Practice Sheet 2

Limit

Problems:

Find the limit of the following functions:

1. $\lim\limits_{x \to 0} \dfrac{x}{\sqrt{x+1} - 1}$

2. $\lim\limits_{x \to 2} \dfrac{2x^2 - 5x + 2}{5x^2 - 7x - 6}$

3. $\lim\limits_{x \to 0} \dfrac{x}{|x|}$

4. $f(x) = \begin{cases} 2 - x & , x < 1 \\ x^2 + 1 & , x > 1 \end{cases}$

   Find $\lim\limits_{x \to 1} f(x)$

5. $\lim\limits_{x \to \infty} \left(\sqrt{x^6 + 5} - x^3 \right)$

6. $f(x) = \begin{cases} \dfrac{1}{x+2} & , x < -2 \\ x^2 - 5 & , -2 < x < 3 \\ \sqrt{x+13} & , x > 3 \end{cases}$

   Find $\lim\limits_{x \to -2} f(x)$ and $\lim\limits_{x \to 3} f(x)$

7. $\lim\limits_{x \to \infty} \dfrac{3x + 5}{6x - 8}$

8. $\lim\limits_{x \to \infty} \left(\sqrt{x^6 + 5x^3} - x^3 \right)$

9. $\lim\limits_{x \to \infty} \sqrt[3]{\dfrac{3x + 5}{6x - 8}}$

10. $f(x) = \begin{cases} x^2 + 1 & , x > 0 \\ 1 & , x = 0 \\ 1 + x & , x < 0 \end{cases}$

    Find $\lim\limits_{x \to 0} f(x)$

11. $f(x) = \begin{cases} 3x - 1 & , x < 1 \\ 3 - x & , x > 1 \end{cases}$

    Find $\lim\limits_{x \to 1} f(x)$

12. Prove that $\lim\limits_{x \to \infty} \left(1 + \dfrac{1}{x} \right)^x = e$

13. $f(x) = \begin{cases} 2x + 1 & , x < 1 \\ 3 - x & , x > 1 \end{cases}$

    Find $\lim\limits_{x \to 1} f(x)$

14. $f(x) = \begin{cases} x^2 & , x < 1 \\ 2.4 & , x = 1 \\ x^2 + 1 & , x > 1 \end{cases}$

    Does $\lim\limits_{x \to 1} f(x)$ exist?

15. $f(x) = \begin{cases} e^{\frac{-|x|}{2}} & , -1 < x < 0 \\ x^2 & , 0 < x < 2 \end{cases}$

    Find $\lim\limits_{x \to 0} f(x)$

16. $\lim\limits_{x \to -\infty} \dfrac{4x^2 - x}{2x^3 - 5}$