Practice Sheet #5

Maxima and minima

1. Find (a) the open intervals on which f is increasing, (b) the open intervals on which f is decreasing, (c) the open intervals on which f is concave up, (d) the open intervals on which f is concave down and (e) the x-coordinate of all inflection points.

(i)
$$f(x) = x^2 - 5x + 6$$
 (ii) $f(x) = 5 + 12x - x^3$

(ii)
$$f(x) = 5 + 12x - x^3$$

(iii)
$$f(x) = x^4 - 8x^2 + 16$$
 (iv) $f(x) = \frac{x^2}{x^2 + 2}$ (v) $f(x) = \sqrt[3]{x + 2}$.

$$(iv) f(x) = \frac{x^2}{x^2 + 2}$$

$$(v) f(x) = \sqrt[3]{x+2}.$$

2. Locate the critical numbers and identify which critical numbers correspond to stationary points.

(i)
$$f(x) = x^3 + 3x^2 - 9x + 1$$
 (ii) $f(x) = x^4 - 6x^2 - 3$

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(iii)
$$f(x) = \frac{x}{x^2 + 2}$$
 (iv) $f(x) = x^{2/3}$

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$$(v) f(x) = x^{1/3}(x+4)$$
 $(vi) f(x) = \cos 3x$.

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3. Find the relative extrema (maxima/ minima) using both the first and second derivative tests.

$$(i) f(x) = 2x^3 - 9x^2 + 12x$$
 $(ii) f(x) = \frac{x}{2} - \sin x$, $0 < x < 2\pi$.

4. Use the given derivative to find all critical numbers of f and determine whether a relative maximum, relative minimum, or neither occurs there.

(i)
$$f'(x) = x^3(x^2 - 5)$$
 (ii) $f'(x) = \frac{x^2 - 1}{x^2 + 1}$.

(ii)
$$f'(x) = \frac{x^2 - 1}{x^2 + 1}$$
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