A Computational Theory of Robust Localization Verifiability in the Presence of Pure Outlier Measurements

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Introduction

- Localization a set of nodes with pairwise relative translation-only measurements corrupted with outliers based on l_1-norm optimization
- Different versions of this problem is applied in different fields like Structure from Motion (SFM), Sensor Networks and Simultaneous Localization And Mapping (SLAM)
- Providing a procedure to compute a priori probability that the recovered solution is completely or partially congruent to the ground truth embedding

Method

- A sensor network is modeled as an oriented graph $G(V, E)$

$$t_{ij} = x_j - x_i + \varepsilon_{ij} \quad (1)$$

- Relative measurements are marred by random outliers with distribution

$$\varepsilon_{ij} = \begin{cases} 0 & w.p.\ 1 - p_{ij}^{+} - p_{ij}^{-} \\ F^{-} & w.p.\ p_{ij}^{-} \\ F^{+} & w.p.\ p_{ij}^{+} \end{cases} \quad (2)$$

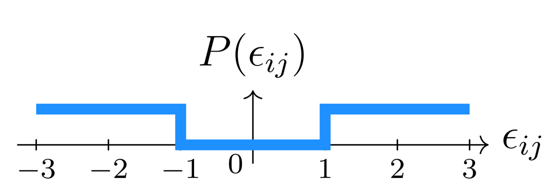

Fig.2. distribution of outlier

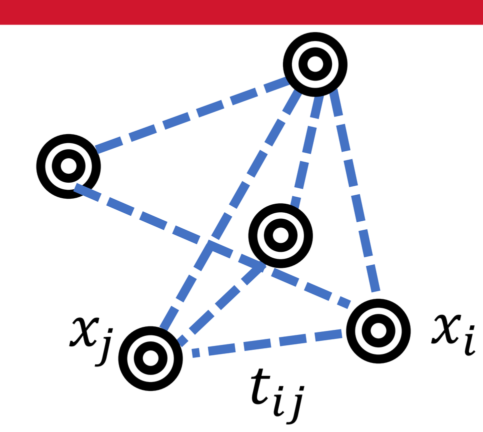

Fig.1. Nodes of the graph represent the sensors and edges of the graph represent the relative measurements

F^{-}, F^{+} are uniform distributions uniform distribution with arbitrary, but finite, non-zero support

- Given t_{ij}, localize set of nodes by find all solution of the robust optimization problem

$$\min \sum_{(i,j)\in E} \left\| x_j - x_i - t_{ij} \right\|_1 \quad (3)$$

- Fixing Translation Ambiguity by choosing a global reference frame such that x_1 moves to origin
- Changing variables to Canonical Form such that the optimal solution is at origin
- Reduction optimization problem to one-dimensional by decomposing the l_1-norm to sum of absolute value across dimensions

$$\min \sum_{d} \sum_{(i,j)\in E} \left| [x_j]_k - [x_i]_k - [\varepsilon_{ij}]_k \right| \quad (4)$$

- Transform the optimization problem to Linear Programming form

Types of Linear programming problem solutions:

Unique optimal solution	$0_V = \chi_{opt}$	:Uniquely Verifiable	Cost-equivalent to ground truth
Multiple optimal solution	$0_V \in \chi_{opt}$	:Verifiable	Cost-equivalent to ground truth
Unique/multiple optimal solution	$0_V \notin \chi_{opt}$	: Not Verifiable	Cost is different from ground truth

- Introducing the maximal verifiable component of a graph as a subset of nodes that are always zero in the set of solutions and congruent to ground truth embedding.
- Defining the function that indicate if the associated localization problem is verifiable

$$VER = \begin{cases} 0 & if\ 0_v \in \chi_{opt} \\ 1 & otherwise \end{cases} \quad (5)$$

- Defining the verifiability probability as the probability of recovering a solution whose cost is the same as the ground truth. $p_{ver} = E[VER(G, E^{\pm})]$ (6)

Results

- The verifiability of a graph is independent from the specific true positions of nodes
- The notion of verifiability depends only on the graph topology G, the support of the outliers E and the sign of the outliers
- For a fixed outlier support E, if we change the scale of the outliers by positive factor, the verifiability of the graph does not change

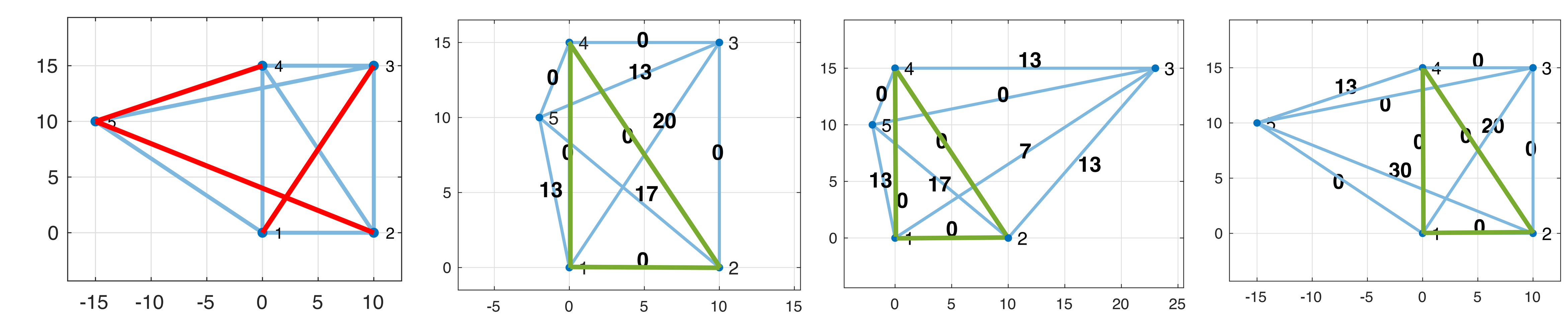

Fig.3. Verifiable graph with 5 nodes and 10 edges, 3 edges are outliers are shown by red color in left-most, the cost of each edge is shown on each and the cost of optimal solution for all embeddings are equal to the cost of the ground truth embedding which is 63

- Verifiable graph with 5 nodes and 10 edges and 3 outliers. The maximal verifiable component is the subgraph $E = \{(1,2), (2,4), (4,1)\}$

#outliers, $\lvert E_{\epsilon}^{\pm}\rvert$	#possible combinations, $\binom{\lvert E\rvert}{\lvert E_{\epsilon}^{\pm}\rvert}$	#verifiable combinations
0	1	1
1	20	20
2	180	180
3	960	920
4	3360	2680
5	8064	4524
6	13440	4560
7	15360	2820
8	11520	1080
9	5120	240
10	1024	24

Table1. Verifiability analysis for all possible cases of outlier support for graph in Fig.3

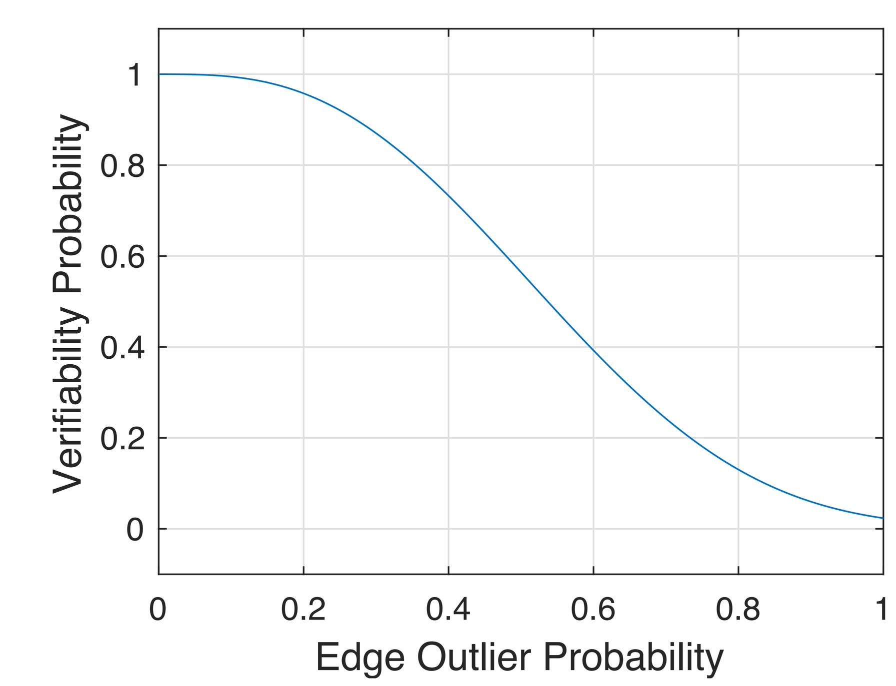

Fig.4.Verifiability probability for the graph in Fig.2

Conclusion

- Estimation of embedding of the nodes through l_1-norm and Linear Programming optimization
- Providing a method to compute verifiability of a graph
- Defining the verifiability probability which characterizes the a priori reliability
- Studying the effect of adding noise to measurements on results and applying Huber-Loss function and piece-wise loss function for the future work

References

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