continuous at $x=1$? Determine whether *f*(*x*) is differentiable at $x=1$.

(*e*) Show that $f(x)=\begin{cases}x^2+1 & ,x\le 1\\ x & ,x>1\end{cases}$ is not continuous and differentiable at $x=1$. Sketch the graph of *f*(*x*).