

Practice Sheet # 4

Techniques of Differentiation

1. Find the differential coefficients of the following functions with respect to x (i.e. $\frac{dy}{dx}$).

$$(i) y = \sin x \sin 2x \sin 3x, \quad (ii) y = \operatorname{cosec}^3 x, \quad (iii) y = \cos 2x \cos 3x, \quad (iv) y = \sin^{-1}(x^2),$$

$$(v) y = \tan(\sin^{-1} x), \quad (vi) \cot^{-1}\left(\frac{1+x}{1-x}\right), \quad (vii) \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right), \quad (viii) \sin^{-1}\left(\frac{2x}{1+x^2}\right),$$

$$(ix) \tan^{-1}\left(\frac{2x}{1-x^2}\right), \quad (x) \tan^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right).$$

2. Find the differential coefficients of:

$$(i) (\sin x)^{\ln x}, \quad (ii) (\sin x)^{\cos x} + (\cos x)^{\sin x}.$$

3. Find the differential coefficients of $\sqrt{\left(\frac{1+x}{1-x}\right)}$.

4. Find $\frac{dy}{dx}$ in the following cases:

$$(i) 3x^4 - x^2y + 2y^3 = 0, \quad (ii) x^3 + y^3 + 4x^2y - 25 = 0, \quad (iii) x^y = y^x.$$

5. Find $\frac{dy}{dx}$ when

$$(i) x = a \cos^3 \theta, y = a \sin^3 \theta, \quad (ii) x = \sin^2 \theta, y = \tan \theta, \quad (iii) x = a \sec^2 \theta, y = a \tan^2 \theta.$$

6. Differentiate the left-side functions with respect to the right-side ones:

$$(i) \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) \text{ with respect to } \tan^{-1}\left(\frac{2x}{1-x^2}\right) \quad (ii) x^{\sin^{-1}(x)} \text{ with respect to } \sin^{-1} x.$$

7. Find the differential coefficients of:

$$(i) \ln \sqrt{\frac{1+\sin x}{1-\sin x}}, \quad (ii) \ln \sqrt{\frac{1-\cos x}{1+\cos x}}, \quad (iii) \tan^{-1} \sqrt{\frac{1-\cos x}{1+\cos x}}, \quad (iv) \tan^{-1} \left(\frac{\cos x - \sin x}{\cos x + \sin x} \right)$$

$$(v) \sin \left(2 \tan^{-1} \sqrt{\frac{1-x}{1+x}} \right).$$