Practice Sheet # 7

Leibnitz's Theorem

1. If $y = \tan^{-1} x$, show that $(1+x^2)y_{n+2} + 2(n+1)xy_{n+1} + n(n+1)y_n = 0.$
2. If $y = \cot^{-1} x,$ show that $(1+x^2)y_{n+2} + 2(n+1)xy_{n+1} + n(n+1)y_n = 0.$
3. If $y\sqrt{1-x^2} = \sin^{-1} x$, show that $(1-x^2)y_{n+1} - (2n+1)xy_n - n^2 y_{n-1} = 0.$
4. If $y = e^{\tan^{-1} x}$, show that $(1+x^2)y_{n+2} + (2nx + 2x - 1)y_{n+1} + n(n+1)y_n = 0.$
5. If $y = e^{m\sin^{-1} x}$, show that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2 + m^2)y_n = 0.$
6. If $y = (\sin^{-1} x)^2$, show that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - n^2 y_n = 0.$
7. If $\log_e y = a\sin^{-1} x$, show that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2 + a^2)y_n = 0.$
8. If $y = e^{m\cos^{-1} x}$, show that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2 + m^2)y_n = 0.$
9. If $\log_e y = \tan^{-1} x$, show that $(1+x^2)y_{n+2} + (2nx + 2x - 1)y_{n+1} + n(n+1)y_n = 0.$
10. If $y = \left(\cos^{-1} x\right)^2,$ show that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - n^2 y_n = 0.$
11. If $\ln y = m\cos^{-1} x,$ show that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2 + m^2)y_n = 0.$
12. If $x = \tan(\ln y),$ show that $(1+x^2)y_{n+2} + (2nx + 2x - 1)y_{n+1} + n(n+1)y_n = 0.$